Results on the π^--Proton Scattering at 1 Bev and a Comparison with the Lindenbaum-Sternheimer Model

I. DERADO AND N. SCHMITZ
Max-Planck-Institut für Physik und Astrophysik, München, Germany
(Received August 6, 1959)

In photographs from the 10-in. hydrogen bubble chamber at Berkeley we have analyzed 640 π^--proton scattering events with two secondary tracks. The primary π^- energy was 1 Bev. The cross sections for the various reactions and especially for the reactions

$$\pi^-+p\rightarrow\pi^-+p,$$
$$\pi^-+p\rightarrow\pi^-+n+\pi^+,$$
$$\pi^-+p\rightarrow\pi^-+p+\pi^0$$

have been determined. The ratio $\sigma(\pi^-+p+\pi^0)/\sigma(\pi^-+n+\pi^+)$ of the two inelastic reactions turned out to be $0.50_{-0.14}^{+0.12}$. The differential cross section of the π^- for the elastic scattering and the momentum and angular distributions of the secondary particles from the two inelastic processes are given.

Our momentum spectra exhibit two maxima which strongly indicate the existence of the isobaric nucleon state. However, our results can hardly be explained by the Lindenbaum-Sternheimer model quantitatively, as has been shown in two independent ways. Perhaps this discrepancy is an indication that those pion productions must not be neglected which do not go through the intermediate state of the isobar but through the direct channel.

INTRODUCTION

Up to this day only a few quantitative conclusions regarding strong interactions can be drawn from the exact meson theory. Therefore, several authors[1,2] have tried to describe pion production in the nucleon-nucleon or pion-nucleon collision by models. The original statistical model of Fermi[3] (i.e., without consideration of the isobaric state of the nucleon) did not agree well with the experiments[2,4,5]; the experimental results were described much better by a statistical theory taking into account the isobaric intermediate state.[2,5,6] Evidence for this resonance state of the nucleon with $J=T=\frac{3}{2}$ came mainly from the resonance-like peak of the π-nucleon scattering cross section at about 140-Mev pion energy in the center-of-mass system.[7,8] In order to describe the pion production in the nucleon-nucleon[8] and pion-nucleon interaction,[9] Lindenbaum and Sternheimer have developed a model (LS model) assuming that the production is possible only via the isobaric intermediate state (isobar) which decays into the ground state (nucleon) and a pion. In this paper we want to compare the predictions of that model with our experimental results which we obtained from 640 π^--proton scattering events at 1 Bev. These events have been observed in photographs taken from the 10-in. hydrogen bubble chamber of the Alvarez Group (Berkeley). The chamber was set up in a magnetic field of *ca* 11 000 gauss. A first discussion of the LS model predictions for π^--proton scattering at 0.93 Bev and 1.37 Bev has already been carried out by Lindenbaum and Sternheimer[9] themselves on the results which Walker *et al.*[10] and Eisberg *et al.*[11] got their measurements from nuclear emulsions and cloud chamber pictures. However, the available statistics were rather poor. (For a compilation of a few further experimental results see the report of the conference on high energy physics at CERN.[12])

MEASUREMENTS

A detailed description of our experimental procedure has been given by Derado *et al.*[13,14] (including elimination of events with short secondary tracks where the measurement of the momentum was difficult or impossible, calculation of a correction factor for each measured event, geometrical and kinematical analysis of an event, distinction between the various reactions of the π^--proton scattering, precision of our measurements). Some remarks should be made about our scanning efficiency:

(1) Several films were scanned twice, and in doing this we found nearly the same number of events. From this it follows that our scanning efficiency was high.

[1] S. J. Lindenbaum, *Annual Review of Nuclear Science* (Annual Reviews, Inc., Palo Alto, 1957), Vol. 7, p. 317.
[2] Belenkii, Maksimenko, Nikishov, and Rozental, Fortschr. Physik **6**, 524 (1958).
[3] E. Fermi, Progr. Theoret. Phys. (Kyoto) **5**, 570 (1950).
[4] R. H. Milburn, Revs. Modern Phys. **27**, 1 (1955).
[5] A. I. Nikishov, J. Exptl. Theoret. Phys. U.S.S.R. **30**, 601 (1956) [translation: Soviet Phys. JETP **3**, 634 (1956)].
[6] S. Z. Belenkii and A. I. Nikishov, J. Exptl. Theoret. Phys. U.S.S.R. **28**, 744 (1955) [translation: Soviet Phys. JETP **1**, 593 (1955)].
[7] L. C. L. Yuan and S. J. Lindenbaum, Phys. Rev. **103**, 404 (1956).
[8] S. J. Lindenbaum and R. M. Sternheimer, Phys. Rev. **105**, 1874 (1957).
[9] S. J. Lindenbaum and R. M. Sternheimer, Phys. Rev. **109**, 1723 (1958).
[10] Walker, Hushfar, and Shephard, Phys. Rev. **104**, 526 (1956).
[11] Eisberg, Fowler, Lea, Shephard, Shutt, Thorndike, and Whittemore, Phys. Rev. **97**, 797 (1955).
[12] 1958 *Annual International Conference on High-Energy Physics at CERN*, edited by B. Ferretti (CERN Scientific Information Service, Geneva, 1958), pp. 65–70.
[13] Derado, Lütjens, and Schmitz, Ann. Physik **7**, 103 (1959).
[14] I. Derado and N. Schmitz, Nuovo cimento **11**, 887 (1959).

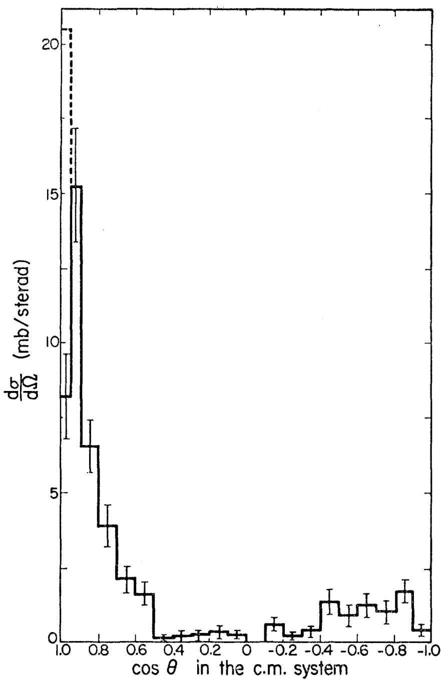

FIG. 1. Differential elastic cross section of the π^- in the center-of-mass system at 1 Bev.

(2) We have plotted the number of elastic events against the azimuth angle of the secondary π^- in the plane which is perpendicular to the primary direction. From this plot, which in the case of 100% scanning efficiency should be isotropic, it could be seen that an elastic event is frequently lost if the π^--proton plane lies nearly in the direction of observation. About 9% of all elastic events were missed due to this effect. In the case of the inelastic events the loss is certainly lower, because here the tracks are generally not coplanar. We have estimated the scanning efficiency to be 96% for the inelastic events with two secondary tracks. The same value has been assumed for events with no visible secondary particle.

(3) From the gap in the $\cos\vartheta$ distribution of the elastically scattered π^- (see Fig. 1, ϑ is the scattering angle in the c.m. system) it followed that small angle scatterings are rather often overlooked in the scanning procedure. Visually extrapolating the experimental distribution to $\cos\vartheta=0$, we found that $(15\pm4)\%$ of all elastic events may have been lost. (This extrapolation is, of course, somewhat arbitrary.) This value and the above value of 9%, however, are not independent of each other because mainly those small angle scatterings are not seen in the pictures where the π^--proton plane lies nearly in the direction of observation. Since also some larger angles will probably have been lost by the effect mentioned under (2), we have assumed that a total of $(17\pm5)\%$ of all elastic events have escaped detection at the scanning stage.

TABLE I. Tabulation of the cross sections and the numbers of events.

Type of reaction	Cross section in mb	Number of events found (without correction factor)
$\pi^-+p\rightarrow\pi^-+p$	22.0 ± 3.1	305 } observed in *ca* 8900 photographs
$\pi^-+p\rightarrow\pi^-+n+\pi^+$	10.4 ± 1.8	190 }
$\pi^-+p\rightarrow\pi^-+p+\pi^0$	5.3 ± 1.2	103 }
Scattering without visible secondary particle: $\pi^-+p\rightarrow\pi^0+n$ $\rightarrow\pi^0+n+\pi^0$ $\rightarrow\pi^0+n+2\pi^0$ etc.	6.4 ± 1.0	63, observed in *ca* 2750 photographs
Multiple production with 2 visible secondary particles: $\pi^-+p\rightarrow\pi^-+p+2\pi^0$ $\rightarrow\pi^-+n+\pi^++\pi^0$ etc.	2.5 ± 0.7	44, observed in *ca* 8900 photographs
Multiple production with 4 visible secondary particles: $\pi^-+p\rightarrow\pi^-+p+\pi^++\pi^-$ etc.	0.6 ± 0.3	6, observed in *ca* 2750 photographs
Total π^--p cross section without production of strange particles	$\sigma_{tot}=(47.2\pm3.5)$ mb	
$R=\dfrac{\sigma(\pi^-+p+\pi^0)}{\sigma(\pi^-+n+\pi^+)}$	$R=0.50^{+0.12}_{-0.14}$	

For the determination of the total π^--proton cross section at our energy of 1 Bev we have followed a π^--track length of about 1600 m and have assumed that the contamination of our primary beam by other particles (μ-mesons, electrons) was $(7\pm2)\%$. This value is based upon information obtained by the Berkeley Group. Of course the amount of the contamination has only an influence on the absolute values

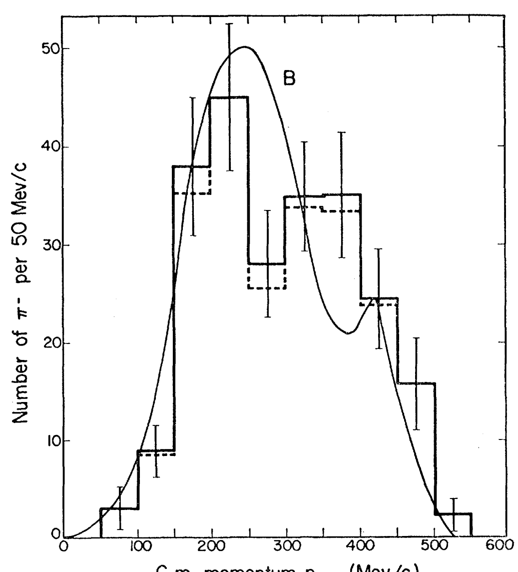

FIG. 2. Center-of-mass momentum distribution of the π^- from the reaction $\pi^-+p\rightarrow\pi^-+n+\pi^+$. Curve B: Lindenbaum-Sternheimer model.

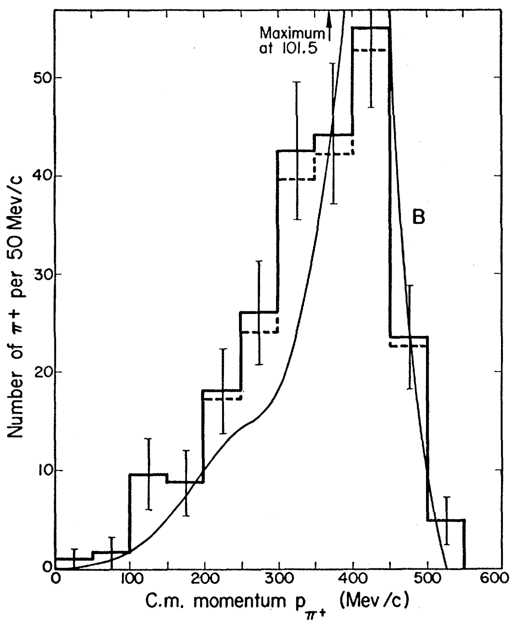

FIG. 3. Center-of-mass momentum distribution of the π^+ from the reaction $\pi^-+p\rightarrow\pi^-+n+\pi^+$. Curve B: Lindenbaum-Sternheimer model.

of the various cross sections, but not on the branching ratios.

Table I gives the cross sections for the various reactions connected with π^--proton scattering at 1 Bev. Our values are in good agreement with the results obtained by other authors in the same energy region.[10,12,15]

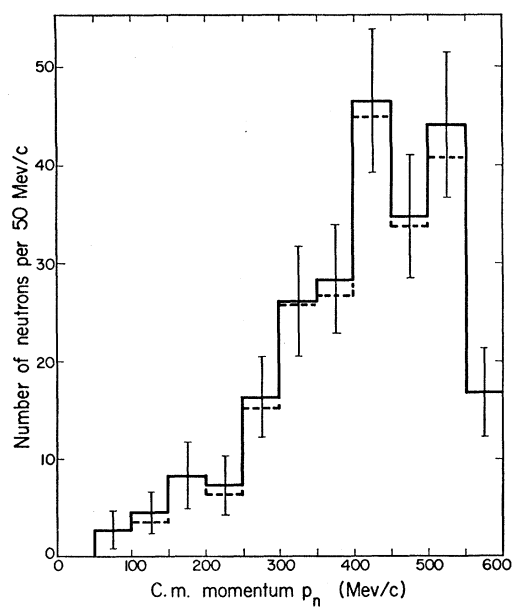

FIG. 4. Center-of-mass momentum distribution of the neutrons from the reaction $\pi^-+p\rightarrow\pi^-+n+\pi^+$.

[15] A. R. Erwin, Jr. and J. K. Kopp, Phys. Rev. **109**, 1364 (1958).

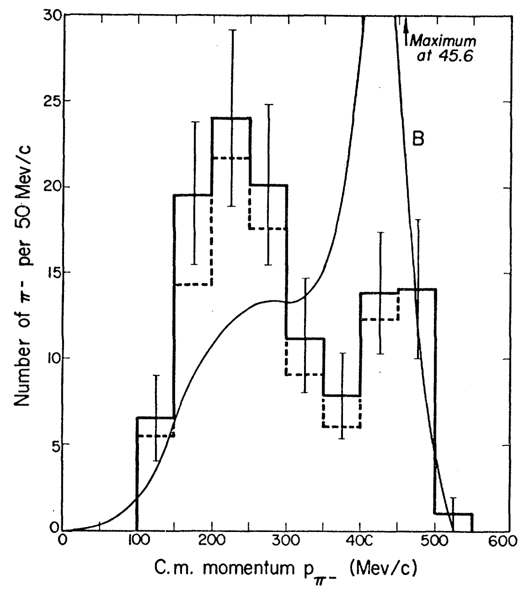

FIG. 5. Center-of-mass momentum distribution of the π^- from the reaction $\pi^-+p\rightarrow\pi^-+p+\pi^0$. Curve B: Lindenbaum-Sternheimer model.

The error intervals include statistical fluctuations and the uncertainties in the scanning efficiency, in the contamination, and in the distinction between the various reaction types. On the other hand, in the histograms of Figs. 1–16 the error intervals mean only the statistical fluctuations.

Figure 1 shows the differential cross section for the elastic scattering

$$\pi^-+p\rightarrow\pi^-+p$$

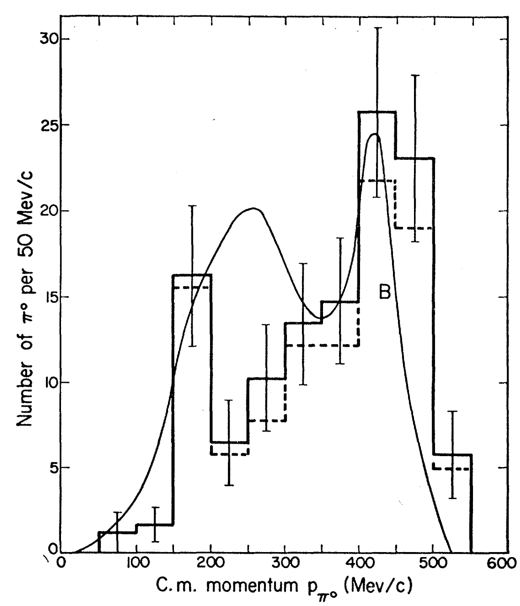

FIG. 6. Center-of-mass momentum distribution of the π^0 from the reaction $\pi^-+p\rightarrow\pi^-+p+\pi^0$. Curve B: Lindenbaum-Sternheimer model.

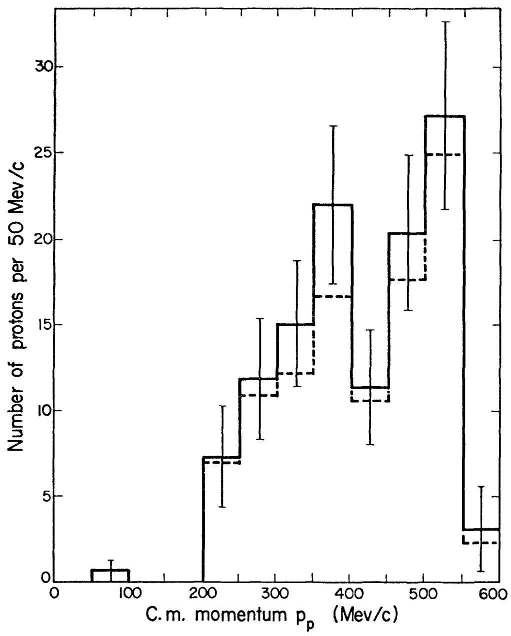

FIG. 7. Center-of-mass momentum distribution of the protons from the reaction $\pi^- + p \rightarrow \pi^- + p + \pi^0$.

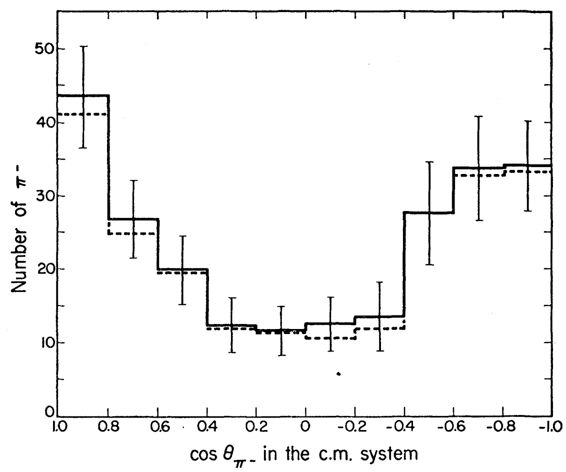

FIG. 8. Center-of-mass angular distribution of the π^- from the reaction $\pi^- + p \rightarrow \pi^- + n + \pi^+$.

in the center-of-mass system. This distribution agrees very well with the result of other authors,[10,12,15] some of whom have in addition tried to perform a phase-shift analysis.

RESULTS OF SINGLE PRODUCTION, AND COMPARISON WITH THE LINDENBAUM-STERNHEIMER MODEL

Figures 2–7 give our momentum distributions and Figs. 8–13 our angular distributions for the secondary particles from the two reactions

$$\text{(I)} \quad \pi^- + p \rightarrow \pi^- + n + \pi^+,$$

$$\text{(II)} \quad \pi^- + p \rightarrow \pi^- + p + \pi^0,$$

in the center-of-mass system of the π^- and proton. The dotted histograms were obtained only from those events which allowed an unambiguous distinction between the two processes (I) and (II). The method by which the questionable events [8.5% of all events of the types (I) and (II)] have been distributed between the two reactions (I) and (II) has been described in reference 13. The curves exhibited in the figures are the theoretical distributions according to the statistical model (A) without taking into account the isobaric nucleon state and according to the LS model (B) (see below) for 1 Bev. These curves are normalized to the area of the experimental histograms.

According to Lindenbaum and Sternheimer the momentum spectra of the secondary pions from the two processes (I) and (II) are given by the formulas (neglecting normalization factors)

$$I_{\pi^-}^{(\mathrm{I})}(p) = x_{\mathrm{I}} J_1(p) + y_{\mathrm{I}} J_2(p), \tag{1}$$

$$I_{\pi^+}^{(\mathrm{I})}(p) = y_{\mathrm{I}} J_1(p) + x_{\mathrm{I}} J_2(p), \tag{2}$$

$$I_{\pi^-}^{(\mathrm{II})}(p) = x_{\mathrm{II}} J_1(p) + y_{\mathrm{II}} J_2(p), \tag{3}$$

$$I_{\pi^0}^{(\mathrm{II})}(p) = y_{\mathrm{II}} J_1(p) + x_{\mathrm{II}} J_2(p). \tag{4}$$

In these equations $J_1(p)$ is the momentum distribution of the pions from the isobar decay and $J_2(p)$ the momentum distribution of the recoil pions. The formulas for calculating these functions are given by Lindenbaum and Sternheimer[9] according to their model. In reference 9 the curves $J_1(p)$ and $J_2(p)$ are presented for the two primary energies 0.93 Bev and 1.37 Bev. The quantities x_{I}, y_{I}, x_{II}, y_{II} are defined by

$$x_{\mathrm{I}} = \tfrac{1}{2} + \tfrac{2}{5}\rho + a, \tag{5}$$

$$y_{\mathrm{I}} = (1/18) + (8/45)\rho - (2/9)a, \tag{6}$$

$$x_{\mathrm{II}} = \tfrac{1}{9} + (1/45)\rho - (1/9)a, \tag{7}$$

$$y_{\mathrm{II}} = \tfrac{1}{9} + (16/45)\rho - (4/9)a. \tag{8}$$

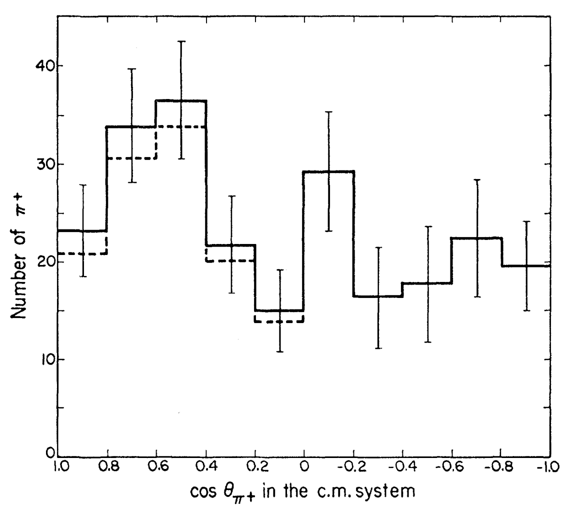

FIG. 9. Center-of-mass angular distribution of the π^+ from the reaction $\pi^- + p \rightarrow \pi^- + n + \pi^+$.

Here

$$2\rho \equiv \sigma_{\frac{3}{2}\mathrm{inel}}/\sigma_{\frac{1}{2}\mathrm{inel}}, \tag{9}$$

i.e., the ratio of the inelastic cross sections for $T=\frac{3}{2}$ and $T=\frac{1}{2}$.

$$a\equiv 2(\rho/5)^{\frac{1}{2}}\cos\varphi, \tag{10}$$

where φ is the phase difference between the matrix elements for pion production in the state $T=\frac{3}{2}$ and $T=\frac{1}{2}$. (In the case of the π^--proton scattering both isospin states occur.)

If one combines the momentum spectra of π^- and π^+ from reaction (I) into one momentum distribution, this distribution is independent of the unknown quantities a and ρ, as one sees from Eqs. (1) and (2). Therefore this joint distribution can be calculated at once, namely (neglecting the normalization factor and the factor $x_{\mathrm{I}}+y_{\mathrm{I}}$)

$$I_{\pi^-}^{(\mathrm{I})}(p)+I_{\pi^+}^{(\mathrm{I})}(p)=J_1(p)+J_2(p).$$

The same is true for the joint momentum spectrum of the π^- and π^0 coming out from reaction (II). Figures 14 and 15 show our experimental $\pi^-+\pi^+$ and $\pi^-+\pi^0$ spectra together with the distributions predicted by the statistical model (A) and the LS model (B). Figure 16 presents the momentum distribution of the protons and neutrons from (I) and (II). In this case the theoretical curves (A) and (B) approximately fall together; therefore in this figure only a mean theoretical curve is given.

We have tried to determine the quantities a and ρ from our experimental spectra and the theoretical distributions (1) to (4) by a best fit. This has been carried out in the following way: From (1) and (2) follows

$$I^{(\mathrm{I})}(p)=[z_{\mathrm{I}}+J(p)]/[1+z_{\mathrm{I}}J(p)], \tag{11}$$

where

$$I^{(\mathrm{I})}(p)=I_{\pi^-}^{(\mathrm{I})}(p)/I_{\pi^+}^{(\mathrm{I})}(p), \quad z_{\mathrm{I}}=x_{\mathrm{I}}/y_{\mathrm{I}},$$
$$J(p)=J_2(p)/J_1(p).$$

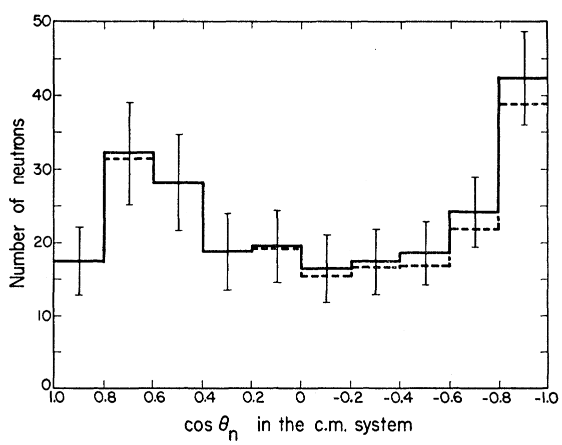

FIG. 10. Center-of-mass angular distribution of the neutrons from the reaction $\pi^-+p\rightarrow\pi^-+n+\pi^+$.

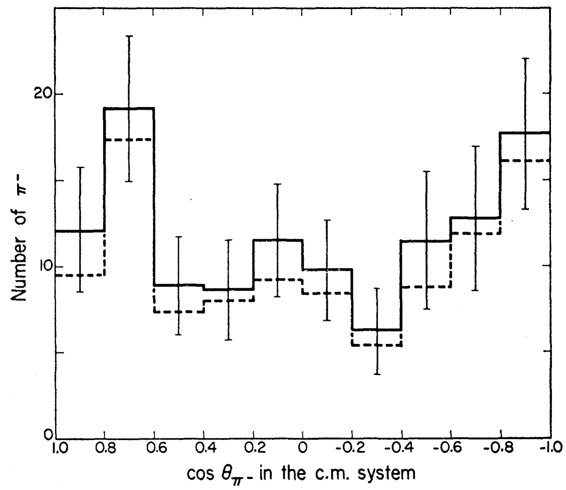

FIG. 11. Center-of-mass angular distribution of the π^- from the reaction $\pi^-+p\rightarrow\pi^-+p+\pi^0$.

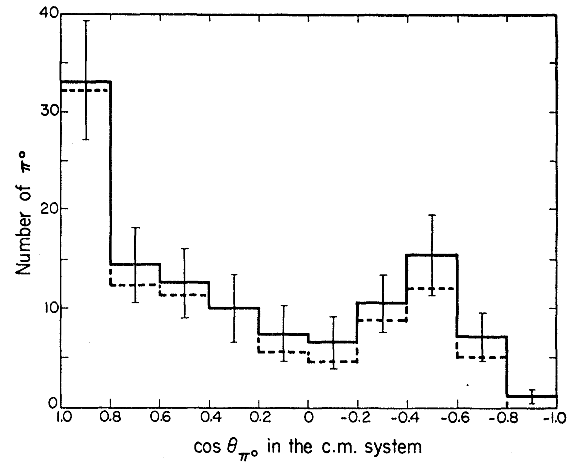

FIG. 12. Center-of-mass angular distribution of the π^0 from the reaction $\pi^-+p\rightarrow\pi^-+p+\pi^0$.

[Equation (11) is independent of unknown normalization factors.] Taking the ratio $I^{(\mathrm{I})}(p)$ from our experimental histograms (Figs. 2 and 3) and the ratio $J(p)$ from the theoretical curves $J_1(p)$ and $J_2(p)$ we have calculated z_{I} for several momentum intervals, i, of our histograms. For each $z_{\mathrm{I}i}$ we have determined an error $\Delta z_{\mathrm{I}i}$ from the statistical errors of our experimental points, applying the error propagation law of Gauss to Eq. (11). Of course all $z_{\mathrm{I}i}$ should be equal if the LS model is correct and if our statistics were sufficiently large. From the various values $z_{\mathrm{I}i}$ which showed large fluctuations we have calculated a weighted mean value taking the quantity $(1/\Delta z_{\mathrm{I}i})^2$ as a weight for $z_{\mathrm{I}i}$. We obtained

$$z_{\mathrm{I}}=1.22\pm0.33.$$

Replacing our histograms by some smooth curves still compatible with the histograms, and taking $I^{(\mathrm{I})}(p)$ from these curves we obtained in the same way (using

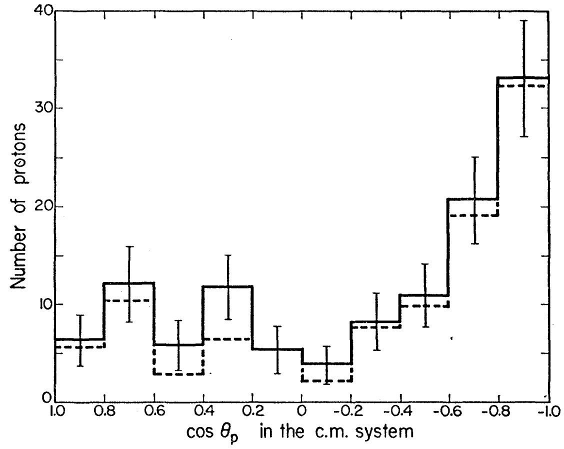

FIG. 13. Center-of-mass angular distribution of the protons from the reaction $\pi^-+p\rightarrow\pi^-+p+\pi^0$.

the same No. of intervals) the mean value

$$z_I=1.90\pm0.30.$$

The same procedure can be applied to the Eqs. (3) and (4) for reaction (II). Making use of the experimental histograms 5 and 6 we found for $z_{II}=x_{II}/y_{II}$ the mean value

$$z_{II}=1.44\pm0.34.$$

From the smooth curves we got

$$z_{II}=1.37\pm0.15.$$

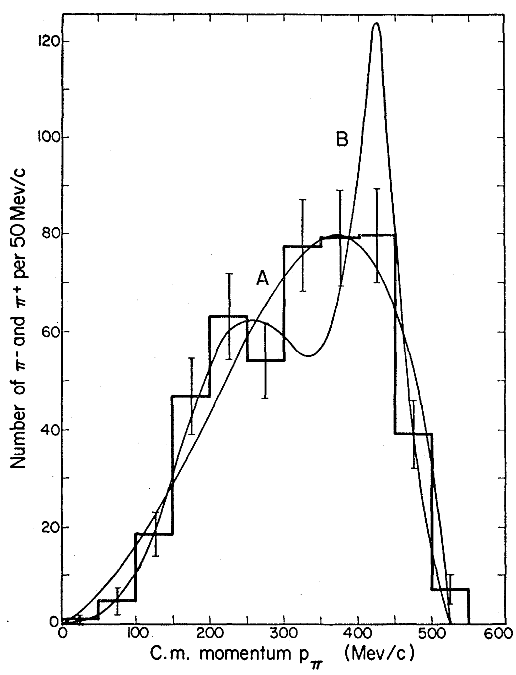

FIG. 14. Center-of-mass momentum distribution of the π^- and π^+ from the reaction $\pi^-+p\rightarrow\pi^-+n+\pi^+$. Curve A: statistical model; Curve B: Lindenbaum-Sternheimer model.

Knowing z_I and z_{II} one can calculate a and ρ by means of Eqs. (5) to (8). However, inserting the above values for z_I and z_{II} into the formula for ρ one finds only negative values for ρ which are physically senseless [see Eq. (9)]. This occurs also in the case that one takes the various limits of the calculated intervals around z_I and z_{II}; even the most favored combination of these error limits is far from yielding a positive ρ. (The values obtained for ρ are between -0.44 and -0.32.) For the calculation of a and ρ one can also go back to the ratio

$$R=\frac{10+17\rho-25a}{25+26\rho+35a}=0.50_{-0.14}^{+0.12} \quad \text{(see Table I)} \tag{12}$$

of the total cross sections for the reactions (I) and (II). Unlike z_I and z_{II} the quantity R can be determined immediately from the experiments and is independent of the special shape of the theoretical curves $J_1(p)$ and $J_2(p)$. If one calculates ρ from the two values R, z_I or R, z_{II}, again one obtains in both cases only negative values even if one inserts the optimal error limits into the formula for ρ (the values are between -5.40 and -0.99, and between -0.28 and -0.01, respectively).

From this one can conclude that our experimental distributions which are based upon rather large statistics cannot be very well described by the LS model.

Another test of the LS model by the momentum distributions of the single secondary pions has been carried out by Lindenbaum and Sternheimer[9] with the comparatively small statistics of Walker *et al.* and Eisberg *et al.* We have done the same with our results:

Making the assumption that

$$\sigma_{\frac{3}{2}\mathrm{inel}}/\sigma_{\frac{1}{2}\mathrm{inel}}=\sigma_{\frac{3}{2}}/\sigma_{\frac{1}{2}} \tag{13}$$

we have calculated ρ from the total cross sections $\sigma(\pi^-+p)$ and $\sigma(\pi^++p)$:

$$\rho=1\bigg/\left[3\frac{\sigma(\pi^-+p)}{\sigma(\pi^++p)}-1\right]. \tag{14}$$

For our energy of 1 Bev, ρ has the value [16,17]

$$\rho=0.200.$$

Inserting this value into formula (12) and taking our value for R we found

$$a=-0.040$$

and [see formula (10)]

$$\varphi=95.3°.$$

With a and ρ we have calculated the distributions (1) to (4) and have plotted them in the Figs. 2, 3 and 5, 6 (curves B). One sees that these theoretical curves do

[16] Cool, Piccioni, and Clark, Phys. Rev. **103**, 1082 (1956).
[17] Burrowes, Caldwell, Frisch, Hill, Ritson, Schluter, and Wahlig, Phys. Rev. Letters **2**, 119 (1959).

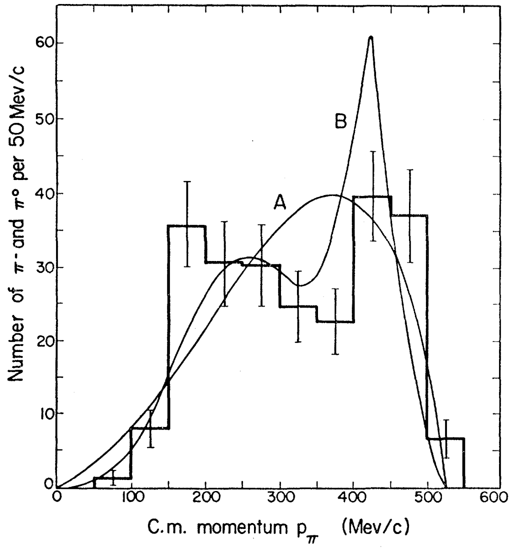

FIG. 15. Center-of-mass momentum distribution of the π^- and π^0 from the reaction $\pi^-+p \rightarrow \pi^-+p+\pi^0$. Curve A: statistical model; Curve B: Lindenbaum-Sternheimer model.

not fit our experimental histograms well. Especially in the case of Figs. 5 and 6, the distributions predicted by the LS model with assumption (13) deviate considerably from our experimental results.

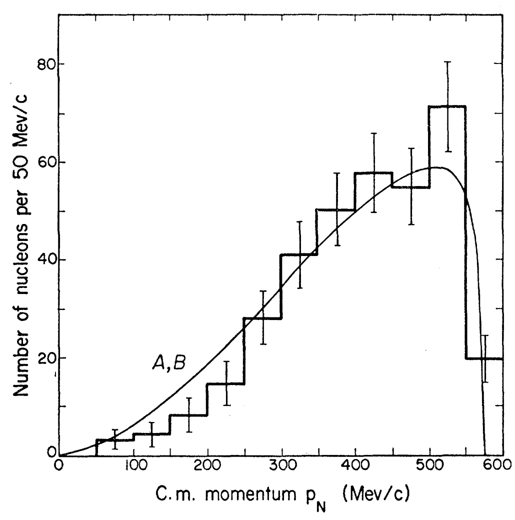

FIG. 16. Center-of-mass momentum distribution of the nucleons from the reactions $\pi^-+p \rightarrow \pi^-+n+\pi^+$ and $\pi^-+p \rightarrow \pi^-+p+\pi^0$. Curve A, B: statistical and Lindenbaum-Sternheimer model.

CONCLUSIONS

Our momentum distributions indicate two pronounced maxima around the momenta 200 Mev/c and 400 or 450 Mev/c. A simple model for the explanation of these two maxima is the LS model which ascribes the first maximum to the pions coming out from the isobar decay and the second maximum to the recoil pions. However, as the preceding chapter has shown, this model cannot explain our results quantitatively. On the other hand, also the statistical model in its simple form without taking into account the isobaric state surely is not a good description. Perhaps one can obtain a better agreement between experiment and theory if one combines the two models in the sense that part of the pion production takes place directly without the isobaric intermediate state and that the other part goes via isobars (see references 2, 5, and 6). Of course, the question remains as to what the frequency ratio for these two reaction channels is. Surely it would be too great a simplification if one calculated this ratio from the statistical weights for 2 and 3 particles. The attempt of combining the statistical and the LS model, of course, can only be successful if the curve of Fig. 16, where the statistical model and the isobar model give practically the same result, is in agreement with the experimental distribution. This is actually the case. Moreover, recent experiments[18–20] on π-proton scattering have revealed the existence of additional resonances at higher energies. A satisfactory theory of pion production would also have to take into account the higher isobaric states corresponding to these resonances. At the moment we are investigating these questions in a more detailed way.

ACKNOWLEDGMENTS

We are very grateful to Dr. K. Gottstein who suggested this work to us, for his continued interest in its progress and for many valuable and helpful discussions. Moreover, we want to thank Professor L. W. Alvarez for the loan of the bubble chamber films, Mr. G. Lütjens for his help in the evaluation, and Dr. B. Zorn and Dr. G. T. Zorn for valuable suggestions and for their help during the initial stages of the measurements. We also thank Dr. G. Höhler for a fruitful discussion.

[18] Reference 12, pp. 86–89.
[19] R. F. Peierls, Phys. Rev. Letters 1, 174 (1958).
[20] Crittenden, Scandrett, Shephard and Walker, and Ballam, Phys. Rev. Letters 2, 121 (1959).