time of the order of 10^{-11} sec, or perhaps somewhat longer, we can imagine that the muonium breaks up for the last time, since the breakup cross section is generally expected to be much greater than the capture cross section. Thus it is also reasonable not to consider large values of τ. In any case, we assume that the repeated captures and breakups can be represented by some *effective* mean life, and find for the final polarization of the muon at the time that it decays the expression

$$P=\left(1-\frac{1}{2}\frac{1}{1+\tau^{-2}+x^2}\right)^n. \quad (5)$$

This equation replaces Eq. (2) proposed by Sens et al.[6] It also contains two parameters, $\tau/2 \rightarrow \tau$, the effective mean life with respect to breakup, and n, the average number of intermediate muonium formations. A convenient procedure for obtaining a fit to the experimental data is to consider that value of the magnetic field strength, x_m, for which the polarization is equal to the geometrical mean of its value for zero and infinite field strengths. x_m is readily estimated from the experimental data. In terms of x_m, τ is determined by the following equation:

$$\tau=[-\tfrac{3}{4}+(\tfrac{1}{16}+x_m^4)^{\frac{1}{2}}]^{-\frac{1}{2}}. \quad (6)$$

n can now be determined from τ and P_0, the zero-field polarization:

$$n=\frac{\ln P_0}{\ln[1-1/2(\tau^{-2}+1)]}. \quad (7)$$

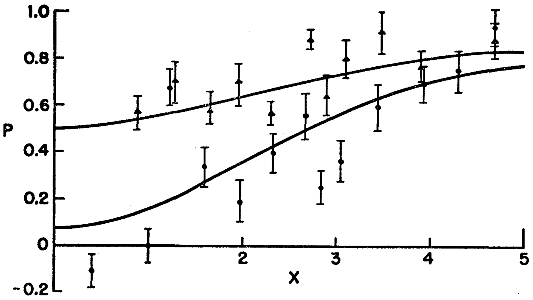

FIG. 1. Dependence of muon polarization P on magnetic field x, measured in units of 1.58 kilogauss. The magnetic quenching data shown are those of Sens et al. (reference 6) for nuclear emulsion (triangles) and fused quartz (circles). The fits achieved by the present theory, which takes into account the finite lifetime of the muonium atom with respect to breakup, are given by the upper and lower curves (nuclear emulsion and fused quartz, respectively).

Figure 1 shows the data of Sens et al.[6] represented by solid triangles for nuclear emulsion and by solid circles for fused quartz. Fits are given to this data by $\tau=0.38$ and $n=10.1$ for nuclear emulsion and by $\tau=0.7$ and $n=13.5$ for fused quartz, and are exhibited by the upper and lower curves, respectively.

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Absorption of Negative Muons in C^{12} Leading to Production of Bound B^{12}*†

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A negative muon beam from the Carnegie Tech synchrocylotron was stopped in a six-inch propane bubble chamber. Since the hydrogen does not form μ-mesonic atoms in the presence of carbon, the pictures yield information on the interaction of stopped muons with carbon. About 30 000 pictures of stopping muons were taken with the bubble chamber kept sensitive for ~20 msec after the beam pulse in order to observe the beta decay of any bound B^{12} nuclei resulting from μ absorption by carbon. The chamber was photographed right after the beam pulse to determine whether a given stopped muon decayed, or was absorbed. Another photograph was taken about 15 msec later to determine if the absorption had led to a nucleus which had beta decayed. A count of μ-e decays in the same film allowed the determination of the probability per unit time of bound B^{12} formation. Forty-six boron decays were observed yielding $(7.6\pm1.2)\times10^3$ sec^{-1} for the rate of bound B^{12} production. Possible interpretation of this result in terms of a universal V-A Fermi interaction is discussed.

INTRODUCTION

THE theory of the universal V-A Fermi interaction[1] leads to a clear cut prediction concerning the rate of the capture process

$$\mu^-+p\rightarrow n+\nu. \quad (1)$$

This theory has been quite successful in explaining the available experimental data, and further verification, in the form of a measurement of the absorption rate

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† This work has been submitted by J. G. Fetkovich to Carnegie Institute of Technology in partial fulfillment of the requirements for the degree of Doctor of Philosophy.

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[1] R. P. Feynman and M. Gell-mann, Phys. Rev. **109**, 193 (1958); E. C. G. Sudarshan and R. E. Marshak, Phys. Rev. **109**, 1860 (1958); J. J. Sakurai, Bull. Am. Phys. Soc. **3**, 10 (1958).

for muons in hydrogen, is desirable. This measurement is difficult to make, however, due to the low probability for the reaction, and due to complications arising from the formation of μ-mesonic molecules.

The coupling constant for (1) can in principle be determined by measuring the absorption rate in heavy nuclei, in which case the above-mentioned experimental difficulties do not occur. In this case, however, there is a large uncertainty in the theoretical prediction due to lack of knowledge of the nuclear matrix elements between the initial and final states. In certain special cases the μ-mesonic absorption by a nucleus goes from an initial to a final nuclear state which are, respectively, the final and initial nuclear states of a known beta decay reaction. In such a case the nuclear matrix element between the two states in question can be evaluated from the known beta decay lifetime. One such special case is the absorption of a μ^- by C^{12} leading to the ground state of B^{12}:

$$\mu^- + C^{12} \rightarrow B^{12} + \nu. \quad (2)$$

The B^{12} is beta active, and decays back to the ground state of C^{12}:

$$B^{12} \rightarrow C^{12} + e^- + \bar{\nu}. \quad (3)$$

From the known probability per unit time, W^{β}, of (3), one can evaluate the nuclear matrix element for the transition between the ground states of C^{12} and B^{12} and use this value to predict W^{μ}, the rate of reaction (2). Unfortunately, there are still some uncertainties in the calculation, stemming mainly from the large momentum transfer in (2) as compared to (3) and from effects of virtual pions.

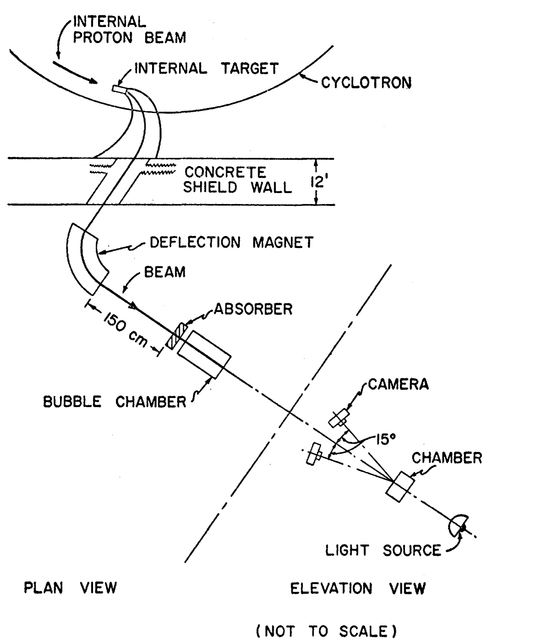

FIG. 1. Diagram of the experimental setup. The elevation view is taken looking along the beam direction.

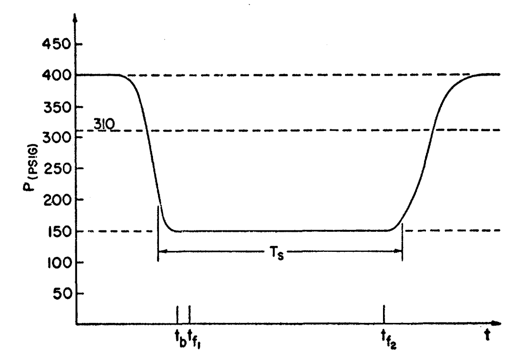

FIG. 2. Pressure vs time in the propane. At the operating temperature the vapor pressure was 310 psig.

The measurement of the rate of reaction (2) was first suggested by Tiomno, and done by Godfrey[2] in 1954 using cosmic ray muons stopped in a liquid scintillation counter. It was deemed desirable, in view of recent advances in the theory of weak interactions, to redo the measurement by a quite different method as a check, and, if possible, to increase the precision of the result.[3]

EXPERIMENTAL METHOD

In this work, the rate of B^{12} production was studied using a propane (C_3H_8) bubble chamber in which were stopped μ-mesons from the Carnegie synchrocyclotron. Figure 1 illustrates schematically the experimental setup. A pulse of beam lasting about 100 μsec was made to penetrate an amount of absorber sufficient to bring the muons to rest in a convenient region of the propane chamber. All the stopping negative muons are captured in atomic orbits around carbon nuclei (the hydrogen is completely ineffective in capturing[4]) and cascade down to the K orbit in a time shorter than about 10^{-11} sec.[5] The weak coupling between the muon and other fermion fields then causes one of several possible reactions to occur. For the purposes of this discussion these reactions may be conveniently grouped into three classes:

$$\text{I.}\ \mu^- + C^{12} \rightarrow C^{12} + e^- + \nu + \bar{\nu},$$

$$\text{II.}\ \mu^- + C^{12} \rightarrow B^{12*} + \nu$$
$$B^{12*} \rightarrow \text{heavy particles},$$

$$\text{III.}\ \mu^- + C^{12} \rightarrow B^{12} + \nu$$
$$B^{12} \rightarrow C^{12} + e^- + \bar{\nu},$$

where B^{12*} means a state whose excitation energy is >3.4 Mev, and B^{12} means one of the five bound states

[2] T. N. K. Godfrey, thesis, Princeton University, 1954 (unpublished).

[3] A preliminary account of this work was given at the *Gatlinburg Conference on Weak Interactions, Gatlinburg, Tennessee, October, 1958* [Bull. Am. Phys. Soc. 4, 81 (1959)]. Further details of the experimental method can be found in J. G. Fetkovich, Atomic Energy Commission Report NYO-2237, 1959 (unpublished).

[4] W. K. H. Panofsky, R. L. Aamodt, and J. Hadley, Phys. Rev. **81**, 565 (1951); J. D. Jackson, Phys. Rev. **106**, 330 (1957).

[5] G. R. Burbidge and A. H. de Borde, Phys. Rev. **89**, 189 (1953).

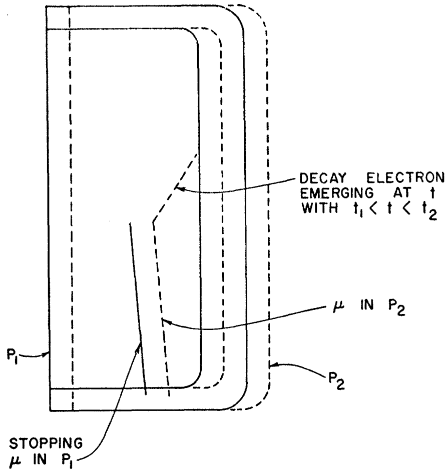

FIG. 3. Drawing illustrating the two pictures, P_1 and P_2, of a boron event. The separation between P_1 and P_2 is proportional to the time interval $t_{f2}-t_{f1}$ and to the speed of film motion in the camera. The boron decay electron appears only in P_2 indicating that the decay occurred between t_{f1} and t_{f2}.

of B^{12}. Reaction I is just the "free" decay of the muon in which the carbon nucleus plays little part and which has a measured probability per unit time $\Lambda_I = 4.5\times10^5$ sec^{-1}. In reaction II the muon is absorbed by the carbon nucleus leading to a particle unstable state of B^{12} which breaks up by the emission of heavy particles, usually neutrons. This reaction has a probability per unit time Λ_{II}. In reaction III, the absorption of the muon leads to a B^{12} nucleus with insufficient excitation to emit particles. The B^{12} may be formed in its ground state or any of its four bound excited states. If the latter occurs, the boron de-excites by γ-ray emission.[2] The probability per unit time of the first arrow in III is represented by Λ_{III} while that of the second is known[6] from other experiments to be $1/29.31$ msec^{-1}. Λ_{III} was measured relative to Λ_I by comparing the number of muon absorptions leading to bound states of B^{12} with the number of observed μ-e decays. The detection of reaction III was accomplished by observing the electron emitted in the B^{12} beta decay. The bubble chamber was kept sensitive for 20 msec after the beam pulse so that a reasonable fraction of the radioactive B^{12} nuclei would be seen to decay before sensitivity was lost. Twenty milliseconds was near the upper limit to the attainable sensitive time imposed by the design of the chamber. As indicated in Fig. 1, the chamber was viewed from above by a stereoscopic pair of cameras, designated T and B, in each of which was registered an image of the chamber each time the light source (an electronic flash) was triggered. The stereo half angle was 10° in the propane. The central rays of the two cameras defined the "camera plane" which was perpendicular to the beam direction.

In order to distinguish reactions I and III, two photographs of the chamber were taken after each beam pulse. The first was taken at time t_{f1}, which was about 1.5 msec, after the time, t_b, of the beam pulse. The second, at t_{f2}, was taken about 15 msec later. Of course t_b, t_{f1}, and t_{f2} were all within T_s, the period of chamber sensitivity (this sequence is illustrated in Fig. 2). The first picture, designated P_1, shows the tracks of all particles which traverse the chamber before t_{f1} (but within T_s), while P_2 shows all particles which traverse the chamber before t_{f2}. Thus a μ-e decay would have both the muon and electron tracks on both P_1 and P_2, while a boron event (wherein the B^{12} decayed between t_{f1} and t_{f2}) would have the muon in P_1 and P_2, but the electron in only P_2. During the interval between t_{f1} and t_{f2}, the film was moved in the cameras to separate the two images. This motion was perpendicular to the beam direction and resulted in photographs of the chamber which appear, schematically, as in Fig. 3 (the T view only is shown). Figures 4 and 5 show the actual T view of a boron event and a μ-e decay event, respectively.

BEAM

The beam emerging from the cyclotron was analyzed by a magnet to select 35-Mev muons (see Fig. 1). There was a contamination of pions and electrons with momentum equal to that of the selected muons. The composition of the beam which entered the chamber

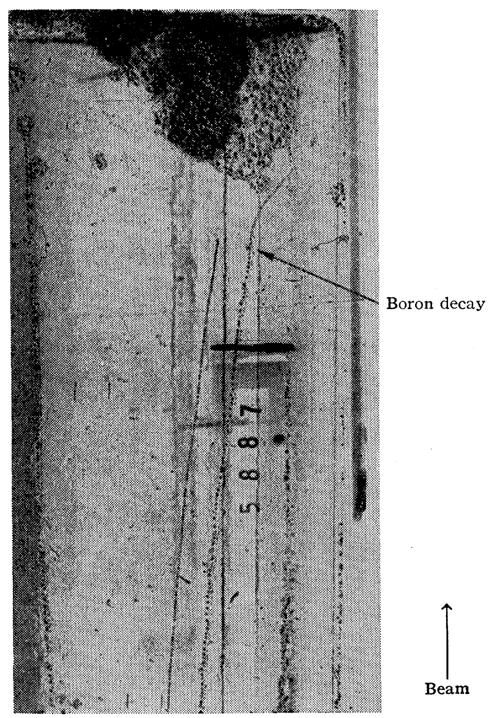

FIG. 4. T view of a boron event with double flash. The electron track is clearly of an age intermediate between the μ_1 and μ_2 tracks.

[6] F. Ajzenberg-Selove, and T. Lauritsen, Revs. Modern Phys. (to be published).

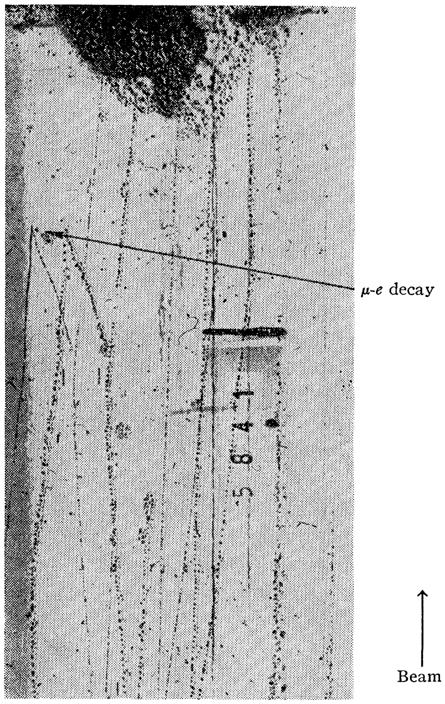

FIG. 5. T view of a μ-e decay with double flash.

was:

$$\pi:\mu:e\cong 1:10:20.$$

In Fig. 6 is shown a plot of the differential range distribution of mesons. The abscissa is distance, along the beam direction, from the upstream end of the chamber. This distance was measured on a projected image which was 1.8 life size. The density of the propane when the chamber was expanded was 0.42 g/cm^3. Curve B clearly shows the separate π and μ peaks. The horizontal bar indicates the acceptance region for boron and μ-e events.

SCANNING PROCEDURES

We could in principle measure Λ_{III} relative to Λ_I by finding all the boron and μ-e decay events on the film. This is extremely difficult because scanner efficiency is not 100% for all events. However, since we are only interested in the ratio of the numbers of the two types of event, we may reject events which are difficult to find without affecting the result, provided we do so through rejection criteria so designed that equal fractions of the two classes of event are omitted. If this is correctly done, there will be high efficiency for finding acceptable events and the efficiency correction may be easily made.

Because the T view was generally easier to scan and measure than the B view (due to accidental optical misadjustment during the run), it was the defining view for application of the acceptance criteria and the lengths and angles referred to in the acceptance criteria are measured in the T view. All measurements were made on the projection. The criteria for acceptable boron and μ-e events were (θ_e is defined in Fig. 7; l_e is the length of the chord to the electron track):

(a) The muon end point must be in the "acceptable" region of the chamber; (b) $l_e \geq 2$ cm (in P_2); (c) $|\theta_e| \leq 165°$ (in P_2); (d) The decay electron (in P_2) does not pass within 2 mm of the muon end point in P_1; (e) Scanned frames must have <18 incident tracks, counting in P_1 and P_2, and have the separation between P_1 and P_2 be ≥ 2 mm; and (f) ($|\theta_e| \geq 20°$) where the expressions in parentheses apply to μ-e events only. Thus in (b) and (c), the measurements were made in P_2 for μ-e events, even though the P_1 tracks were more clearly defined, in order to parallel the procedure for boron events which have no P_1 electron. Criterion (d) was necessary for boron events because if the decay electron had passed directly over the muon end point in P_1, one could not tell whether a decay electron in P_1 existed. Application of (a) insured that accepted events occur in easily scannable regions of the chamber, not at walls or windows, and not where there were too many stopping pions (Fig. 6).

Certain corrections to the raw data were necessitated for the following reasons: Criterion (a) includes a limitation on the depth in the chamber of the event (to prevent acceptance of events wherein the muon penetrated a glass window). To save time, the depth of μ-e events was not measured. Thus, some unacceptable events were included in the μ-e scan. Criterion (f) was applied to μ-e events because of difficulty in detecting forward μ-e decays (there was not always a sufficiently

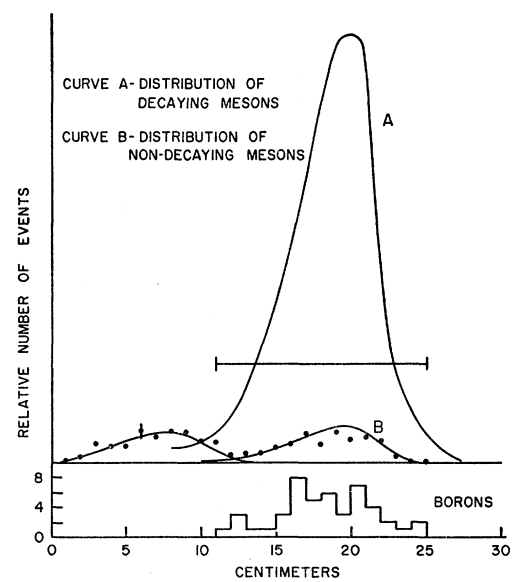

FIG. 6. Curve A is the differential range curve of μ-e decay events in the propane. Curve B is the distribution of meson absorptions showing a $\pi-$ and a $\mu-$ peak. The histogram at the bottom shows the distribution of the accepted boron events. The horizontal bar indicates the acceptance region for μ-e decays and boron events. The upstream end of the chamber is at 0 cm, and the downstream end at 28 cm.

great difference in bubble density between mesons and electrons to tell them apart. This was due to difficulties in controlling chamber sensitivity in the long-sensitive-time mode of operation). This criterion was not applied to boron events. Finally, (b) rejects a greater fraction of boron events than μ-e decays because of the low energy of the boron decay electrons compared to μ-e electrons. These and other corrections are considered below.

In all, about 28 000 frames were scanned for boron events and μ-e decays. About 17 000 of these were scanned for boron events at least twice. In these multiply-scanned frames 34 events were found and only one of these was missed by a scanner. Thus the efficiency correction for boron events was negligible. The average efficiency for μ-e decays was measured to be $(97.7 \pm 0.3)\%$.

DATA REDUCTION

The data obtained in this experiment are as follows: In 28 000 frames, 8908 μ-e decays (corrected for scanner efficiency) and 46 B^{12} decays were observed.

We now allow for the finite observation period. If $n_B(t)$ is the number of undecayed B^{12} nuclei present t seconds after the beam pulse we have:

$$n_B \equiv n_B(t_{f1}) - n_B(t_{f2}) = N_B(t_b)[\exp(-t_{f1}/\tau) - \exp(-t_{f2}/\tau)],$$

where n_B is the number of observed decays. Thus:

$$N_B(t_b) = n_B/[\exp(-t_{f1}/t) - \exp(-t_{f2}/t)],$$

yielding 120 for $N_B(t_b)$, the number originally formed. Because the track bubbles require a finite time to grow to visibility after the passage of the charged particle, the times, t_{f1} and t_{f2}, used in the above calculation were not the flash times, but were less by a small amount δ. Measurements, in this experiment, of visibility versus track age led to the choice $\delta = 0.3 \pm 0.2$ msec. The uncertainty of 0.2 msec in δ contributes a 1% uncertainty to N_B, while a measuring error of $\pm\frac{1}{4}$ msec in the flash times contributes negligibly thereto (since they were measured 28 times).

As stated previously, criterion (b) rejected a greater fraction of boron events than μ-e events due to the lower energy of boron electrons. We calculate, from experimental data on attenuation of electrons passing through matter, a correction factor f such that the corrected number of boron events is fN_B. We find $f = 1/(0.92 \pm 0.05)$.

The number, N_μ, of μ-e events found on the film must now be corrected for the effects of criteria (a) and (f). By measuring the depth distribution of a sample of the μ-e decay points, it was determined that a fraction $C = 0.24 \pm 0.015$ of the otherwise acceptable μ-e events did not satisfy the depth part of criterion (a).

The number of events rejected by (f) was determined by plotting $dN_\mu/d\theta_e$ versus θ_e for a sample of events

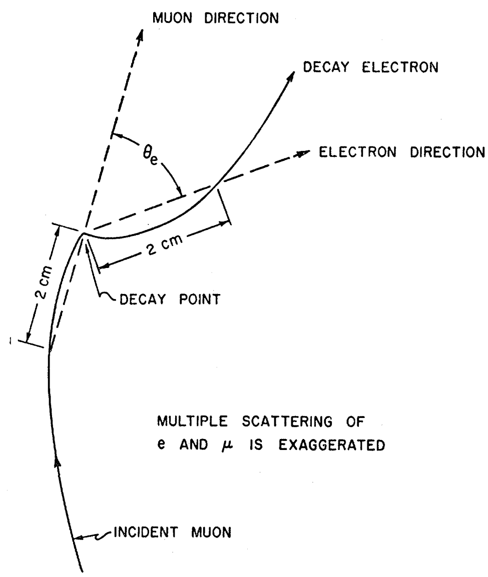

FIG. 7. Diagram of a μ-e decay or a boron event showing the definition of θ_e, the decay electron angle.

in which the decay electron emerged to the right in the T view. This plot was extrapolated to $\theta_e = 0$ to determine the average intensity of events in the angular region $-20° < \theta_e < 20°$. The number of events in this angular interval was expressed as a fraction, b, of $N_{\mu R}$, the number of μ-e decays to the right [only those events wherein the decay is to the right ($\theta_e > 0$) were used because criterion (d) rejects a variable fraction of left decays since the P_1-P_2 separation varies from frame to frame]. The result is $b = 0.26 \pm 0.03$.

The number of μ-e events satisfying all criteria except part of (a) is N_μ. We add to this $bN_{\mu R}$ [correcting for criterion (f)] to get $N_\mu + bN_{\mu R}$. We subtract from these events the fraction C of them to obtain $\mathfrak{N}_\mu$, the number of μ-e events to be compared to fN_b, the number of boron events. Thus:

$$\mathfrak{N}_\mu = (1-C)(N_\mu + bN_{\mu R}),$$

and:

$$\Lambda_{\mathrm{III}} = \Lambda_{\mathrm{I}} \frac{fN_b}{(1-C)(N_\mu + bN_{\mu R})}.$$

To recapitulate:

$$(1-C) = 0.76 \pm 0.015,$$
$$b = 0.26 \pm 0.03,$$
$$N_\mu = 8908 \pm 90,$$
$$N_{\mu R} = 5272 \pm 70,$$
$$N_B = 120 \pm 17,$$
$$1/f = 0.92 \pm 0.05,$$

which yields:

$$\Lambda_{\mathrm{III}}/\Lambda_{\mathrm{I}} = (1.67 \pm 0.26)\%,$$

or:

$$\Lambda_{\mathrm{III}} = (7.6 \pm 1.2) \times 10^3 \text{ sec}^{-1}.$$

The error assigned to this result arises almost entirely from the statistical error on the 46 boron events. The major corrections applied to the raw data are represented by the factors b and C, the net correction being about 25%. This is moderately large; however, it is felt that the correction factors are well known and that their precision is well represented by the errors assigned them.

There are several possible sources of systematic error in this measurement. There is a background of boron and boron-like events due to absorption of π^- by carbon. There is also a background of boron-type events due to accidental space coincidences of compton electrons with absorbed muons. Finally, the results would be affected if a significant number of captures were on elements other than carbon. All these effects have been considered, and found to affect the result negligibly.

DISCUSSION

This experiment measures $\Lambda_{\rm III}$, the transition probability per unit time for μ^- absorption leading to any bound state of B^{12}. This is very difficult to predict theoretically. There are several calculations of W^μ, the probability per unit time for μ^- absorption leading to the ground state of B^{12}. Fujii and Primakoff[7] have done the calculation using a nonrelativistic hydrogenic wave function for the muon, neglecting higher than s-wave neutrinos, and neglecting terms of order $(v/c)^2$ in the nucleons. Wolfenstein[8] has done a similar calculation wherein, however, he includes up to d-wave neutrinos. Flamand and Ford[9] have further refined the calculation by using the relativistic muon wave function obtained when the finite nuclear extension is taken into account. The results of these calculations are listed in Table I. The three cases are as follows:
Case A neglects all effects of virtual pions. Case B includes the expected virtual pion effects but without the assumption of a conserved vector current.[10] Case C includes the conserved vector current assumption. The uncertainty in these calculated values has been estimated[7–9] to be 10% to 20%. This is due mainly to uncertainty in the magnitude of effects of virtual pions, and to imprecise knowledge of the nuclear wave functions of C^{12} and B^{12}.

TABLE I. Theoretical transition rate.

Assumption	$W^\mu \times 10^{-3}$ (sec^{-1}) Fujii and Primakoff	Wolfenstein	Flamand and Ford
A	...	7.3	...
B	6.34	5.9	5.3
C	7.86	7.4	7.3

Before comparing the experimental number with the calculations we need to know what fraction of absorptions to bound states of B^{12} are to excited states. This experiment gives no information on this, but there is other evidence[11] that the fraction is about 10%. If we use this number we obtain:

$$W_{\rm exp}{}^\mu = (9/10)\Lambda_{\rm III} = (6.8\pm 1.1)\times 10^3 \text{ sec}^{-1},$$

where it should be emphasized that the quoted error does not reflect the uncertainty in the 9/10 factor.

This experiment has also been done by several groups using counter techniques. Their results (including the correction for absorption leading to bound excited states, which is again assumed to happen 10% of the time) are:

$$W^\mu = (9.05\pm 0.95)\times 10^3 \text{ sec}^{-1},^{11}$$
$$W^\mu = (9.18\pm 0.5)\times 10^3 \text{ sec}^{-1},^{12}$$
$$W^\mu = (6.8\pm 1.5)\times 10^3 \text{ sec}^{-1},^{13}$$
$$W^\mu = (5.9\pm 1.5)\times 10^3 \text{ sec}^{-1}.^{2}$$

It appears that the spread in the present experimental values of W^μ is comparable to or perhaps a little larger than the theoretical uncertainty in W^μ, and is certainly larger than the effect of a conserved vector current term in the interaction.

ACKNOWLEDGMENTS

We wish to thank Professor L. Wolfenstein for many stimulating discussions. It is a pleasure to thank J. Deahl, K. Derrick, M. Derrick, and G. Pewitt for assistance in various phases of the experiment. We are indebted to B. Cherry and the scanning group for efficient scanning of the film.

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[13] W. A. Love, S. Marder, I. Nadelhaft, R. T. Siegel, and A. E. Taylor, Bull. Am. Phys. Soc. **4**, 81 (1959).

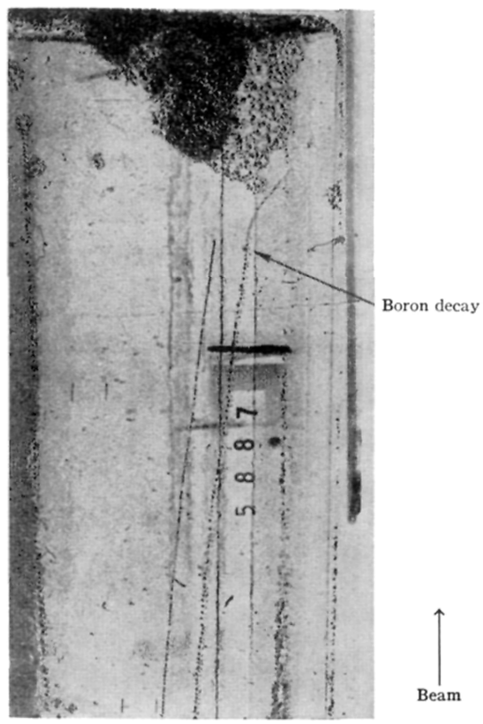

FIG. 4. T view of a boron event with double flash. The electron track is clearly of an age intermediate between the μ_1 and μ_2 tracks.

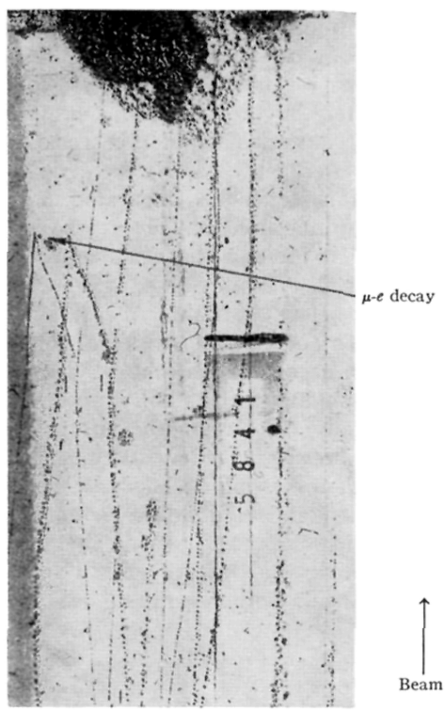

FIG. 5. T view of a μ-e decay with double flash.