Deviations from the $\Delta T=0$ Isotopic Spin Selection Rule in Fermi Transitions

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Experimental deviations from the $\Delta T=0$ isotopic spin selection rule have been observed experimentally in $J\rightarrow J$ beta transitions. In the theory of a vector interaction with a conserved current these deviations have to be explained only in terms of isotopic spin impurities, while in the conventional theory exchange mesonic currents may also induce Fermi transitions with $\Delta T\neq 0$. In this paper an attempt is made to estimate the contribution of the isotopic spin impurities arising from the Coulomb interaction between the protons. The j-j coupling shell model is used to calculate the relevant Coulomb matrix elements. When all the nucleons outside the core are in the same orbit the main contribution comes from the Coulomb interaction between the protons outside the core. A comparison between the empirical Fermi matrix element M_F and the calculated one is performed in the case of Mn^{52}, Sc^{44}, and Na^{24}. The two quantities agree fairly well for the manganese-52. No such an agreement is found in the two other cases; the predicted M_F being too large for the sodium-24, too small for the scandium-44 at least by a factor ten. This discrepancy may reflect the inadequacy of the j-j shell model to describe the Coulomb effects or the presence of mesonic effects. More experiments are needed to make a choice between these two possibilities.

I. INTRODUCTION

THE nonconservation of parity in β decay has made possible a direct determination of the nuclear Fermi matrix element in $J\rightarrow J$ transitions. The formulas for the β asymmetry from polarized nuclei and the β-γ (circularly polarized) angular correlation contain interference terms between the Fermi and Gamow-Teller amplitudes. Using the values of the β-decay coupling constants taken from the study of $0^{(+)}\rightarrow 0^{(+)}$ transitions[1] and the neutron decay experiments,[2] it is possible to obtain the ratio between the Fermi and Gamow-Teller matrix elements. The absolute value of the Fermi matrix element is derived by comparison with the experimental ft value. Such a determination has been performed for transitions between nuclei belonging to different isotopic spin multiplets. It has been pointed out by several authors[3] that a careful analysis of the experimental data should give some information about the mesonic effects in β decay. The theory of a vector interaction with a conserved current[4] leads to the following expressions for the Fermi matrix element

$$\left\langle f\left|\int\left\{\bar{\psi}\tau_{(+)}\psi+i\left[\phi^*\rho_{(+)}\frac{\partial\phi}{\partial t}-\frac{\partial\phi^*}{\partial t}\rho_{(+)}\phi\right]\right\}d^3x\right|i\right\rangle,$$

while in the conventional Fermi theory the corresponding expression is: $\langle f|\int\bar{\psi}\tau_{(+)}\psi d^3x|i\rangle$. In other words the Fermi matrix element is in the first case just the matrix element of the total isotopic spin operator $T_{(+)}$ and in the second case the nucleon isotopic spin operator $\mathcal{T}_{(+)}$, between eigenstates of the total isotopic spin. No transition with $\Delta T\neq 0$ should occur in the theory with a conserved current but in the conventional theory exchange currents inside the nucleus may induce Fermi transitions with $\Delta T\neq 0$. (If the nucleus is described as a collection of independent clothed nucleons the selection rule remains valid in both theories.)

In practice the initial and final states of the β transition are not exact eigenstates of the total isotopic spin, since electromagnetic interactions between nucleons destroy the validity of the isotopic spin quantum number. Let us consider for instance a β transition from a state of isotopic spin T to a state of isotopic spin $T+1$. The initial state should contain a small admixture of $T+1$ state which gives rise to a nonvanishing Fermi matrix element. In this paper an attempt is made to estimate the contribution of the isotopic impurities induced by the Coulomb interaction between nucleons to the Fermi matrix element of $J\rightarrow J(\Delta T=\pm 1)$ allowed transitions. Our purpose is to see whether the observed deviations from the isotopic spin selection rule can be explained in terms of Coulomb effects. Unfortunately since we have used a specific nuclear model (the j-j coupling shell model) the results remain somewhat ambiguous, although there is an indication that the results do not depend strongly upon the coupling scheme. In a simple case we have made a calculation on the basis of an L-S coupling and a j-j coupling. The two coupling schemes lead practically to the same result (Appendix).

We have compared our estimate of the Fermi matrix element induced by the isotopic spin impurities and the experimental data in the β decay of Mn^{52}, Sc^{44} and Na^{24}. For the Mn^{52} the computed value and the experimental[5] one seems to agree fairly well. In the two

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[1] J. B. Gerhart, Phys. Rev. **109**, 807 (1958); C. Vander Leun, thesis, Rijkuniversiteit te Utrecht, 1958 (unpublished).

[2] M. T. Burgy, V. E. Krohn, T. B. Novey, G. R. Ringo, and V. L. Telegdi, Phys. Rev. **110**, 1214 (1958); A. N. Sosnovskij, P. E. Spivak, Yu. A. Prokofiev, I. E. Kulikov, and Yu. P. Drobynin, *1958 Annual International Conference on High-Energy Physics at CERN*, edited by B. Ferretti (CERN Scientific Information Service, Geneva, 1958).

[3] E. P. Wigner, Proceedings of the R. Welch Foundation Conference on Chemical Research, November, 1957 (unpublished); J. Bernstein and R. Lewis, Phys. Rev. **112**, 232 (1958).

[4] S. S. Gershtein and J. B. Zeldovich, Zhur. Eksptl. i Teoret. Fiz **29**, 698 (1955) [translation: Soviet Physics-JETP **2**, 576 (1957)]; R. P. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1958).

[5] E. Ambler, R. W. Hayward, D. D. Hopes, and R. P. Hudson, Phys. Rev. **110**, 787 (1958).

other cases no such an agreement is found. For Sc^{44} the experiment of Boehm and Wapstra[6] gives a rather large Fermi matrix element of about 3×10^{-2}. Our theoretical estimate gives a value smaller than 10^{-3}. The situation seems to be reversed in Na^{24}. Three of the four experiments[7] performed on Na^{24} are consistent only with a very small Fermi matrix element ($M_F<10^{-3}$), while the theory predicts a number of the order of 10^{-2}. We believe that it is still premature to conclude from this analysis that Coulomb effects alone cannot explain the deviations from the $\Delta T=0$ selection rule. It is possible that the discrepancy reflects simply the inadequacy of our nuclear model to describe the Coulomb effects. In any case new experiments are needed so that the general trend of the deviations may be compared with the theoretical predictions.

II. GENERAL THEORY

We begin with some general considerations. Let us consider the $\beta^{(+)}$ decay of a nucleus of spin J and isotopic spin T into a nucleus of same spin J but isotopic spin $T+1$. Under the influence of a charge-dependent interaction the initial and final states $|\alpha TT_\zeta\rangle$ and $|\alpha' T+1T_\zeta\rangle$ are perturbed:†

$$\begin{aligned}|i\rangle&=a_{\alpha T}(\alpha T)\,|\alpha TT_\zeta\rangle\\&\quad+a_{\alpha T}(\alpha' T+1)\,|\alpha' T+1T_\zeta\rangle+\cdots,\\|f\rangle&=a_{\alpha' T+1}(\alpha' T+1)\,|\alpha' T+1T_\zeta+1\rangle\\&\quad+a_{\alpha' T+1}(\alpha'' T+2)\,|\alpha'' T+2T_\zeta+1\rangle+\cdots.\end{aligned}\tag{1}$$

The indices α, α', α'' represent the sets of quantum numbers necessary to get a complete characterization of the states of isotopic spin T, $T+1$, $T+2$, $\cdots$. Neglecting the second order terms in $a_{\alpha T}$ the contribution of the isotopic spin impurities to the Fermi matrix element is given by

$$\begin{aligned}M_F&=a_{\alpha T}(\alpha' T+1)\langle\alpha' T+1\;T_\zeta+1\,|\,T_{(+)}\,|\,\alpha' T+1T_\zeta\rangle\\&=a_{\alpha T}(\alpha' T+1)[(T+1-T_\zeta)(T+2+T_\zeta)]^{\frac{1}{2}}.\end{aligned}\tag{2}$$

The amplitude $a_{\alpha T}(\alpha' T+1)$ is given in the first order perturbation theory by

$$a_{\alpha T}(\alpha' T+1)=-\frac{\langle\alpha' T+1T_\zeta\,|\,H_1\,|\,\alpha TT_\zeta\rangle}{E(\alpha' T+1)-E(\alpha T)}.\tag{3}$$

In the above formula H_1 is the charge-dependent perturbation, $E(\alpha' T+1)$, $E(\alpha T)$ are the unperturbed energies. Let us first consider the energy denominator. In first approximation the difference between the unperturbed energies which appears in the denominator of (3) is equal to the difference between the actual energies $E(\alpha' T+1T_\zeta)-E(\alpha TT_\zeta)$. The latter difference can be written as follows:

$$\begin{aligned}&E(\alpha' T+1T_\zeta)-E(\alpha TT_\zeta)\\&\quad=E(\alpha' T+1T_\zeta)-E(\alpha' T+1\;T_\zeta+1)\\&\qquad-[E(\alpha TT_\zeta)-E(\alpha' T+1\;T_\zeta+1)].\end{aligned}\tag{4}$$

The first difference is just the energy separation inside the multiplet $(\alpha T+1)$. This quantity is given at least with a ten percent accuracy by the simple formula:

$$\begin{aligned}&E(\alpha' T+1T_\zeta)-E(T+1T_\zeta+1)\\&\quad=(3e^2/5R)(A-2T_\zeta-2)-(m_n-m_p)c^2,\end{aligned}\tag{5}$$

with $R=1.3A^{\frac{1}{3}}\times10^{-13}$ cm. The second difference is the energy available in the transition $E_\beta+m_ec^2$, where E_β is the maximum kinetic energy of the positron which is in general known from experiment.

We shall sketch the computation of the matrix element $M=\langle\alpha' T+1T_\zeta\,|\,H_1\,|\,\alpha TT_\zeta\rangle$. This will be done in the framework of the j-j coupling shell model. We shall restrict ourselves to the charge dependent perturbation coming from the electrostatic interaction between the protons, so that H_1 is just:

$$H_1=\sum_{i<j}^{A}\tfrac{1}{4}(1-\tau_{\zeta i})(1-\tau_{\zeta j})\frac{e^2}{r_{ij}}-\frac{3e^2}{5R}Z(Z-1).\tag{6}$$

By subtracting $(3e^2/R)Z(Z-1)$, we have practically removed from the Coulomb interaction its diagonal elements. Without this subtraction the perturbations series for $a_{\alpha T}(\alpha' T+1)$ are not always convergent. The wave function of the nucleus will be written as the product of the wave function of the core and the wave function of the extra core nucleons. Unlike the case of the $\Delta T=0$ transitions[8] the core impurities have practically no influence on the Fermi matrix element.

Following MacDonald[9] we write the Coulomb perturbation acting on the k nucleons outside the core as a sum of two terms

$$\begin{aligned}&\mathcal{C}=\mathcal{U}+\mathcal{C}^{(k)},\\&\mathcal{U}=\sum_{i=1}^{i=k}\tfrac{1}{2}(1-\tau_{\zeta i})V(r_i),\\&\mathcal{C}^{(k)}=\sum_{i<j}^{k}\tfrac{1}{4}(1-\tau_{\zeta i})(1-\tau_{\zeta j})\frac{e^2}{r_{ij}}.\end{aligned}\tag{7}$$

In the first term $V(r)$ is an effective potential which describes the nonexchange electrostatic interaction of one nucleon outside the closed shell with all the protons of the core. The second term $\mathcal{C}^{(k)}$ is the mutual electrostatic interaction of the extra-core nucleons.

[6] F. Boehm and A. H. Wapstra, Phys. Rev. **109**, 456 (1958).

[7] H. Schopper, Phil. Mag. **2**, 710 (1957); R. M. Steffen and P. Alexander, *Proceedings of the Rehovoth Conference on Nuclear Structure*, edited by H. J. Lipkin (North Holland Publishing Company, Amsterdam, 1958), p. 419; F. Boehm and A. H. Wapstra, Phys. Rev. **109**, 456 (1958); T. Mayer-Kuckuk and R. Nierhaus, Z. Physik **154**, 383 (1959).

† In the following formula it is assumed that T_ζ is positive and that the empiric rule $T=|T_\zeta|$ is valid for the initial state.

[8] W. M. MacDonald, Phys. Rev. **101**, 271 (1956).

[9] W. M. MacDonald, Phys. Rev. **110**, 1420 (1958).

(a) Effect of Electrostatic Interaction Between the Protons Outside the Core

In our analysis of the experimental data we will have to deal with four-hole configurations. The prescription given by Visscher and Ferrell[10] is to use a four-particle wave function with the same value for T and T_ζ and to make in $\mathcal{C}^{(k)}$ the substitution:

$$(1-\tau_{\zeta i})(1-\tau_{\zeta j}) \rightarrow (1+\tau_{\zeta i})(1+\tau_{\zeta j}).$$

The angular momentum J and the isotopic spin are in general insufficient to give a complete characterization of a four-particle configuration. We have used the seniority number s which is defined as the number of particles left after all antisymmetric pairs with $T=1$ and $J=0$ have been removed from the state. At least for short-range nuclear forces the seniority number is approximately a good quantum number.

In our calculations we have chosen for the final and initial states those of lowest seniority number ($s=2$). With this assumption and using the recoupling coefficients of four angular momenta (9_j symbols), the following formula was obtained:

$$M=\left\langle j^4 s=2 J J_z T+1 T \left| \sum_{i<j} \frac{e^2}{r_{ij}} \tfrac{1}{4}(1+\tau_{\zeta i})(1+\tau_{\zeta j}) \right| j^4 s=2 J J_z T T \right\rangle$$

$$=C(T,j)[\langle j^2 J J_z T=T_\zeta=1 \,|\, (e^2/r_{12}) \,|\, j^2 J J_z T=T_\zeta=1\rangle - \langle j^2 J=0\ T=T_\zeta=1 \,|\, (e^2/r_{12}) \,|\, j^2 J=0\ T=T_\zeta=1\rangle]. \quad (8)$$

The formula is valid only for even J and

$$C(T,j)=\tfrac{1}{2}[(2j-3)/(2j+1)]^{\frac{1}{2}} \quad \text{for} \quad T=1$$
$$C(T,j)=(1/\sqrt{6})[(2j+3)/(2j+1)]^{\frac{1}{2}} \quad \text{for} \quad T=0. \quad (9)$$

A method of computing the matrix element of a central two-body interaction between two particle states, is given in the book of Edmonds.[11] The result is expressed in terms of the familiar Slater integral $F^{(k)}$. The Slater integral $F^{(0)}$, which arises from the part of r_{12}^{-1} acting like a central potential, does not appear in the expansion of the matrix element M. This fact, already noted by MacDonald[8] will be discussed in detail in the next paragraph.

(b) Effect of the Electrostatic Potential of the Core

Let us proceed to the proof of the following rule: The electrostatic potential of the core cannot induce Fermi transitions with $\Delta T \neq 0$ as long as the extra core nucleons are all in the same orbit. This statement is rigorously true to the first order in the perturbation of Coulomb potential of the core and only approximately true to higher orders.

Let us expand the k nucleons wave functions in term of Slater determinants:

$$|(nl)^k \alpha T\rangle = \sum_{l\mu\sigma\tau} \langle \mu_1\sigma_1\tau_1 \cdots \mu_k\sigma_k\tau_k | \alpha T\rangle$$
$$\times\{(1/\sqrt{k!}) \sum_P \epsilon_P P[\Psi_{nl}{}^{\mu_1\sigma_1\tau_1}(1)\cdots\psi_{nl}{}^{\mu_k\sigma_k\tau_k}(k)]\}. \quad (10)$$

In this formula $\psi_{nl}{}^{\mu\sigma\tau}$ is the wave function a nucleon in a state of orbital momentum, spin and isotopic spin specified by the numbers $\mu\sigma\tau(\sigma, \tau=\pm 1)$; P stands for any permutation of the k particles and ϵ_P is the signature of P. Let us consider the matrix element of the operator $\mathcal{U}$ defined by

$$\mathcal{U}=\sum_{i=1}^{i=k} \frac{1-\tau_{\zeta i}}{2} V(r_i)$$

between the states $|(nl)^k \alpha T T_\zeta\rangle$ and $|(nl)^k \alpha' T+1 T_\zeta\rangle$ of the configuration $(nl)^k$. Writing the two wave functions in the form (10), one sees immediately that $\mathcal{U}$ has the same matrix as the operator $W^{(1)}$ defined by

$$W^{(1)}=\sum_i \frac{1-\tau_{\zeta i}}{2} \langle nl|\, V(r)\,|nl\rangle = \langle nl|\, V(r)\,|nl\rangle \left(\frac{k}{2}-T_\zeta\right).$$

Hence follows the first part of our rule.

Let us look next into the second order terms. Two kinds of terms have to be considered: the quadratic terms in $a_{\alpha T}(\gamma T+1)\ a_{\alpha' T+1}(\gamma T+1)$ and the second order contribution to the amplitude $a_{\alpha T}(\alpha' T+1)$. However, if the final and initial states are those of T_ζ maximum, the quadratic terms do vanish. The argument goes as follows: the only states of isotopic spin $T+1$ that the electrostatic potential of the core can mix to the first order with $|(nl)^k \alpha T T_\zeta\rangle$ are the states $|(nl)^{k-1}(n+ql)_\gamma\rangle$ obtained by promoting a proton from the orbit (nl) into the orbit $(n+ql)$. These states can always be written as linear combinations of the states obtained by a vector coupling of a state $|(nl)^{k-1}\alpha'' T'' T_\zeta''\rangle$ belonging to the configuration $(nl)^{k-1}$ with a single proton state in the orbit $(n+ql)$. If $T_\zeta=T$ one must have $T''=T_\zeta''=T+\frac{1}{2}$. Similarly the states $|(nl)^{k-1} \times (n+ql) T+1\rangle$ mixed with $|(nl)^k \alpha' T+1\ T+1\rangle$ are built from the states of the configuration $(nl)^{k-1}$ having an isotopic spin $T''=T+\frac{3}{2}$. Consequently no quadratic terms can appear.

Let us now consider the second order contribution to the amplitude $a_{\alpha T}(\alpha' T+1)$. The only possible intermediate states are the states $|(nl)^{k-1}(n+ql)\gamma\rangle$ already introduced. The amplitude $a_{\alpha T}{}^{(2)}(\alpha' T+1)$ is given by:

[10] W. Visscher and R. Ferrell, Phys. Rev. **107**, 781 (1957).
[11] A. Edmonds, *Angular Momentum in Quantum Mechanics* (Princeton University Press, Princeton, 1957), p. 114.

$$a_{\alpha T}{}^{(2)}(\alpha' T+1) = \sum_{q\gamma} \frac{\mathfrak{A}_\gamma{}^q[V(r)]}{[E(\alpha' T+1)-E(\alpha T)][E^q(\gamma)-E(\alpha T)]}, \quad (11)$$

where

$$\mathfrak{A}_\gamma{}^q[V(r)]=\langle (nl)^k\alpha' T+1T_\xi|\mathcal{U}|(nl)^{k-1}(n+ql)\gamma\rangle \times\langle (nl)^{k-1}(n+ql)\gamma|\mathcal{U}|(nl)^k\alpha T T_\xi\rangle. \quad (12)$$

We want to prove that $\sum_\gamma \mathfrak{A}_\gamma{}^q=0$ for each value of q. Physically this relation means that in the approximation where the spreading of the spectrum of the states $|(nl)^{k-1}(n+ql)\gamma\rangle$ is neglected compared to the energy separation between the shell (nl) and the shell $(n+ql)$ the second order contribution vanish. Writing again the states $|(nl)^{k-1}(n+ql)\gamma\rangle$ as linear combinations of the states obtained by a vector coupling of the states $|(nl)^{k-1}\alpha''T''\rangle$ with a one nucleon state of the shell $(n+ql)$ and introducing the fractional parentage coefficients of the states $|(nl)^kT\alpha\rangle$ and $|(nl)^k\alpha' T+1\rangle$ in the latter states, we get for the sum $\sum_\gamma \mathfrak{A}_\gamma{}^q$ the following expression

$$\begin{aligned}\sum_\gamma \mathfrak{A}_\gamma{}^q=&[\langle nl|V(r)|n+ql\rangle]^2\\ &\times k(\tfrac{1}{2}-\tfrac{1}{2}T+\tfrac{1}{2}T_\xi+\tfrac{1}{2}|TT_\xi)\\ &\times(\tfrac{1}{2}-\tfrac{1}{2}T+\tfrac{1}{2}T_\xi+\tfrac{1}{2}|T+1T_\xi+1)\\ &\times\sum_{\alpha''}((nl)^k\alpha' T+1\}(nl)^{k-1}\alpha'' T+\tfrac{1}{2};nl)\\ &\times(nl;T+\tfrac{1}{2}\alpha''(nl)^{k-1}\}(nl)^k\alpha T). \end{aligned} \quad (13)$$

On this expression one sees clearly that it is sufficient to prove that $\sum_{\gamma q}\mathfrak{A}_\gamma{}^q=0$.

Using a closure relation one can rewrite the latter sum in the following way:

$$\sum_{\gamma q}\mathfrak{A}_\gamma{}^q=\langle (nl)^k\alpha' T+1T_\xi|(\mathcal{U})^2|(nl)^k\alpha T T_\xi\rangle.$$

We remark that $(\mathcal{U})^2$ has the same matrix elements between two states belonging to the configuration $(nl)^k$ as the operator $W^{(2)}$ defined by the following equation:

$$\begin{aligned}W^{(2)}=&\langle n+ql|V^2(r)|nl\rangle\sum_{i=1}^{i=k}\left(\frac{1-\tau_{\xi i}}{2}\right)^2\\ &+2\langle n+ql|V(r)|nl\rangle^2\sum_{i<j}\frac{(1-\tau_{\xi i})(1-\tau_{\xi j})}{4}.\end{aligned} \quad (14)$$

It is easy to write $W^{(2)}$ as a polynomial of second degree in T_ξ:

$$\begin{aligned}W^{(2)}=&\langle n+ql|V^2(r)|nl\rangle(\tfrac{1}{2}k-T_\xi)\\ &+2(\langle n+ql|V(r)|nl\rangle)^2\\ &\times\left[\frac{k(k-2)}{8}-\frac{k-1}{2}T_\xi+\tfrac{1}{2}T_\xi{}^2\right].\end{aligned} \quad (15)$$

From the above equation it follows immediately that:

$$\sum_{q\gamma}\mathfrak{A}_\gamma{}^q=0.$$

It is possible to prove that $a_{\alpha T}{}^{(2)}(\alpha' T+1)$ vanishes when the effect of the residual interaction is neglected in the intermediate states without any explicit calculation.

Let us rewrite $a_{\alpha T}{}^{(2)}(\alpha' T+1)$ in the following way:

$$a_{\alpha T}{}^{(2)}=\frac{1}{E(\alpha T)-E(\alpha' T+1)} \times\left\langle (nl)^k\alpha' T+1\left|\mathcal{U}\frac{1}{E(\alpha T)-H}\mathcal{U}\right|(nl)^k\alpha T\right\rangle, \quad (16)$$

where H is the charge-independent Hamiltonian. (Since the state $|(nl)^k\alpha T\rangle$ is not a possible intermediate state no special care is needed for the operator $1/[E(\alpha T)-H]$.) We consider next the operator P which describes the permutation of all the nucleons variables $nl\mu\sigma\tau$ and decompose it in the following way

$$P=P_{\mu\sigma}P_{nl\tau},$$

where $P_{\mu\sigma}$ acts on the variables $\mu\sigma$ and $P_{nl\tau}$ on the variables $nl\tau$ only. We recall that the wave function $|(nl)^k\alpha T\rangle$ belongs to the irreducible representation $\mathbf{D}^{(1/2k-T)}(P)$ of the symmetric group with regard to the operator P_τ which interchanges the isotopic spin coordinates.[12] Now we remark that since the k nucleons are in the same orbit P_{nl} acts like the unit operator. Therefore the state $|(nl)^k\alpha T\rangle$ belongs to the irreducible representation $\overline{\mathbf{D}}^{(k/2-T)}$ associated with $\mathbf{D}^{(k/2-T)}$ with regard to the operator $P_{\mu\sigma}$.[12] Similarly $|(nl)^k\alpha' T+1T_\xi\rangle$ belongs to the representation $\overline{\mathbf{D}}^{(k/2-T-1)}$ of the operators $P_{\mu\sigma}$. If we neglect in H the residual interaction, the operator $\mathcal{U}\{1/[E(\alpha T)-H]\}\mathcal{U}$ does not act on the variable $\mu\sigma$. (We forget about the spin-orbit potential since this proof can be easily adapted to the case of j-j coupling.) Consequently $a_{\alpha T}{}^{(2)}(\alpha T+1)$ which is the scalar product of two wave functions belonging to inequivalent representations of the group of operators $P_{\mu\sigma}$, do vanish. The extension of the proof to any order is straightforward.

In practice the energy levels of the intermediate states are spread out. Let us define $\gamma_0, \gamma_1, \cdots, \gamma_p$ so that $E^q(\gamma_0)<E^q(\gamma_1)\cdots<E^q(\gamma_p)$. Using the relation $\sum_\gamma \mathfrak{A}_\gamma{}^q=0$, we rewrite $a_{\alpha T}{}^{(2)}(\alpha' T+1)$ in the following way:

$$a_{\alpha T}{}^{(2)}(\alpha' T+1)=-\frac{1}{E(\alpha' T+1)-E(\alpha T)}\sum_{q\gamma}\mathfrak{A}_{\gamma_i}{}^q\lambda_i{}^q, \quad (17)$$

with

$$\lambda_i{}^q=\frac{E^q(\gamma_i)-E^q(\gamma_0)}{E^q(\gamma_0)-E(\alpha T)}. \quad (18)$$

From this formula it appears that $a_{\alpha T}{}^{(2)}$ and more precisely its sign is very sensitive to the ordering of the spectrum. It is possible to get an idea of the order

[12] E. P. Wigner, *Group Theory* (Academic Press, Inc., New York, 1959), p. 258.

TABLE I. Lists of the quantities entering in the evaluation of amplitude of the state $T=2$ mixed with the state $T=1$ in Sc^{44}.

$\alpha''=J_3$	7/2	5/2	3/2	11/2	9/2
$A(J_3)$	$-\sqrt{2}/3$	$11\sqrt{2}/80$	$9\sqrt{2}/280$	$5\sqrt{2}/40$	$13\sqrt{2}/336$
$E(J_3)-E(7/2)$ Mev	0	0.24	0.98	1.74	1.82
$\lambda_{J_3}=[E(J_3)-E(7/2)]/2\hbar\omega$	0	0.01	0.042	0.075	0.08

of magnitude of $a_{\alpha T}{}^{(2)}(\alpha' T+1)$ in the framework of the j-j coupling model if one makes the following assumptions:

(a) The residual interaction of the $(n+1lj)$ protons with (nlj) nucleons is weak compared with the interaction of the (nlj) nucleons with each other. In that case the states $|(nlj)^{k-1}(n+qlj)\gamma\rangle$ are obtained by the vector coupling of any state $|(nlj)^{k-1}\alpha'T''\rangle$ of the configuration $(nlj)^{k-1}$ with a single nucleon state of the subshell $(n+qlj)$. (Actually only the states with $T''=T+\frac{1}{2}$ do occur in the expression of $a_{\alpha T}{}^{(2)}(\alpha' T+1)$.) The set of quantum numbers γ is now composed of the quantum numbers $\alpha''T''$ labelling the states of the configuration $(nlj)^k$ combined with the total angular momentum and the isotopic spin. The differences between the energies $E(\gamma)$ will be equal to the differences between the energies $E(\alpha''T'')$ of the corresponding states of the configuration $(nlj)^{k-1}$.

(b) The energy difference $E^1(\gamma_0)-E(\alpha T)$ can be replaced by the energy separation $2\hbar\omega$ between the shell $(n+1lj)$ and (nlj) in a harmonic oscillator well.

(c) Only intermediate states obtained by promoting a proton from the shell (nlj) to the shell $(n+1lj)$ have to be considered.

Using the assumption (a), it is possible to derive the following expression for $\mathfrak{A}_\gamma{}^q$:

$$\mathfrak{A}_\gamma{}^q=(\langle n+ql|V(r)|nl\rangle)^2 A(\alpha''), \tag{20}$$

with

$$\begin{aligned}A(\alpha'')=k(&T+\tfrac{1}{2}\ T_\zeta+\tfrac{1}{2}\ \tfrac{1}{2}-\tfrac{1}{2}|TT_\zeta)\\ &\times(T+\tfrac{1}{2}\ T_\zeta+\tfrac{1}{2}\ \tfrac{1}{2}-\tfrac{1}{2}|T+1T_\zeta)\\ &\times((nlj)^k\alpha' T+1\}(nlj)^{k-1}\alpha'';nlj)\\ &\times(nlj;\alpha''(nlj)^{k-1}\}(nlj)^k\alpha T).\end{aligned}$$

As an example let us discuss the case of the $(1f_{7/2})^4$ configuration with $T=1$ $J=2$ which is assigned to the ground state of Sc^{44}. The quantum number α'' is the total angular momentum of the three $1f_{7/2}$ neutrons. In Table I we listed the different values of $A(\alpha''=J_3)$ and the corresponding energy differences $E(J_3 f_{7/2}{}^3)-E(jf_{7/2}{}^3)$ calculated by Lawson and Uretsky[13] from the experimental energy spectrum of the $(1f_{7/2})^2$ configuration. In the third row of the same table we give a set of values for the number λ_i obtained by the following formula:

$$\lambda_{J_3}=\frac{E(J_3 f_{7/2}{}^3)-E(\frac{7}{2}f_{7/2}{}^3)}{2\hbar\omega}. \tag{21}$$

We have taken the excitation energy $2\hbar\omega$ equal to the separation between the states $(n+1lj)$ and (nlj) in the harmonic oscillator well, ω having the value obtained by Talmi and Thieberger[14] from their analysis of the binding energy of light nuclei. We have used harmonic oscillator wave functions corresponding to the same ω to calculate the matrix element $\langle n+1l|V(r)|nl\rangle$. (We have approximated the effective Coulomb potential of the core by the potential of a uniform spherical charge distribution.) Using the numbers λ_i of Table I we get the following numerical value for $a_T{}^{(2)}(T+1)$

$$a_T{}^{(2)}(T+1)=3\times10^{-4}.$$

This number corresponds to a Fermi matrix element equal to:

$$M_F=2a_T{}^{(2)}(T+1)=6\times10^{-4}.$$

This number is smaller than the experimental value by a factor 50. One must keep in mind that there is a large amount of arbitrariness in some of the hypotheses we have used to get this number. First the interaction of the $2f_{7/2}$ proton with the $1f_{7/2}$ is not completely negligible compared with those of the $1f_{7/2}$ neutrons with each other. Secondly, although the harmonic oscillator well has been used successfully in the analysis of low-energy states, it will give probably a poor description of the excited states we are dealing with. However we hope that the number we have obtained has at least the correct order of magnitude.

III. ANALYSIS OF THE EXPERIMENTAL DATA

(a) Manganese-52

Ambler and co-workers[15] have made an experimental determination of the ratio between the Fermi and Gamow-Teller intensities in the positron decay of the $(J=6^{(+)}\ T=1)$ state of the manganese-52 to the $(J=6^{(+)}\ T=2)$ state of chromium-52, by combining the results of three independent experiments. Using the value of C_V/C_A taken from the neutron decay experiment[2] and the experimental ft value, we have derived an experimental value for Fermi matrix element

$$M_F{}^{ex}=0.0065\pm0.015.$$

The ground state of Mn^{52} is very likely to belong to the configuration $(f_{7/2})^{-4}$. We have computed the Fermi

[13] R. D. Lawson and J. L. Uretsky, Phys. Rev. **106**, 1369 (1956).

[14] I. Talmi and R. Thieberger, Phys. Rev. **103**, 718 (1956).

[15] There is another experiment on Mn^{52}: F. Boehm, Phys. Rev. **109**, 1018 (1958). His result is in contradiction with the more precise experiments of Ambler et al. He finds a ratio $M_F/M_{GT}<0$ while Ambler *et al.* give $M_F/M_{GT}>0$. A j-j coupling calculation of the ratio M_F/M_{GT} gives a positive sign.

matrix element $M_F{}^c$ induced by the electrostatic interaction of the extra core protons using the methods described in part II.

$$M_F{}^c=\frac{1}{\sqrt{2}[E(2)-E(1)]}\times\left(\frac{20}{63}F^{(2)}+\frac{12}{77}F^{(4)}+\frac{10^3}{3^2\times11\times13^2}F^{(6)}\right). \quad (22)$$

Using the formula (5) we have found $E(2)-E(1)=6.5$ Mev. The Slater integrals have been calculated using harmonic oscillator wave functions. We have expressed the results in terms of the parameter $e^2(\nu/\pi)^{\frac{1}{2}}$ introduced by Talmi; taking the value of $e^2(\nu/\pi)^{\frac{1}{2}}$ from the work of Talmi and Thieberger[13] on nuclear binding energies we have obtained a number very close to the experimental value

$$M_F{}^c=0.010.$$

(b) Scandium-44

Boehm and Wapstra[6] have measured the β-γ circular polarization correlation in the $\beta^{(+)}$ transition of the ground state $T=1$ $J=2^{(+)}$ of Sc^{44} to the $J=2^{(+)}$ $T=2$ excited state of Ca^{44}. We have deduced from the result of their experiment the following value for

$$M_F=0.035\pm0.010.$$

If we assume a $(1f_{7/2})^4$ configuration for the ground state of Sc^{44} the only possible Coulomb effects are those coming from the electrostatic potential of the core. From the considerations of part II a very small Fermi matrix element is expected: $M_F\simeq6\times10^{-4}$. Although our calculation is somewhat questionable we believe that there is a serious discrepancy. A possible way to reduce the discrepancy is to assume that there is a large admixture of $(1f_{7/2})^3(2p_{3/2})$ but unlike the case of nuclear moments, the effect will be proportional to the square of the coefficient of the admixed states. If we assume that the configuration is entirely $(1f_{7/2})^3(2p_{3/2})$, one can show easily that the maximum $M_F{}^{\max}$ of the Fermi matrix element induced by Coulomb effect is given by:

$$M_F{}^{\max}=\frac{\sqrt{3}}{2}\frac{1}{E(2)-E(1)}\times(\langle2p|V(r)|2p\rangle-\langle1f|V(r)|1f\rangle). \quad (23)$$

Using again harmonic oscillator wave functions, we have found $M_F{}^{\max}\simeq0.017$. One should remark that there is no Coulomb matrix element between the $T=2$ states of the configuration $(1f_{7/2})^3(2p_{3/2})$ and the $T=1$ states obtained by coupling a one particle state $2p_{3/2}$ with a $T=\frac{1}{2}$ state of the $(1f_{7/2})^3$ configuration. Since these energetically favored states will be present with large amplitudes the actual value of M_F is expected to be far from its maximum value.

TABLE II. Comparison of the experimental values of the anisotropy coefficient A with the theoretical predictions in the decay of Na^{24}.

	A
Boehm and Wapstra	0.07 ± 0.04
Steffen and Alexander	0.05 ± 0.04
Mayer Kuckuk and Nierhaus	0.12 ± 0.03
Schopper	-0.07 ± 0.05
Pure Gamow-Teller	0.08
Theoretical $M_F/M_{GT}>0$	0.23
Theoretical $M_F/M_{GT}<0$	-0.07

(c) Sodium-24

Several groups[7] have investigated the β-γ circular polarization in the $\beta^{(-)}$ decay of the $T=1$ $J=4^{(+)}$ state of Na^{24} to the $T=0$ $J=4^{(+)}$ state of Mg^{24}. In Table II we have listed the different experimental values of the anisotropy coefficient A which is given in the allowed approximation by

$$A=\frac{1}{1+y^2}\left(\frac{1}{12}+\frac{\sqrt{5}}{3}y\right),$$

where

$$y=\left|\frac{C_V}{C_A}\right|\times\frac{M_F}{M_{GT}}.$$

The Coulomb interaction between the extra core protons introduces a $T=1$ impurity in the $T=0$ state of Mg^{24}. Assuming a $(1d_{5/2})^{-4}$ configuration, and using the methods of part II we have found the following expression for the Fermi matrix element induced by Coulomb effect:

$$M_F=\frac{2}{3}\frac{1}{E(1)-E(0)}\left(\frac{12}{35}F^{(2)}+\frac{1}{9}F^{(4)}\right). \quad (25)$$

Again evaluating the Slater integrals with harmonic oscillator wave functions, we have obtained the following number $M_F=0.013$. Three of the experiments quoted seem to indicate a very small Fermi matrix element. If we calculate A using our theoretical value of M_F, the experimental M_{GT} and the sign of M_F/M_{GT} suggested by a j-j coupling calculation, we found a value of A which is in contradiction with the four experiments quoted. With the opposite choice for the sign of the new value of A agrees with the experiment of Schopper but remains in contradiction with the three others which are consistent only with a Fermi matrix element smaller than 10^{-3}.

CONCLUSION

The preceding analysis leads us to the following observations: Coulombs effects, calculated with the nuclear wave functions given by the j-j coupling shell model, are unable to explain the experimental deviations from the $\Delta T=0$ selection rule in the β decay of Na^{24}

and Sc^{44} while a reasonable agreement is obtained for Mn^{52}. Two interpretations are possible:‡

(a) The j-j shell model fails to describe correctly the Coulomb effects in Na^{24} and Sc^{44}.

(b) Exchange mesonic effects do induce Fermi transitions with $\Delta T=\pm 1$.

New experiments are necessary to decide between these two alternatives. An interesting case will be the decay of the $T=\frac{3}{2}$ $J=\frac{5}{2}^+$ state of O^{19} to the $T=\frac{1}{2}$ $J=\frac{5}{2}^+$ state of F^{19} since in that case a more reliable calculation of the Coulomb matrix element is perhaps possible. It will be also instructive to compare the general trend of the deviations with the rule derived in part II stating that the electrostatic potential of the core cannot induce Fermi transitions with $\Delta T=\pm 1$ as long as the extra-core nucleons are in the same orbits.

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APPENDIX

We shall compare the results of two calculations of the Coulomb matrix element between a $T=\frac{1}{2}$ $J=\frac{3}{2}$ state and $T=\frac{3}{2}$ $J=\frac{3}{2}$ state, both belonging to the configuration $(p)^5$. The first of these will be based on the j-j, the second on the L-S model. In the j-j coupling model we have to deal with three hole configurations of the $p_{\frac{3}{2}}$ shell. No quantum number other than J,T are needed. We have found for the Coulomb matrix element:

$$p_{jj}=\langle(p_{\frac{3}{2}})^{-3}J=\tfrac{3}{2}\ T=\tfrac{1}{2}|\sum_{i<j}\tfrac{1}{4}(1-\tau_i\tau_i)\times(1-\tau_i\tau_j)r_{ij}^{-1}|(p_{\frac{3}{2}})^{-3}J=T=\tfrac{3}{2}\rangle \quad (26)$$
$$=(4/15\sqrt{5})F^{(2)}.$$

In L-S coupling we shall label the two states with the usual quantum numbers LST and a partition $[\lambda]$ which describe the symmetry properties of the wave function. We shall compute the matrix element between two states characterized by $S=T=\frac{1}{2}$, $[\lambda]=[41]$ and $S=\frac{1}{2}$, $T=\frac{1}{2}$, $[\lambda]=[32]$. We have used the fractional parentage coefficients calculated by Elliott, Hope, and Jahn.[16] We get the following result:

$$p_{LS}=\langle[41]^{22}P|\sum_{i<j}\tfrac{1}{4}(1-\tau_i\tau_i)(1-\tau_i\tau_j)r_{ij}^{-1}|[32]^{24}P\rangle \quad (27)$$
$$=(3/36)(\tfrac{5}{2})^{\frac{1}{2}}F^{(2)}.$$

The two numbers p_{jj} and p_{LS} are surprisingly close to each other ($p_{jj}/p_{LS}\simeq 0.9$). This result may be considered as an indication that the Coulomb matrix elements do not depend critically on the coupling scheme.

‡ There is perhaps a third interpretation. It is to consider other charge dependent pertubation: for instance the effect of the mass splitting between the neutral and charge pions induces a difference between the neutron-neutron and proton-proud interaction which may lead to appreciable effects.

[16] J. P. Elliot, J. Hope, and H. A. Jahn, Phil. Trans. Roy. Soc. (London) **A246**, 241 (1953).