with

$$D_2=(\tfrac{2}{5})^{\frac{1}{2}}\left(\frac{d}{dr}-\frac{1}{r}\right),\quad D_1=0,\quad D_0=-\left(\frac{d}{dr}+\frac{2}{r}\right). \tag{A9}$$

Similarly,

$$[0\,1\,1\,p]=\sum_l \pi^{-1}\cdot 3^{\frac{1}{2}}(2l+1)(l\,1\,0\,0|1\,0)\times[W(\tfrac{3}{2}\,1\,\tfrac{1}{2}\,1,\tfrac{1}{2}\,l)]^2\times\int \phi e^{-\alpha Z m_\mu' r}j_l(qr)(D_l\phi)r^2dr. \tag{A10}$$

$$[1\,l\,1]=-\pi^{-1}\cdot 3[3(2l+1)]^{\frac{1}{2}}\times(l\,1\,0\,0|1\,0)\begin{Bmatrix}1 & l & 1\\ \frac{1}{2} & 1 & \frac{3}{2}\\ \frac{1}{2} & 1 & \frac{1}{2}\end{Bmatrix}\times\int\phi^2 e^{-\alpha Z m_\mu' r}j_l(qr)r^2dr. \tag{A11}$$

Integrating over radial coordinate, we have the results given in (68)–(73) of Sec. 10.

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Application of Dispersion Relations to K_{e3} and $K_{\mu3}$ Decays*†

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The decay modes, K_{e3} and $K_{\mu3}$, are studied by means of a dispersion relation. It is assumed that the fundamental couplings involved are the strong pion and K-meson couplings to the baryons and a weak four-field coupling connecting nucleon, hyperon, and the lepton pair. The baryon-antibaryon pair contribution to the absorptive part of the decay amplitude is expressed in terms of the imaginary part of the pion propagator in the same approximation. The decay rate is determined in terms of the various coupling constants and the quantity Z which renormalizes the pion propagator. Comparison with experiment is made for the case $g_\pi^2/g_K^2=15$. The results are consistent with a hyperon leptonic decay coupling constant an order of magnitude less than the beta-decay strength.

1. INTRODUCTION

It is not known whether four-Fermi forms are appropriate to represent the fundamental weak interactions. However, many authors[1] have suggested various weak couplings of the form $\mathcal{L}_{\text{weak}}=\lambda J_\mu J_\mu$ where J_μ consists of vector and axial-vector combinations of various baryon and lepton pairs. So far evidence for a universal coupling constant is found only in the strangeness conserving decays; present data on hyperon decays into a nucleon plus leptons indicate that if nucleon-hyperon terms are present in J_μ, they are present in a reduced amount.[2]

Aside from the leptonic hyperon decays the most direct evidence bearing on the strangeness nonconserving "current" comes from the $K^+\rightarrow\mu^++\nu$, $K^+\rightarrow\mu^++\nu+\pi^0$ and $K^+\rightarrow e^++\nu+\pi^0$ decay modes, and the similar K^- decay modes. The partial lifetimes for these processes are fairly well known, and if it were possible to connect them to the strength of a strangeness violating Fermi interaction one would have an indication of the consistency of such a form.

In a perturbation calculation the divergence problem renders this connection impossible. Either one must choose a numerical value for a cutoff or one must introduce counter-terms in the form of fundamental K decay couplings for each process. Nevertheless there is some hope that another calculation procedure could avoid this problem; that in this case the divergence is really a consequence of the perturbation expansion.

Recently Goldberger and Treiman[3] have used a dispersion relation approach to the similar problem of $\pi\rightarrow\mu+\nu$ decay. An answer for the decay rate was obtained in terms of the pi-nucleon and mu capture coupling constants. The same method is applicable to the $K\rightarrow\mu+\nu$ mode,[4] except that here the coupling constants involved are unknown.

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In the present work a similar method is applied to the $K^+ \to \mu^+ + \nu + \pi^0$ and $K^+ \to e^+ + \nu + \pi_0$ decay modes. It is hoped that such an analysis, together with the $K \to \mu + \nu$ analysis and whatever data emerge on leptonic hyperon decays, will serve to check the consistency of the four-field coupling as the basis for strange particle decays.

The procedure to be followed, while parallel to that of reference 3, will be stated in somewhat different terms. A dispersion relation will be conjectured. In this dispersion relation perturbation theory will be used to evaluate the absorptive part arising from baryon-antibaryon intermediate states. The calculation can be modified to the extent of introducing an exact pion-nucleon vertex function (as in reference 3). Whereas Goldberger and Treiman express their result in terms of an integral involving a complex nucleon-antinucleon phase shift, we prefer to relate our approximate answer directly to functions involved in a similar approximation to the pion propagation function.[5] In this way it will turn out that all unknown information can be expressed in terms of the renormalization constant for the pion propagator. This also will make the neglect of higher physical states more plausible. Subsidiary dispersion relations for the intermediate processes are avoided while Goldberger and Treiman's "damping due to the one pion state" is still obtained by a careful treatment of processes on the external pion line.

The strong K coupling will be assumed pseudoscalar (though the calculation may be done for scalar coupling). Now it is the vector (rather than the axial vector) part of the Fermi coupling of nucleon to hyperon which contributes to $K_{\mu 3}$ and K_{e3} decays. That is, the relevant weak interaction presumably looks like a sum of terms of the form,

$$\mathcal{L}_W = \lambda(\bar{\psi}_{\text{hyperon}}\gamma_\mu\psi_{\text{nucleon}}) \times (\bar{\psi}_{\text{lepton}}\gamma_\mu(1+i\gamma_5)\psi_{\text{lepton}}) + \text{H.c.} \quad (1)$$

In order to avoid for as long as possible the problem of what particle combinations and what factors should be inserted into (1) we shall discuss the dynamics first. To this end we abbreviate the sum of contributions $\bar{\psi}_{\text{hyperon}}\gamma_\mu\psi_{\text{nucleon}}$ to (1) by the symbol K_μ.

2. DISPERSION RELATION FOR $K_{\mu 3}$, K_{e3} DECAY

The transition probability per unit time may be written as

$$\Gamma_{K\to\pi\mu\nu} = (2\pi)^{-5} \int \frac{m_\mu}{2E_\mu E_\nu \omega_\pi \omega_K} \times \Big(\sum_{\text{spin}} |M|^2\Big)\delta^4(p)d^3p_\pi d^3p_\mu d^3p_\nu. \quad (2)$$

M is obtained from the reduction formulas,[6]

$$M = \bar{u}_\nu \gamma_\mu(1+i\gamma_5)u_\mu f_\mu[-(q-k)^2, q^2], \quad (3)$$

where

$$f_\mu[-(q-k)^2, q^2] = i(2\omega_K)^{\frac{1}{2}} \int e^{-iq\cdot x}\theta(-x) \times \langle 0|[J(0), K_\mu(x)]|K\rangle. \quad (4)$$

Here $K_\mu(x)$ is, as explained above, the baryon contribution to the strangeness nonconserving weak coupling. J equals $(\Box - \mu^2)\varphi_\pi$, again leaving out charge indices. k is the K particle four momentum, q is the sum of the μ and ν four momenta. Use has been made of the local Fermi coupling; this insures that p_μ and p_ν enter only in their sum. The variable, $(q-k)^2$ is the pion four-momentum squared. It is this that we shall allow to become unphysical. q^2 will be held fixed at its physical value (depending on the point in the decay energy spectrum). In reducing the time ordered product to a retarded commutator, use has been made of the fact that there exist no physical states with strangeness $+1$, number of nucleons equal to zero, and four momentum p such that $p^2 = q^2$ for the range of q defined by the real decay processes.

The quantity f_μ, defined by (4), is essentially a three-point or vertex function. f_μ may be written in terms of two invariant functions,

$$f_\mu(\xi,\eta) = k_\mu a(\xi,\eta) + q_\mu b(\xi,\eta), \quad (5)$$

with

$$\xi = -(q-k)^2, \quad \eta = q^2.$$

Now we conjecture a dispersion relation for the functions a and b, in the variable ξ. Perturbation theory supports this conjecture in a model in which the decay proceeds through the heavy fermions.[7] The relation of the external and intermediate masses is indeed such that, as anticipated from Eq. (3), the cut in the ξ plane will start from the first physical state connected by the strong interactions to the single pion. There being no direct coupling, $K \to \pi + \mu + \nu$, we shall assume dispersion relations without subtractions. That is, it is assumed that the $K_{\mu 3}$ rate is calculable in terms of a fundamental Fermi interaction strength and various strong couplings. The dispersion relations take the form

$$\text{Re}a(\xi,\eta) = \frac{P}{\pi}\int_{\xi_0}^{\infty} \frac{\text{Im}a(\xi',\eta)}{\xi'-\xi} \quad (6)$$

and similarly for $b(\xi,\eta)$. From (3) one obtains

$$\text{Im}f_\mu(\xi,\eta) = \pi(2\omega_K)^{\frac{1}{2}} \sum_n \delta^4(k-q-p_n) \times \langle 0|J(0)|n\rangle\langle n|K_\mu(0)|K\rangle, \quad (7)$$

where the states, n, include 3π, $N\bar{N}$, $Y\bar{Y}$, etc. We shall

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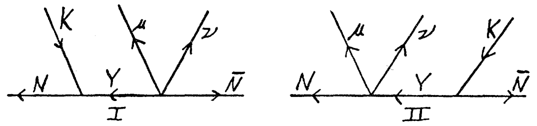

FIG. 1. Two kinds of contributions to $\langle N\bar{N}|K_\mu(0)|K\rangle$.

take all baryon masses equal and, following reference 3, investigate the contribution of the nucleon-antinucleon (or hyperon-antihyperon) state alone.

To find the imaginary parts of a and b in this approximation the two quantities needed are $\langle 0|J(0)|n\rangle$ and $\langle N\bar{N}|K_\mu(0)|K\rangle$. The first factor is the pion-nucleon vertex function and may be written

$$\langle 0|J(0)|N\bar{N}\rangle = gF[-(p_N+p_{\bar{N}})^2]\bar{u}_{\bar{N}}\gamma_5 u_N. \qquad (8)$$

In the summation, (7), $F[-(p_n+p_{\bar{n}})^2]$ may be replaced by $F(\xi)$. Goldberger and Treiman have discussed this function in further approximations. However we shall not need to know this form.

The second factor in (7) is the quantity $\langle N\bar{N}|K_\mu(0)|K\rangle$. This will be treated in lowest order, assuming the strangeness nonconserving interaction (1) and a K coupling to nucleon and hyperon. There are two dynamically different contributions to the factor, $\langle N\bar{N}|K_\mu(0)|K\rangle$, shown in Fig. 1. N and $\bar{N}$ are meant to stand for any baryon pair connected to the pion. For the moment we shall consider the contribution of one process of type I and one of type II separately. When these are introduced into the sum, (7), and all intermediate baryon pairs are included, some combination of type I and II contributions will appear, depending on the specific couplings used. The possibilities for this combination will be considered later.

The lowest order result is,

$$(2\omega_K)^{\frac{1}{2}}\langle N\bar{N}|K_\mu(0)|K\rangle = \lambda(M/E)g_K\bar{u}_N O_\mu u_{\bar{N}}, \qquad (9)$$

where

$$O_\mu{}^{\mathrm{I}} = \gamma_5\frac{\gamma\cdot(p_N-p_K)-M}{(p_N-p_K)^2+M^2}\gamma_\mu,$$

$$O_\mu{}^{\mathrm{II}} = \gamma_\mu\frac{\gamma\cdot(-p_{\bar{N}}+p_K)-M}{(p_{\bar{N}}-p_K)^2+M^2}\gamma_5.$$

M is the baryon mass and E is the baryon energy in the center of mass of the pair. Inserting these expressions and (8) into (7) we find

$$\mathrm{Im}f_\mu{}^{\mathrm{I}} = -\mathrm{Im}f_\mu{}^{\mathrm{II}} = (1/4\pi) \times (k_\mu/M)|p|\lambda g_K g F(\xi)\theta(\xi-4M^2), \qquad (10)$$

where

$$E=\tfrac{1}{2}\sqrt{\xi}, \quad p=[(\xi/4)-M^2]^{\frac{1}{2}}.$$

Terms of order η/M^2 compared to (10) have been neglected.

Instead of substituting these expressions directly into the dispersion relations we want to compare them to a function related to the pion propagator calculated in the same intermediate state approximation.

Defining

$$P(\xi) = i\int e^{-i(k-q)\cdot x}\theta(x_0)\langle 0|[J(x),J(0)]|0\rangle, \qquad (11)$$

we have

$$\mathrm{Im}P(\xi) = \pi\sum \delta^4(p_N+p_{\bar{N}}-k+q) \times\langle 0|J(0)|N\bar{N}\rangle\langle N\bar{N}|J(0)|0\rangle. \qquad (12)$$

All isotopic spin factors will be included later when the problem is considered of what states should be inserted for $N\bar{N}$ in (12). For the sake of consistency with the approximation to $\mathrm{Im}f_\mu$, we again put in the expression (8) for $\langle 0|J(0)|N\bar{N}\rangle$, but take the second factor in lowest order. Then one obtains,

$$\mathrm{Im}P(\xi) = -(g^2/2\pi)(\xi/4)(|p|/M)F(\xi)\theta(\xi-4M^2). \qquad (13)$$

From (5), (10), and (13) one sees that in the present approximation,

$$\mathrm{Im}b=0,$$

$$\frac{\mathrm{Im}a_{\mathrm{I}}(\xi)}{\mathrm{Im}P(\xi)} = -\frac{\mathrm{Im}a_{\mathrm{II}}(\xi)}{\mathrm{Im}P(\xi)} = \frac{-2\lambda g_K}{g_\pi}\xi. \qquad (14)$$

Now in the Goldberger and Treiman treatment of pion decay, an expression similar to (7) was derived and approximated by the nucleon pair intermediate state. Their term $\langle N\bar{N}|\bar{\psi}\gamma_\mu\gamma_5\psi|0\rangle$ was subjected to a further dispersion relation and approximated by a μ capture coupling constant (subtraction) plus a contribution from a single pion intermediary. This second (one pion) contribution supplies a damping in the result which is of critical importance. Its existence is evidently connected to the possibility of self-energy processes on the external pion line.

For the real external pion these contribute only a renormalization constant, which has already been taken into account (by the use of the renormalized field in the reduction formula and the renormalized coupling in $\langle N\bar{N}|J|0\rangle = g\bar{u}\gamma_5 u$). But for the unphysical momenta involved in the dispersion relation there are additional effects. Since our dispersion relation for $K_{\mu 3}$ decay involves unphysical pion momenta in exactly the same manner there is a similar phenomenon here. To obtain the damping effect (without subsidiary dispersion relations) we must follow a more accurate treatment of the external pion processes.

All our various expectation values of Heisenberg field operators may be expressed in terms of appropriate S-matrix elements, $\langle\alpha(in)|\beta(out)\rangle$. These in turn may be calculated by perturbation theory in the interaction representation. We shall denote modified matrix elements with a subscript P (for proper) to indicate the contribution from Feynman diagrams not containing a single intermediate pion line.

The external pion processes are separated explicitly

when the amplitude, f_μ, is written in the form,

$$f_\mu(\xi,\eta)=Z_3(-\xi+\mu^2)\Delta(-\xi)f_\mu'(\xi,\eta), \qquad (15)$$

where f_μ' is the contribution from the diagrams which contain no single pion intermediate lines. Here $\Delta(-\xi)$ is the renormalized pion propagator and f_μ' is given by (4) with the modification that when (4) is expanded in the interaction representation only proper contributions are counted. We write this

$$f_\mu'(\xi,\eta)=i(2\omega_K)^{\frac{1}{2}}\int e^{-iq\cdot x}\theta(-x) \times\langle 0|[J(0),J_\mu(x)]|K\rangle_P. \qquad (16)$$

From (15) we may infer the same analytic properties for f_μ' as for f_μ.

Just as the K-decay matrix element was written in terms of the proper contribution, the pion propagator itself is to be written in terms of proper contributions,

$$\Delta=\frac{1}{k^2+\mu^2}\times\frac{1}{1-(k^2+\mu^2)R(-k^2)}. \qquad (17)$$

$(k^2+\mu^2)^2R(-k^2)$ here is the proper meson self energy part and,

$$\mathrm{Im}R(-k^2)=Z_3^2(k^2+\mu^2)^{-2}\,\mathrm{Im}\,i\int e^{-ik\cdot x}\theta(x_0) \times\langle 0|[J(x),J(0)]|0\rangle_P. \qquad (18)$$

The function $f_\mu'(\xi,\eta)$ is to be divided as was f_μ;

$$f_\mu'(\xi,\eta)=k_\mu a'(\xi\eta)+q_\mu b'(\xi\eta). \qquad (19)$$

The imaginary parts of f_μ' and R will be calculated in the intermediate baryon pair approximation. In analogy to (7) and (12) one has,

$$\mathrm{Im}f_\mu'=\pi(2\omega_K)^{\frac{1}{2}}\sum\delta(p_N+p_{\bar N}-k+q) \times\langle 0|J(0)|N\bar N\rangle_P\langle N\bar N|K_\mu(0)|K\rangle_P, \qquad (20)$$

and

$$\mathrm{Im}R(\xi)=Z_3^2(-\xi+\mu^2)^2\sum\delta(p_N+p_{\bar N}-k+q) \times\langle 0|J(0)|N\bar N\rangle_P\langle N\bar N|J(0)|0\rangle_P. \qquad (21)$$

Evidently any improper contribution to the intermediate matrix elements would produce an improper contribution to f_μ'; hence the subscripts P in (20) and (21).

We shall concern ourselves only with the ratio of $\mathrm{Im}a'$ and $\mathrm{Im}b'$ to $\mathrm{Im}R$. Again the form of the first factor $\langle 0|J(0)|N\bar N\rangle_P$, common to all three quantities, $\mathrm{Im}a'$, $\mathrm{Im}b'$, and $\mathrm{Im}R$, is irrelevant. In (2) we again take lowest order (with renormalized couplings) for the factor $\langle N\bar N|j_\mu(0)|K\rangle_P$. In taking only a coupling constant for $\langle N\bar N|J(0)|0\rangle_P$ however we must remember that the processes on the outgoing pion line have been omitted. These would contribute a factor Z_3 to the amplitude $\langle N\bar N|J(0)|0\rangle$ (as well as a correction in form). Therefore we take,

$$\langle N\bar N|J(0)|0\rangle_P\cong Z_3^{-1}g\bar u_N\gamma_5 u_{\bar n}. \qquad (22)$$

This leads to the result,

$$\frac{\mathrm{Im}a_{\mathrm{I}}'}{\mathrm{Im}R}=-\frac{\mathrm{Im}a_{\mathrm{II}}'}{\mathrm{Im}R}=-2\frac{(-\xi+\mu^2)^2}{\xi}\frac{\lambda g_K}{g}. \qquad (23)$$

$$\mathrm{Im}b'=0.$$

This ratio, the relation [from (15) and (17)]

$$a(\xi,\eta)=\frac{1}{1+(\xi-\mu^2)R(\xi)}a'(\xi,\eta), \qquad (24)$$

and the dispersion relation for f, (6), will be sufficient to determine the $K_{\mu 3}$ decay rate in terms of the various coupling constants and the pion field renormalization constant.

Henceforth the pion mass, μ, will be set zero in all expressions for the imaginary parts of R and f', i.e., μ^2 is neglected compared to $4M^2$. Also the decay amplitude will be evaluated at zero pion mass. Equation (23) then becomes,

$$\begin{aligned}\mathrm{Im}a_{\mathrm{I}}'(\xi,\eta)&=-\mathrm{Im}a_{\mathrm{II}}'(\xi\eta)=C\xi\,\mathrm{Im}R(\xi\eta),\\ C&=-2\lambda g_K/g.\end{aligned} \qquad (25)$$

From (24) and the assumed dispersion relation (6) we have (with $\mu=0$)

$$a(0)=\frac{P}{\pi}\int_{4M^2}^{\infty}\frac{d\xi}{\xi} \times\frac{\mathrm{Im}a(\xi)[1+\xi\,\mathrm{Re}R(\xi)]-\xi\,\mathrm{Im}R(\xi)\,\mathrm{Re}a(\xi)}{|1+\xi R(\xi)|^2}. \qquad (26)$$

Substituting in (26) the two relations,

$$\mathrm{Re}R(\xi)=\frac{P}{\pi}\int d\xi'\frac{\mathrm{Im}R(\xi')}{\xi'-\xi}, \qquad (27)$$

$$\mathrm{Re}a'(\xi)=a'(0)+\frac{1}{\pi}\xi P\int\frac{\mathrm{Im}a'(\xi')}{\xi'(\xi'-\xi)}d\xi', \qquad (28)$$

and using (25) one obtains,

$$a(0)=\frac{1}{\pi}\int_{4M^2}^{\infty}\frac{d\xi}{\xi}\frac{C\xi\,\mathrm{Im}R(\xi)-\xi a(0)\,\mathrm{Im}R(\xi)}{|1+\xi R(\xi)|^2}. \qquad (29)$$

The use of a subtracted form in (28) is for algebraic convenience. By this procedure [in view of (25)] the integrals under the integral in (26) are seen to cancel. The subtraction term $a(0)$ appears under the integral in exactly the same way as in reference 3. However in achieving an equivalent result we have avoided subjecting the subsidiary quantities, $\langle N\bar N|K_\mu|K\rangle$, to a

dispersion treatment. The analytic properties of $a(\xi\eta)$ are merely used over again.

Making use of the fact that $a'(0)=a(0)$ and that

$$\mathrm{Im}\Delta(-\xi)=\frac{\mathrm{Im}R(\xi)}{|1+\xi R(\xi)|^2}, \qquad (30)$$

one find

$$a(0)=\frac{1}{\pi}\int_{4M^2}^{\infty} d\xi[C-a(0)]\,\mathrm{Im}\Delta(-\xi). \qquad (31)$$

Provided the indicated integral converges one may write[7]

$$\frac{1}{\pi}\int_{4M^2}^{\infty} \mathrm{Im}\Delta(-\xi)d\xi=Z_3^{-1}-1. \qquad (32)$$

The solution to (31) is thus

$$a_{\mathrm{I}}=-a_{\mathrm{II}}=C(1-Z_3). \qquad (33)$$

3. NUMERICAL ESTIMATES

It remains to determine the coefficients of the contributions' a_{I} and a_{II} to the decay amplitude of the real K particle. For simplicity we assume a globally symmetric pi coupling. Now the total function (summed over $n\bar{n}$, $p\bar{p}$, $\Sigma\bar{\Sigma}$ etc.) will be just 8 times its value for a single $n\bar{n}$ intermediary. This result depends on the universality of the coupling constant magnitude and not on relative signs (each g being squared).

In the calculation of Ima', we must contend with contributions of both signs, and depending on the exact interaction there may or may not be large cancellations. However, just from the number of intermediate processes, and some kind of universal (or average) order of magnitude coupling constants we may infer the maximum decay rate. The coupling constants required are those for the various processes (for K^+ decay), $K^+\to p+\bar{\Lambda}, \bar{\Sigma}^0$, $K^+\to n+\bar{\Sigma}^+$, $K^+\to\Sigma^++\bar{\Xi}^0$, $K^+\to\Lambda, \Sigma^0+\bar{\Xi}^+$ and for the weak couplings $p+\bar{\Lambda}\to\mu+\nu$ etc.

It turns out that the sign assignments in the globally symmetric pi coupling taken with the (dynamical) result, $a_{\mathrm{I}}=-a_{\mathrm{II}}$, insure no contribution from the loops beginning $K^+\to\bar{\Sigma}^++n$ and $K^+\to\bar{\Xi}^0+\Sigma^+$. There are in total eight additional remaining processes whose signs are unconstrained by the global symmetry or by charge independence in the K coupling. Our maximum ratio,

$$\Sigma a'(\xi\eta)/\Sigma R(\xi),$$

is the same as a'/R in Eq. (25) where only one intermediate channel was considered for both a and R. However, with another choice of signs in the weak coupling this ratio could be reduced to zero. Now assuming $0<Z_3\ll 1$ we may calculate the maximum transition rate for the K decay. From (33), (5), (3), and (2) is obtained

$$\Gamma_{\max}\begin{Bmatrix}K_{\mu3}\\K_{e3}\end{Bmatrix}=\frac{1}{\pi^3}(m_\pi^2\lambda)^2\left(\frac{g_K}{g}\right)^2 m_\pi\begin{Bmatrix}2.67\\3.35\end{Bmatrix}. \qquad (34)$$

4. CONCLUSION

With $g^2/g_k^2=15$ we find, after fitting $\Gamma_{\max}$ to the total leptonic plus pion decay rate (7% of the total K^+ rate[8]), $\lambda_{\min}^2\simeq(1/20)g_V^2$ where g_V is the vector coupling constant in β decay.

On the basis of the above analysis one could conclude either that the fundamental strangeness nonconserving coupling constant is substantially smaller than the beta-decay coupling constant or that there is considerable cancellation among the contributions of various baryon pair intermediate states. The same conclusion is obtained in a similar analysis of $K_{\mu2}$ decay.[4] The present leptonic hyperon decay data strongly indicate the reduced coupling constant.

The strongest conclusion is that the frequency of the leptonic modes of K decay is not so large as to require excessive leptonic decays of hyperons if the Fermi interaction is taken as the fundamental weak coupling.

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