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Significance of Potentials in Quantum Theory

W. H. FURRY AND N. F. RAMSEY
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts
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The effects of the scalar and vector potentials in quantum mechanics, which were pointed out recently by Aharonov and Bohm, are discussed from the point of view of the consistency of the quantum-mechanical description of interference experiments. A well-known requirement for this consistency is that if any measuring device is introduced that can be used to determine which path the particle has taken, it must have the effect of eliminating the interference phenomenon. Two conceptual experiments are discussed, corresponding to the two phase effects noted by Aharonov and Bohm. In each case it is found that the phase effect is of just the magnitude required to destroy the interference pattern when the circumstances are such that no pattern should be observed.

1. INTRODUCTION

IN classical physics the electromagnetic field strengths are regarded as basic physical quantities, and the potentials as mathematical auxiliary quantities. This same attitude has been carried over into many discussions of quantum theory. In both kinds of theory it is customary to motivate and interpret the invariance of the equations under gauge transformations in terms of this attitude.

Recently Aharonov and Bohm[1] have pointed out some effects of potentials in quantum theory which have appeared surprising to many physicists, and which show that a more careful discussion of the part played by potentials is required. The purpose of the present note is to discuss these effects from a different point of view, which we believe can help in understanding their significance.

Aharonov and Bohm have discussed the possibility of observing these effects experimentally, particularly the second (magnetic) effect, and have given references to work that is of interest in this connection. We are concerned here with the bearing of these effects on discussions of the consistency of quantum mechanics. Our diagrams are schematized accordingly, and depart rather far from what might actually be feasible; but the principles involved are just those discussed by Aharonov and Bohm.

We shall be dealing with the principle of complementarity as applied to a two-slit experiment on the interference of de Broglie waves. As has been emphasized in many discussions,[2] the key point here for the consistency of the theory is that the interference phenomenon can be observed only when a wave picture, with waves passing through both slits, can legitimately be used to describe the process. The introduction of any device that can tell which slit the particle went through must have some effect that will cause the destruction of the interference pattern.

We shall discuss in turn the two effects of potentials pointed out by Aharonov and Bohm. For each, we sketch an arrangement that in principle could be used to demonstrate the effect, by observation of an interference pattern. We then point out how a *different* procedure with the same arrangement could be used to tell which slit the particle has gone through. For this procedure it is found that the uncertainty in the phase effect of the potential is of just the magnitude required to eliminate the interference pattern.

[1] Y. Aharonov and D. Bohm, Phys. Rev. **115**, 485 (1959). We wish to thank Professor Purcell for acquainting us with these arguments before the appearance of the Aharonov-Bohm paper, and for stimulating discussions.

[2] N. Bohr, in *Albert Einstein, Philosopher-Scientist*, edited by P. A. Schilpp (Tudor Publishing Company, New York, 1951), especially pp. 217–218; W. Pauli, *Handbuch der Physik* (Verlag Julius Springer, Berlin, 1957), Vol. 5, Part 1, pp. 1, 2; D. Bohm, *Quantum Theory* (Prentice-Hall, Inc., Englewood Cliffs, 1951), pp. 118, 119, and 124–129.

2. THE SCALAR-POTENTIAL EFFECT

If, in a two-slit experiment, the slits are a few wavelengths wide, and if plane waves fall on them, then the waves emerging from the slits diverge only slightly in angle; they will, however, overlap and produce an interference pattern if the screen on which they fall is far enough from the slits.

The use of only slightly divergent waves makes it possible in principle to let the waves travel some part of the distance beyond the slits inside two metal pipes P' and P'' (Fig. 1). We now use not an infinite wave train, but a finite train or "wave packet" which is very long compared to the wavelength but short compared to the length of the pipes. There will then be a time interval during which the waves are well inside the pipes. During a time T in the middle of this interval we apply a potential difference V to the tubes; for definiteness, let us keep P' at potential zero, and raise the potential of P'' to the value V for the time T. Before the beginning of the time interval T, and after its end, the potential is zero everywhere.

With this procedure the waves travel only in regions where the field is zero at the time of their passage. The waves in pipe P'' show no change in their probability distribution, either in coordinate space or momentum space. The only difference from the case $V=0$ is that during the time interval T the frequency of each monochromatic component of the packet, being equal to the *total* energy divided by $\hbar$, is increased for the waves in pipe P'' by the amount $eV/\hbar$. Thus the wave function of the packet that emerges from the pipe P'' is changed by the phase factor $e^{-i\varphi}$, with $\varphi=eVT/\hbar$. The general nature of the interference pattern on the screen is not changed, but if $eVT/\hbar$ is not a multiple of 2π there will be a shift of the interference fringes. In principle, though presumably not in practice, one could demonstrate and study the phase effect by observing the patterns obtained with various precisely fixed values of the product VT.

Let us now consider a different use of this apparatus. It provides in principle a method for obtaining some information about the process of the passage of a particle through the two-slit arrangement. For this purpose we must use a wave train that is the wave function of a single particle, and suppose that the pattern is obtained by many repetitions of the experiment. The pipes P', P'' provide electrostatic shielding for the regions inside them, and allow us to use an electric field produced by the particle to get information about its location, without subjecting it to any field produced by the test body.

The test body, of charge q, is between two condenser plates separated by a distance l (Fig. 1). It is held fixed half-way between them ($x=l/2$) until the waves are inside the tubes, and is brought back to this position before the waves emerge; thus it produces no field between the pipes at any time when the field could act on the particle. The test body is free to move during a time interval T when the waves are certainly inside the pipes, and by observing the direction in which it is accelerated during this time T we can find out which tube contains the particle.

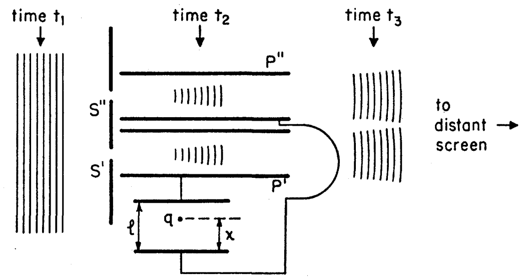


FIG. 1. Electrostatic effects.

The potential difference produced by the presence of the particle in one tube or the other is $v=\pm e/(2C)$, where C is the total capacity of the condenser and attached pipes. The magnitude of the field strength is thus[3]

$$|\mathbf{E}|=e/(2lC). \tag{1}$$

The force on the test body is $q\mathbf{E}$. If its direction is to be determined, it must produce a change of the momentum of the test body that is larger than the uncertainty in that momentum:

$$q|\mathbf{E}|T>\Delta p. \tag{2}$$

Displacement of the test body from its central position at $x=l/2$ produces a potential difference[4]

$$V=(q/C)\cdot(x-l/2)/l,$$

and the uncertainty of the potential difference is

$$\Delta V=(q/lC)\Delta x. \tag{3}$$

Substituting Eq. (1) in Eq. (2) and multiplying by Eq. (3), we have

$$qeT\Delta V/(2lC)>(q/lC)\Delta p\Delta x>(q/lC)\cdot\hbar/2.$$

Therefore

$$eT\Delta V>\hbar, \tag{4}$$

and the uncertainty in the phase difference φ caused by the potential produced by the test body is

$$\Delta\varphi=eT\Delta V/\hbar>1. \tag{5}$$

[3] The discrete nature of the mobile charges in the conducting materials would of course deprive the argument of all meaning, if e is the charge of an electron. A basic principle accepted in all such arguments is that the materials used for the apparatus are to be treated as ideal continua, and no significance is to be attached to the actual numerical values of the charges and masses of particles [see N. Bohr and E. Rosenfeld, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. 12, No. 8 (1933)].

[4] This formula results from the application of Green's electrostatic reciprocity theorem to the parallel-plate geometry; the argument is a familiar one in calculations on the collection of ions. The reciprocity theorem can be used to show that the final result of Eq. (4) holds for any arrangement for producing a fairly uniform field in the region where the test body is accelerated.

We get fairly certain information as to which slit the particle went through if we make $q|\mathbf{E}|T$ at least three times Δp; $\Delta\varphi$ then has a value $\gtrsim 3$, which is precisely enough to wipe out the interference pattern.

3. THE VECTOR-POTENTIAL EFFECT

We consider the same sort of two-slit experiment as before, with slits wide enough so that the waves from them remain fairly well separated until they have travelled a considerable distance beyond the slits, after which they overlap and interfere. Beyond the slits, in the space between the separated beams of waves, is a region R in which there can be a nonvanishing magnetic flux Φ. Aharonov and Bohm speak of either a long, closely-wound solenoid or a ferromagnetic rod or "whisker." We shall consider the case of a ferromagnetic rod. It can be thought of as infinitely long in the direction perpendicular to the plane of the diagram (Fig. 2), so that there is no stray flux outside it, and thus no field in the regions traversed by the de Broglie waves. Actually a yoke could be used to close the magnetic circuit and secure the same effect.

Although there is no field outside the rod R, the line integral of the vector potential $\mathbf{A}$ around any path inclosing R is equal to the flux Φ. In hydrodynamical language, the vector potential has zero rotation but a circulation equal to Φ. Aharonov and Bohm point out that there is a resulting phase difference $\varphi=(e/\hbar c)\Phi$ between the waves that have passed above and below R. This means that in principle a phase effect of the vector potential can be demonstrated by observation of a shift of the interference fringes, in a case in which the particle is never subject to a field; apparently the actual demonstration may not be beyond the range of experimental possibilities.

We now consider an apparatus in which a search coil of N turns is wound around R and connected to the condenser plates C. Passage of a particle through S' and on to the screen involves a transient current that flows counterclockwise with respect to R; passage through S'' means a transient clockwise current. By Lenz's law, in either case a current in the opposite direction is induced in the search coil. Observation of the sign of the resulting charge on C will then reveal which slit the particle went through.

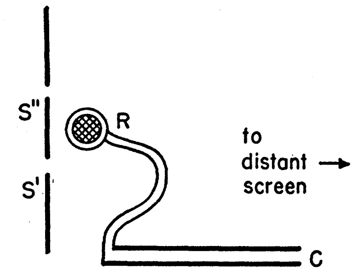
For the greatest convenience in the discussion and the most favorable conditions for the proposed observation, we make two assumptions:

(a) The permeability of the rod R is enormous, and the resistance of the search coil and plates is zero. The current induced in the search coil is then just that required to prevent any change in the flux Φ. Passage of the particle through either slit and on to the screen is tantamount to flow of the charge e through one-half turn; accordingly, the charge delivered to C is[5]

$$Q_i=\pm e/2N. \tag{6}$$

[5] The comment of footnote 3 applies with the same force here.

FIG. 2. Magnetic effects.



(b) The characteristic time of the circuit[6]

$$T=(LC)^{\frac{1}{2}}/c$$

is very long compared with the time of passage of the wave packet through the apparatus. Thus we can have the advantage, as compared with the case of the scalar-potential experiment, of ample time for the determination of the sign of Q.

The apparatus now under consideration, unlike the purely electrostatic apparatus of Fig. 1, necessarily involves a nonvanishing inductance.[7] The circuit thus has two canonically-conjugate dynamical variables, the charge Q and the flux linkage $N\Phi$, which appear in the Hamiltonian for the equivalent harmonic oscillator,

$$H=Q^2/2C+(N\Phi)^2/2L,$$

and satisfy the uncertainty relation

$$\Delta Q\cdot N\Delta\Phi>\tfrac{1}{2}\hbar c. \tag{7}$$

If we are to determine the sign of Q_i, we must have

$$|Q_i|>\Delta Q. \tag{8}$$

From this and Eqs. (6) and (7) we get

$$e\Delta\Phi>\hbar c, \tag{9}$$

and the uncertainty in the phase difference φ caused by the vector potential associated with the flux Φ is

$$\Delta\varphi=(e/\hbar c)\Delta\Phi>1. \tag{10}$$

Thus if φ is well enough determined so that we can (in repeated experiments) observe an interference pattern, then ΔQ is so large that we cannot learn which slit a particle has gone through by observing the sign of Q_i; and if the oscillating circuit is prepared in such a way that we can learn which slit the particle has gone through, the phase effect from the uncertainty in the flux will eliminate the interference pattern.

4. CONCLUSION

In view of the discussion given by Aharonov and Bohm, and the discussion in the present paper, the

[6] Like Aharonov and Bohm, we are expressing e in esu and Φ in Maxwells. L and C are in cm. The factor c in the uncertainty relation (7) is also characteristic of the use of these electrical units.

[7] The deflecting plates in Fig. 1 could of course be much more directly connected to the pipes; in principle they could be parts of the outer surfaces of the pipes.

following statements can be made about the effects of the potentials in quantum theory:

1. These effects change only the phase of the wave function, and the phase changes are independent of the kinetic energy and kinetic momentum of the particle. Thus they are intrinsically quantum-mechanical effects, with no analog in classical theory.

2. These effects do not affect the gauge invariance of the theory.

3. These effects can have objective meaning only when they act differently on different parts of the wave function of the same particle. A classical particle could show effects of an electrostatic potential difference or a magnetic flux only by following a path passing through a region of nonvanishing field. A quantum-mechanical particle need not enter such a region to detect its existence; the de Broglie waves can pass on either side of it and receive a relative phase shift that is in principle observable.

4. These effects are not accidental results of a particular way of formulating the theory, and they do not constitute any paradox or inconsistency in the theory. On the contrary, our discussions of conceptual experiments have shown that these effects prevent paradoxes and are essential for the consistency of the theory.

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Electric Field Distributions in an Ionized Gas. II*

BERNARD MOZER† AND MICHEL BARANGER
Carnegie Institute of Technology, Pittsburgh, Pennsylvania
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A method previously described is used to calculate the probability distribution of the low-frequency component of the electric field at a neutral point, the distribution of the low-frequency component at an ion, and that of the high-frequency component at an electron. The results are compared with those obtained by other authors.

1. INTRODUCTION

IN a previous paper,[1] a method was described by which the Holtsmark[2] distribution can be systematically corrected for the correlations between the particles producing the field, provided that this correction is not too large. The result was to put the Fourier transform $F(\mathbf{k})$ of the field distribution in the form

$$F(\mathbf{k})=\exp\left[\sum_{P=1}^{\infty}(n^P/P!)h_P(\mathbf{k})\right], \tag{1}$$

where n is the density of particles and the functions $h_P(\mathbf{k})$ correspond to increasing orders in a cluster expansion. Those can in turn be expressed in terms of other functions g_P,

$$h_P(\mathbf{k})=\int \varphi_1\varphi_2\cdots\varphi_P \times g_P(\mathbf{x}_1,\mathbf{x}_2,\cdots,\mathbf{x}_P)d^3x_1d^3x_2\cdots d^3x_P, \tag{2}$$

where

$$\varphi_i=\exp(i\mathbf{k}\cdot\mathbf{E}_i)-1, \tag{3}$$

and $\mathbf{E}_i$ is the electric field produced by the particle of coordinates $\mathbf{x}_i$. The present calculations include only the first two terms of the series in (1). The corresponding g functions will be rewritten here explicitly,

$$g_1(\mathbf{x})=\mathcal{V}P_1(\mathbf{x}), \tag{4}$$

$$g_2(\mathbf{x}_1,\mathbf{x}_2)=\mathcal{V}^2[P_2(\mathbf{x}_1,\mathbf{x}_2)-P_1(\mathbf{x}_1)P_1(\mathbf{x}_2)], \tag{5}$$

P_1 and P_2 being the single-particle and pair distribution functions, respectively, and $\mathcal{V}$ being the volume of the container. The field distribution itself, $W(\mathbf{E})$, is obtained from

$$W(\mathbf{E})=(2\pi)^{-3}\int \exp(-i\mathbf{k}\cdot\mathbf{E})F(\mathbf{k})d^3k. \tag{6}$$

The distinction was made in I between the *low-frequency component* and the *high-frequency component* of the electric field in an ionized gas or plasma. The former is that part of the field whose time variation is governed by the motion of the ions. It is obtained by averaging the total field over a time long compared to typical electronic relaxation times, but short compared to ionic times. Therefore, it consists of the sum of the fields from the ions, each field being shielded by a cloud of electrons. Only the case of singly charged ions will be considered. Other cases would be equally easy to treat. The electron and ion densities are then equal and both denoted by n. Thermal equilibrium will also be assumed. One can then use the shielded field given by

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† Now at Brookhaven National Laboratory, Upton, New York. The material in this article is part of a thesis submitted by Bernard Mozer in partial fulfillment of the requirements for the Degree of Doctor of Philosophy at the Carnegie Institute of Technology.

[1] M. Baranger and B. Mozer, Phys. Rev. 115, 521 (1959), referred to in the following as I.

[2] J. Holtsmark, Ann. Physik 58, 577 (1919).