Multilevel Analysis of the Slow Neutron Cross Sections of U^{233}†

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A multilevel analysis of the slow neutron total and fission cross sections of U^{233} from thermal energies to 11 ev has been carried out, under the assumption that in one of the two spin states the nucleus undergoes fission primarily through a small number of channels. The cases of one and of two fission channels for this spin state have been investigated in detail. Although the analysis does not yield an exact value for the number of available fission channels, the data are adequately fit with a two fission channel formula. It is not necessary to invoke any negative energy resonances in the analysis. The presence of a noninterfering component in the cross sections, having a $1/v$ energy variation, is indicated. At thermal energies, this component accounts for approximately 80% of the total and fission cross sections. An interpretation of this component as being due to one spin state of the compound nucleus is presented. Under this assumption, the fission characteristics of the two spin states of the compound nucleus formed by s-wave neutrons are quite different.

I. INTRODUCTION

HIGH resolution measurements of the slow neutron fission and total cross sections of U^{233} are reported in a companion paper.[1] Marked asymmetries, which cannot readily be described by the single level Breit-Wigner formula, are observed in most of the resonances. Such asymmetries have also been reported for resonances in U^{235} [2] and Pu^{239}.[3] It has been reasonably well established that these asymmetries are due to interference among the resonance levels, in which case a multilevel treatment is necessary. Multilevel approaches have recently been applied to the analysis of the U^{235} fission and total cross sections.[2,4]

The existence of interference among fission resonances implies that there are but few open fission channels. This assertion is further supported by the large fluctuation in the sizes of the fission widths, which, according to Porter and Thomas,[5] implies that the number of channels available to the process is small. The present analysis of the resonance structure of U^{233} is based on a specialization of the dispersion theory formalism of Wigner and Eisenbud[6] given in a previous paper.[7] In the present analysis it is assumed that, in one of the two possible spin states formed by s-wave neutrons, fission proceeds through either one or two channels. Each of these possibilities is considered in detail. The data are quite adequately fit by the two-channel formula. While the more important features of the data are also described by the one-channel formula, it appears that the assumption of only one fission channel for U^{233} is somewhat restrictive. Both of these approaches yield markedly better fits to the data than does the Breit-Wigner single level formula.

II. ANALYSIS

The multilevel approach used in the analysis has been discussed in a previous paper,[7] henceforth called I. This treatment assumed one neutron channel, an arbitrary number of fission channels, and an essentially infinite number of radiative capture channels. In particular, for the case of one fission channel in a given spin state, explicit expressions for the multilevel scattering, fission, and radiative capture cross sections were presented. These have been used in obtaining the single fission channel fit to the cross-section data. For the case of two fission channels, Eq. (7) of I forms a convenient starting point. Explicit algebraic expressions, in terms of the reduced width parameters and characteristic energies, for the elements S_{11}, S_{12}, and S_{13} of the collision matrix can readily be obtained. The subscript 1 refers, as before, to the neutron channel, and the subscripts 2 and 3 refer to the two assumed fission channels. In terms of these elements of the collision matrix, the s-wave neutron scattering and fission cross sections, for resonances of spin J, can be calculated from the formulas

$$\sigma_{n,n}{}^J=\pi\lambda\!\!\!^{-}{}_n{}^2 g_J|1-S_{11}|^2, \quad (1)$$

$$\sigma_{n,f}{}^J=\pi\lambda\!\!\!^{-}{}_n{}^2 g_J(|S_{12}|^2+|S_{13}|^2), \quad (2)$$

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[1] M. S. Moore, L. G. Miller, and O. D. Simpson, preceding paper [Phys. Rev. **118**, 714 (1960)].

[2] V. L. Sailor, *Proceedings of the International Conference on the Peaceful Uses of Atomic Energy, Geneva, 1955* (United Nations, New York, 1956), Vol. IV, p. 199; F. J. Shore and V. L. Sailor, *Proceedings of the Second United Nations International Conference on the Peaceful Uses of Atomic Energy, Geneva, 1958* (United Nations, Geneva, 1958), A/CONF.15/P 648; F. J. Shore and V. L. Sailor, Phys. Rev. **112**, 191 (1958).

[3] L. M. Bollinger, in Atomic Energy Research Establishment, Harwell Report NP/R 2076 (revised), edited by N. J. Pattenden, 1957 (unpublished), p. 21.

[4] E. Vogt, Phys. Rev. **112**, 203 (1958).

[5] C. E. Porter and R. G. Thomas, Phys. Rev. **104**, 483 (1956).

[6] E. P. Wigner and L. Eisenbud, Phys. Rev. **72**, 29 (1947).

[7] C. W. Reich and M. S. Moore, Phys. Rev. **111**, 929 (1958).

where $\lambda\!\!\!^{-}_n$ is the neutron wavelength divided by 2π, and g_J is the statistical weight factor for the resonances of spin J. For two fission channels, the direct calculation of the radiative capture cross section is extremely tedious. Instead, in the numerical calculations, the radiative capture cross section was obtained by subtracting the calculated scattering and fission cross sections from the total cross section. The expression (which follows from the unitary nature of the collision matrix) used for the total cross section is

$$\sigma_{n,t}{}^J = 2\pi\lambda\!\!\!^{-}_n{}^2 g_J \,\mathrm{Re}(1-S_{11}), \tag{3}$$

where the symbol Re denotes the real part of the quantity in parentheses. Equations (1), (2), and (3) give the contribution of one spin state to the various cross sections. The various measured cross sections include contributions from the other spin state as well, in which the number of assumed fission channels may be quite different.

The formulas for both the single fission channel and two fission channel expressions have been programed for use with the IBM-650 data processing machine. All the numerical calculations have been carried out with the use of this machine. The expressions for the two fission channel case have also been programed in the IBM Fortran language, and are thus compatible with the IBM 704.

In the single level approximation,[6] the relation between the reduced width parameters $\gamma_{\lambda j}$, which occur in the general resonance theory, and the more familiar observed widths $\Gamma_{\lambda j}$ is $\Gamma_{\lambda j} = 2B_j{}^2\gamma_{\lambda j}{}^2(1+C_j{}^2)$, where λ denotes the resonance level and j the channel. As indicated in I, the channel quantities C_j either vanish identically or may be set equal to zero in the analysis of slow neutron fission resonances. This, along with the definition of B_j in the neutron channel, leads to the following relationships which were used in the analysis:

For the neutron width,

$$\Gamma_{\lambda n} = \Gamma_{\lambda n}{}^0(E)^{\frac{1}{2}} = 2k_n\gamma_{\lambda 1}{}^2; \quad k_n = 1/\lambda\!\!\!^{-}_n.$$

For the fission width,

$$\Gamma_{\lambda f} = 2\beta_{\lambda 2}{}^2 \quad \text{(one fission channel)},$$

$$\Gamma_{\lambda f} = 2(\beta_{\lambda 2}{}^2 + \beta_{\lambda 3}{}^2) \quad \text{(two fission channels)},$$

where

$$\beta_{\lambda j} = B_j\gamma_{\lambda j}.$$

In Eqs. (1), (2), and (3), the interference among the resonance levels, λ, of a given spin state occurs through summations of the form $\sum_\lambda \beta_{\lambda j}\beta_{\lambda k}/(E_\lambda - E - \frac{1}{2}i\Gamma_{\lambda\gamma})$ where $\Gamma_{\lambda\gamma}$ is the radiative capture width and E_λ is the characteristic energy. The $\beta_{\lambda j}$ are real quantities, but may be either positive or negative. The choice of sign for the product $\beta_{\lambda j}\beta_{\lambda k}$ for a given level, relative to that of other levels, depends upon the type of interference (constructive or destructive) observed, and thus this choice is not completely arbitrary. With no loss of generality, all of the reduced neutron width parameters in the analysis were taken as positive. The reduced fission width parameters were then given those signs which predicted the proper type of interference. The fits to the data were obtained by trial-and-error adjustments of the characteristic energies and the reduced width parameters.

One Fission Channel Analysis

The single fission channel multilevel fit to the cross-section data in the various energy regions is shown as the dashed curve in Figs. 1 through 4. The parameters used in obtaining this fit are given in Table I. With the exceptions of the regions of 0.2 and 3.6 ev, the fit was considered adequate. The following features of the fit should be emphasized:

(1) The fullness below the 1.8-ev resonance was assumed to be due to a broad resonance near 1.5 ev. While it might appear that this fullness is due to

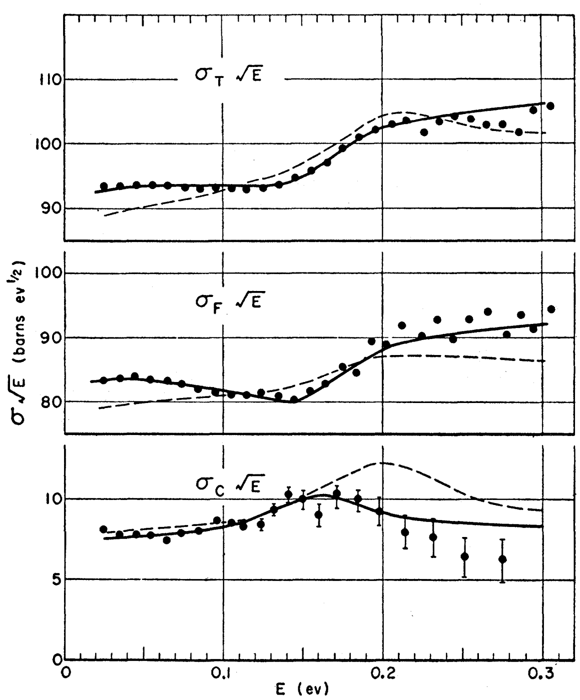

FIG. 1. The total, fission, and radiative capture cross sections of U²³³ below 0.3 ev. The ordinate has been plotted as the respective cross section, multiplied by the square root of the neutron energy, in order to remove the $1/v$ dependence. The dashed and solid curves show the results of the one and two fission channel approaches, respectively. The capture cross-section data have been obtained by a subtraction from the measured total cross section of the measured fission cross section and the scattering cross section as calculated from the multilevel parameters. It should be noted that, while the contribution from this resonance to the radiative capture cross section is symmetric, the position of the resonance peak does not correspond to the characteristic energy. This resonance is detectable only through its interference with the other levels. A single level contribution from a resonance with these parameters would not have been detected experimentally, because of the small neutron width. Indeed, a fit which was only slightly poorer than that presented above was obtained under the assumption that the neutron width was zero.

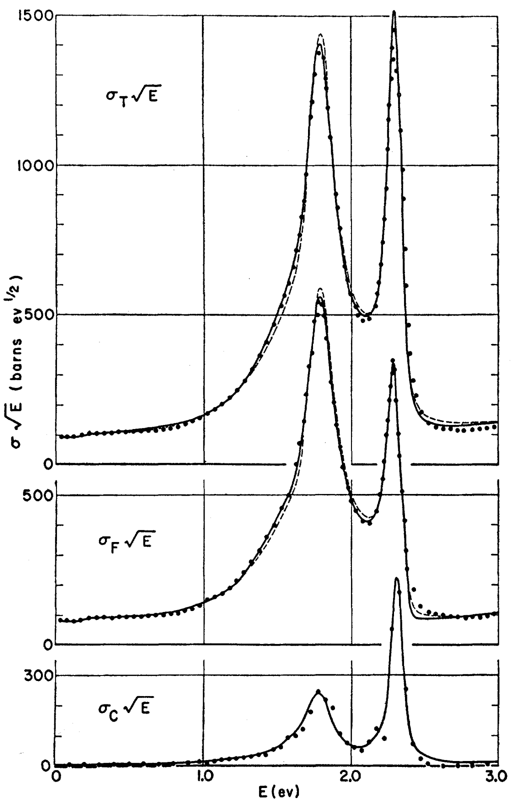

FIG. 2. The U^{233} total, fission, and radiative capture cross sections, multiplied by the square root of the neutron energy, below 3 ev. The dashed and solid curves show the results of the one and two fission channel approaches, respectively. The effects of Doppler and resolution broadening have been included in the calculated curves. The radiative capture cross-section data were obtained as described in Fig. 1.

strong constructive interference involving the 1.8-ev resonance and some other resonance, none of the observed resonances are sufficiently strong to account for this shape. A combination of negative energy resonances each with highly anomalous parameters can be postulated which will give an adequate fit in this region. However, such a combination gives a poor fit in other energy regions, notably near thermal. A simpler explanation is that there is a fairly broad resonance in the region of 1.5 ev. The parameters of such a resonance are quite reasonable, and, when such a resonance is assumed, it is unnecessary to postulate the existence of a negative energy resonance. There remains the question of whether or not this 1.5-ev resonance interferes with those at 1.8 and 2.3 ev. In the single fission channel analysis, it was assumed that it does not, for the following reasons: The contribution of the 1.8- and 2.3-ev levels was calculated, and subtracted from the data. The remainder was found to be adequately described by a single level formula. If one assumes that this resonance interferes with the 1.8- and 2.3-ev resonances, the calculated cross section is grossly distorted.

(2) The sharp drop-off on the high-energy side of the 2.3-ev resonance is adequately described on the basis of interference with the 1.8-ev resonance. The single fission channel formula predicts a very small cross section in the region of 2.5 ev as a result of this destructive interference. In order to describe the observed cross sections in this energy region, it is necessary to postulate the existence of a noninterfering component.

(3) The quite asymmetric shape of the 3.6-ev resonance is not well described on the basis of interference with the 1.8- and 2.3-ev resonances. Attempts to correct this poor fit by the inclusion of the 4.7-ev resonance in the interference create further difficulties in the regions of 4.0 and 5.0 ev. The 4.7-ev resonance was therefore assumed not to interfere. It should be noted that the 4.7- and 1.5-ev resonances are, by themselves, not sufficient to account for the large noninterfering component necessary near 2.5 ev.

(4) The regions above and below ~5.5 ev were found to be relatively independent of one another. Small changes in the parameters of the resonances above 5.5 ev had a negligible effect below 5.5 ev, and *vice versa*. The single fission channel formula gives a a reasonable fit to the resonance structure above 5.5 ev, as shown in Fig. 4. However, a noninterfering component in the region of 8 ev was required.

(5) The final fit was obtained by assuming that all the resonances except those at 1.5 and 4.7 ev interfere in a single fission channel, and that the resonances at 1.5 and 4.7 ev are described by Breit-Wigner single level formulas. A noninterfering component was also required to account for ~80% of the cross section at

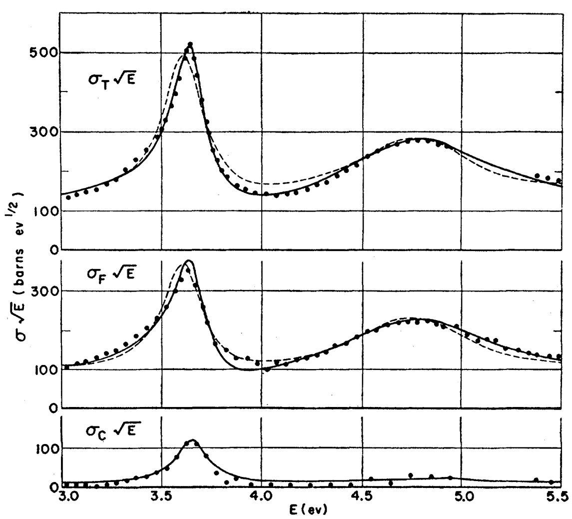

FIG. 3. The U^{233} total, fission, and radiative capture cross sections, multiplied by the square root of the neutron energy, from 3 to 5.5 ev. The dashed and solid curves show the results of the one and two fission channel approaches, respectively. The effects of Doppler and resolution broadening have been included in the calculated curves. The presentation of the total cross-section data is precluded in the region of ~5 ev because of the effects of a 1% U^{234} contaminant in the sample.

thermal energies. It is perhaps significant that the magnitude of this component in the three energy regions, 0.02, 2.5, and 8 ev, is consistent with a $1/v$ energy variation. If the noninterfering component were due to resonances whose widths are large compared to their spacings, such a $1/v$ energy dependence would be expected.

While the asymmetric shape of the 3.6-ev resonance is not well described by the assumption of a single fission channel, by far the most serious objection to this approach occurs for the resonance near 0.15 ev. The inadequacy of this fit was emphasized when high precision data in this region became available.[1] Consequently, a less restrictive approach, invoking two fission channels, was adopted.

Two Fission Channel Analysis

The fit resulting from the two fission channel analysis is given by the solid curves in Figs. 1, 2, and 3. The associated resonance parameters are listed in Table I. Since the single fission channel approach gave an adequate fit above 5.5 ev, and since the regions above and below this energy were found to be relatively independent, no serious attempt was made to obtain a two fission channel fit to the data above 5.5 ev. Since some contribution to the interference below 5.5 ev arises from the strong resonances at 6.8 and 10.5 ev, the effects of these resonances were included in the two fission channel calculations. Their parameters, however, were left unchanged. Contributions from the smaller resonances above 5.5 ev were neglected.

The results of the single fission channel analysis formed the starting point of the two-channel analysis. In the latter, however, all the observed resonances, including those at 1.5 and 4.7 ev, were allowed to interfere. Even under this assumption, a noninterfering component is required. This component has a $1/v$ energy dependence and has the same magnitude as in the one fission channel analysis. For the most part, the resonance parameters are virtually unchanged from those of the one-channel analysis.

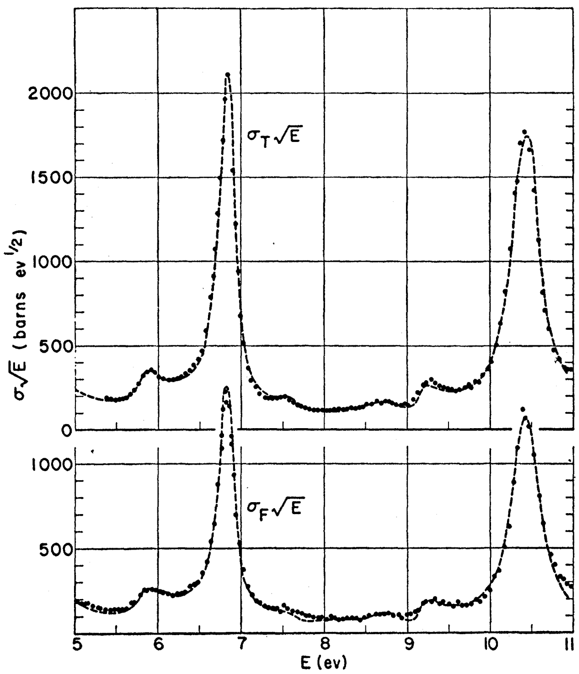

FIG. 4. The U^{233} total and fission cross sections, multiplied by the square root of the neutron energy, from 5.0 to 11 ev. The dashed line represents the results of the one fission channel analysis. The effects of Doppler and resolution broadening have been included in the calculated curves.

TABLE I. U^{233} resonance parameters. These parameters were used in obtaining the fits presented in Figs. 1 through 4. The radius, a, in the neutron channel was calculated from the relation: $a=1.45\times10^{-13}(A^{\frac{1}{3}}+1)$ cm and had the value of 10^{-12} cm. The magnitude of the additive noninterfering component required in order to achieve a fit to the total and fission data, as discussed in the text, was 73 barns $ev^{\frac{1}{2}}$ and 68 barns $ev^{\frac{1}{2}}$, respectively.

	One fission channel parameters						Two fission channel parameters								
Level (λ)	E_λ (ev)	g_J[a]	$\Gamma_{\lambda n}^0$ (10^{-3} ev)	$\Gamma_{\lambda\gamma}$ (10^{-3} ev)	$\Gamma_{\lambda f}$ (10^{-3} ev)	Relative sign of $\beta_{\lambda1}\beta_{\lambda2}$	E_λ (ev)	g_J[a]	$\Gamma_{\lambda n}^0$ (10^{-3} ev)	$\Gamma_{\lambda\gamma}$ (10^{-3} ev)	$\Gamma_{\lambda f}$ (10^{-3} ev)	$2\beta_{\lambda2}^2$ (10^{-3} ev)	$2\beta_{\lambda3}^2$ (10^{-3} ev)	Relative signs of $\beta_{\lambda1}\beta_{\lambda2}$	$\beta_{\lambda1}\beta_{\lambda3}$
1	0.195	7/12	0.00059	44	60	+	0.150	7/12	0.00002	30	60	35	25	−	+
2	1.55	5/12	0.165	60	562	...[b]	1.56	7/12	0.104	54	420	0	420	...	−
3	1.76	7/12	0.155	36	182	−	1.755	7/12	0.162	36	186	186	0	−	...
4	2.31	7/12	0.086	34.6	48	+	2.305	7/12	0.086	34.6	48	45.6	2.4	+	−
5	3.61	7/12	0.060	62	174	+	3.65	7/12	0.057	53	149	129	20	+	+
6	4.75	5/12	0.130	80	718	...[b]	4.825	7/12	0.105	80	850	750	100	+	−
7	5.82	7/12	0.047	80	316	−	...[c]	...	...	...	...	...	...		
8	6.82	7/12	0.300	55	146	+	6.82[d]	7/12	0.300	55	146	0	146	...	−
9	7.6	7/12	0.007	48	125	−	...[c]	...	...	...	...	...	...		
10	8.7	7/12	0.010	40	300	−	...[c]	...	...	...	...	...	...		
11	9.2	7/12	0.019	50	180	−	...[c]	...	...	...	...	...	...		
12	10.47	7/12	0.411	85	270	+	10.47[d]	7/12	0.411	85	270	0	270	...	−

[a] Based on considerations presented in the text (discussion).
[b] Sign immaterial, since the contribution was calculated using a single-level formula.
[c] Not included in the two fission channel analysis.
[d] Carried over from the one fission channel analysis.

III. DISCUSSION

From the analysis of the cross sections of U^{233}, based on the assumption that, within one spin state, fission of the compound nucleus proceeds primarily by way of one or two channels, one is led to the following conclusion: Two components, which do not interfere with one another, exist in the U^{233} cross sections. One of these components is characterized by sharp, well-defined resonances, most of which exhibit strong interference effects, and which can be treated as though they all interfere with one another. The other component is characterized by a cross section which varies as $1/v$ at these neutron energies, and might be due to broad resonances which do not interfere appreciably with one another.

Uniqueness cannot be claimed for any theoretical fit, since even such fundamental information as the resonance spins is not currently available. Thus any assumption concerning which resonances interfere with one another is subject to question. In fact, even in the present analysis, adequate fits were obtained by alternately assuming that the 1.5- and 4.7-ev resonances do and do not interfere with the other resonances. Furthermore, using still different assumptions, Vogt has also obtained a fit to the U^{233} cross-section data.[8]

One would expect two components which do not interfere with one another to be present in the cross section, one for each spin state of the compound nucleus. Since all the resonances observed have a fission component, it may be concluded that both spin states contribute to the fission cross section. It is tempting to interpret the results of this multilevel analysis as an indication that the two distinct contributions to the U^{233} cross sections arise from the two spin states formed in the compound nucleus. There is some recent experimental evidence which supports this conclusion. For U^{233} direct measurements of η, the average number of neutrons emitted per neutron absorbed, have been made in this energy region.[9] These data show that the ratio of the average fission cross section to the capture cross section is much lower at the large resonances than at 0.025 ev and in the valleys between the resonances. This indicates that the ratio of the fission width to the radiative capture width for the prominent resonances is much less than for those levels which contribute to the off-resonance cross section. The radiative capture widths are not expected to vary markedly, even from one spin state to another. The observed variation in η is consistent with the assumption that the fission widths of the strong resonances are considerably smaller than the fission widths of the levels which contribute to the valleys.

[8] E. Vogt, following paper [Phys. Rev. **118**, 724 (1960)]. The authors are indebted to Dr. Vogt for kindly making available prior to publication a copy of his manuscript in which he arrived independently at several of the conclusions presented here.

[9] J. R. Smith and E. H. Magleby, Atomic Energy Commission Report IDO-16505, 1958 (unpublished), p. 54.

Wheeler[10] has suggested that the mass distribution of fission fragments might depend on the spin of the state in which fission occurs. On the basis of the present analysis it would appear that the fission of U^{233} by slow neutrons is ideal for investigating this suggestion. At thermal neutron energies, the fission cross section may be due primarily to one spin state, while at the interfering resonances it may be due to the other. Radiochemical studies[11] have shown that there is indeed a difference between the mass distributions of the products from thermal fission and from fission at the 1.8- and 2.3-ev resonances.

According to Porter and Thomas,[5] the distribution of widths for a given process provides an indication of the number of channels available to that process. The distribution of neutron widths for the resonances in U^{233}, which have been assumed to be of one spin state, is given in Fig. 5. The solid lines give the expected Porter-Thomas distributions for one, two, and four degrees of freedom. The neutron width distribution is not inconsistent with the assumption that these resonances are of the same spin state. The distribution of fission widths for these resonances is given in Fig. 6. The solid lines again give the Porter-Thomas distribu-

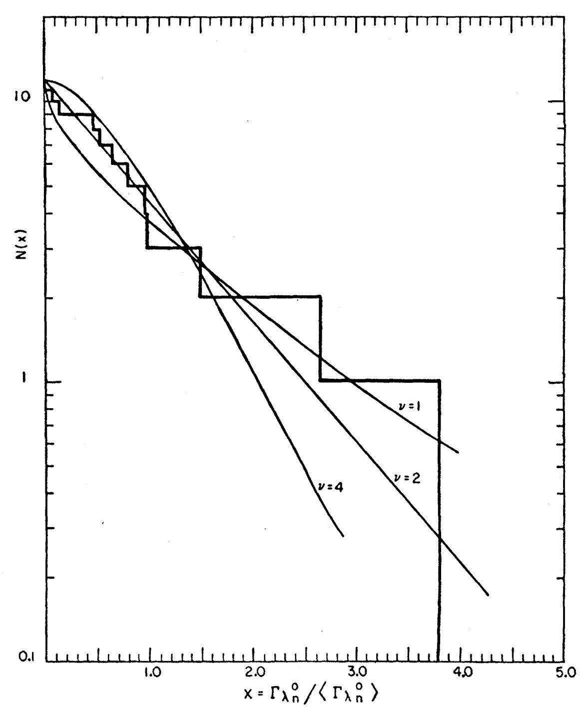

FIG. 5. The number, N, of levels having a value of $x=\Gamma_{\lambda n}{}^0/\langle\Gamma_{\lambda n}{}^0\rangle$ greater than x vs x. All twelve of the levels listed in Table I are included. The parameters for the levels below 5.5 ev are those of the two fission channel analysis. The solid curves are the Porter-Thomas distributions for one, two, and four degrees of freedom.

[10] J. A. Wheeler, Physica **22**, 1103 (1956).

[11] R. B. Regier, W. H. Burgus, and R. L. Tromp, Phys. Rev. **113**, 1589 (1959).

tions for one, two, and four degrees of freedom. To the extent that those fission widths constitute a representative sample, it may be concluded that the number of fission channels is small, although probably greater than one.

Relatively large variations are to be noted in the sizes of the radiative capture widths. Similar large variations in the radiative capture widths of the fissionable nuclide U^{235} have been reported.[2,4] The variations are larger than would be expected for isotopes having the level densities of U^{233} and U^{235} and are much larger than those observed in the radiative capture widths of resonances in nonfissile heavy

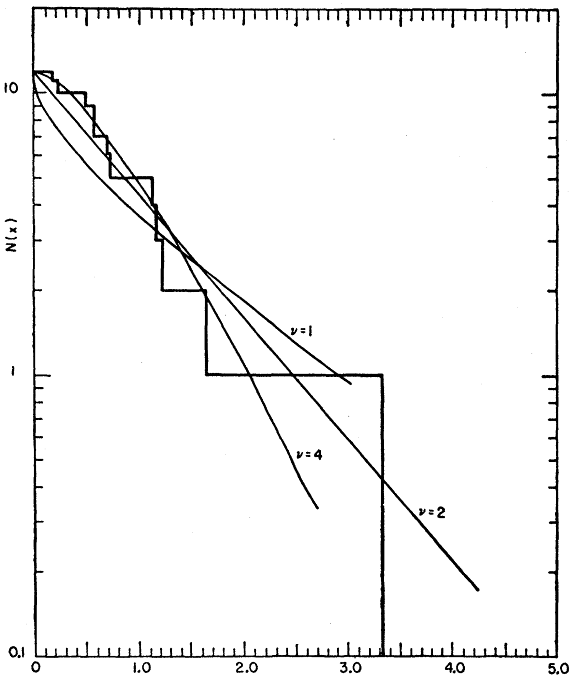

FIG. 6. The number, N, of levels having a value of $x=\Gamma_{\lambda f}/\langle\Gamma_{\lambda f}\rangle$ greater than x vs x. All twelve of the levels listed in Table I are included. The parameters for the levels below 5.5 ev are those of the two fission channel analysis. The solid curves are the Porter-Thomas distributions for one, two, and four degrees of freedom.

nuclei.[5,12] Furthermore, the levels in U^{233} which show these fluctuations are generally those having parameters which should be the most accurately known, since these levels are prominent and well separated. The possibility exists that radiative capture can compete favorably with fission in only a few capture channels. An indication of the number of such channels is given in Fig. 7, where the distribution of capture widths is plotted. The solid lines give the Porter-Thomas distributions for 10, 20, 30 and an infinite number of degrees of freedom.

The values of the statistical weight factor shown in Table I were chosen, somewhat speculatively, as a

[12] J. S. Levin and D. J. Hughes, Phys. Rev. **101**, 1328 (1956).

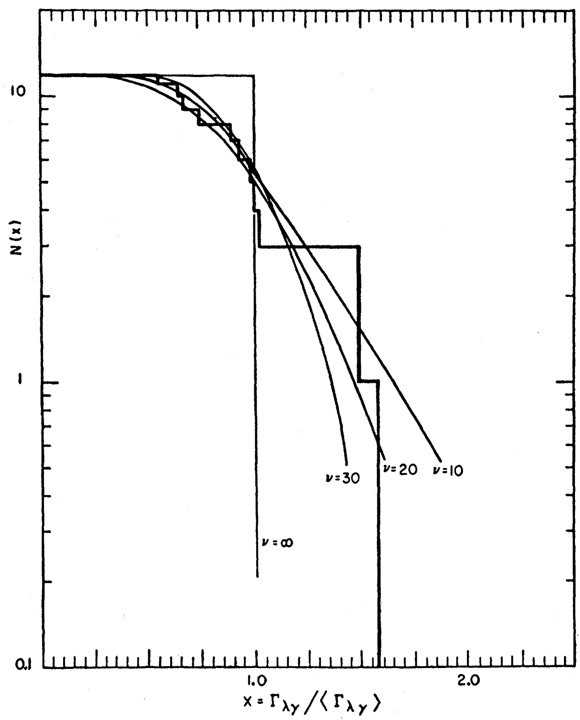

FIG. 7. The number, N, of levels having a value of $x=\Gamma_{\lambda\gamma}/\langle\Gamma_{\lambda\gamma}\rangle$ greater than x vs x. All twelve of the levels listed in Table I are included. The parameters for the levels below 5.5 ev are those of the two-channel analysis. The solid curves are the Porter-Thomas distributions for ten, twenty, thirty, and an infinite number of degrees of freedom.

result of the following considerations. The ground-state spin of U^{233} is $\frac{5}{2}$, and the parity is even.[13] The two possible spin states of the compound nucleus are then 2+ and 3+, both of which are expected to contribute to the fission yield. According to the collective model, the states through which the compound nucleus U^{234} passes in undergoing fission may have considerably different fission thresholds for the two possible spins.[10] One might then expect the states with the lower fission threshold to be characterized by broad levels, with a less well-defined resonance structure than for the other. Since for U^{234} the collective model predicts a rotational band containing a 2+ state at a lower excitation than one containing a 3+ state, one might tentatively assign the spin 3+ to the interfering levels, and 2+ to the noninterfering component.

The results of the present analysis might be given the following physical interpretation: In one spin state, the resonances are sharp, and interference effects are pronounced. There are but few open fission channels, and the "threshold for fission" for this spin state of U^{234} lies reasonably close to the excitation provided by the binding energy of a neutron in U^{234}. In the other spin state, the resonances are wide and overlapping, and exhibit no detectable interference effects. There

[13] D. Strominger, J. M. Hollander, and G. T. Seaborg, Revs. Modern Phys. **30**, 585 (1958).

are many open fission channels, and the "threshold for fission" for this spin state in U^{234} lies lower in excitation than the other.

At present, the understanding of slow neutron induced fission is far from complete. The analysis presented here is reasonably self-consistent, but it is certainly not unique. In the analysis of U^{235}, it has been found necessary to postulate an anomalously large negative energy resonance in order to explain the high thermal cross section.[2,4] In the present analysis, a negative energy resonance was not necessary. The presence of a noninterfering component, possibly arising from broad overlapping levels, was required and accounts for the large thermal cross section as well as for the high value of η observed at thermal energies. The consequences of the possible existence in U^{233} of two distinct types of resonances, belonging to different spin states, would seem to warrant further study.

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Low-Energy Neutron Cross Sections of Fissionable Nuclei

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The method of analysis developed in a previous paper is applied to the low-energy neutron cross sections of the common fissionable isotopes. Further evidence is presented to show that U^{235} possesses the unusual negative energy level required by the previous analysis. However, good fits are obtained for the cross sections of both U^{233} and Pu^{239} without such an unusual bound level, suggesting that the neutron resonance cross sections of the fissionable isotopes do not exhibit a basic anomaly. The size of the level interference effects in each of the isotopes implies that the fission process involves more than one but no more than a few fission channels.

I. INTRODUCTION

IN a previous paper[1] the general Wigner-Eisenbud resonance theory was used to derive a method of fitting the low-energy neutron resonance cross sections of the fissionable nuclei. The method was capable of describing the interference between levels of the same spin and parity using only a few parameters to describe the interference and without any assumption concerning the number of fission channels. Rather, the number of fission channels is found from the average value of the interference parameters. The method was employed, in I, to describe the low-energy neutron cross sections of U^{235} measured by Shore and Sailor.[2]

The purpose of the present paper is to apply the methods derived, in I, to the analysis of the cross-section data of the other common fissionable isotopes, U^{233} and Pu^{239}. In doing so we shall attempt to answer the question which originally motivated this investigation. For many years it has been asserted that the low-energy neutron cross sections of the common fissionable isotopes involved some basic peculiarities: for example, that each isotope had an anomalously large resonance just below the neutron binding energy. The question is: are the peculiarities real or have the assertions rested on inadequate applications of the resonance theory to the data? We shall also try to determine what the cross-section analysis has to say about the number of channels involved in fission.

The next section gives a brief review of the results obtained in I for U^{235} together with some amplifications of those results and some minor improvements in the method of analysis. The review and the amplifications serve two purposes. First of all it displays the variety of evidence, in U^{235}, for a negative energy resonance with a large neutron reduced width. Secondly, since the data on U^{235} are the most detailed and the analysis least ambiguous of all the fissionable isotopes, the discussion of U^{235} is helpful in showing how the method of I is to be applied to the other fissionable isotopes.

The analysis of U^{233} and Pu^{239} is given in Secs. III and IV.

II. U^{235}

The fit obtained to the cross sections of U^{235} is shown in Fig. 1 and the constants employed in the fit are given in Table I. The data employed and the results found are those previously reported in I except that $1+\alpha$, the ratio of the absorption cross section to the fission cross section, has now also been calculated and compared to the experimental data of the Brookhaven neutron cross-section compilation.[3]

Two minor improvements have been added to the method of analysis outlined in I which account for the slight change of the fit to U^{235} and the corresponding parameters from those given in I. The first improvement

[1] E. Vogt, Phys. Rev. **112**, 203 (1958), hereafter referred to as I.
[2] F. J. Shore and V. L. Sailor, Phys. Rev. **112**, 191 (1958).
[3] D. J. Hughes and R. B. Schwartz, *Neutron Cross Sections*, Brookhaven National Laboratory Report BNL-325 (Superintendent of Documents, U. S. Government Printing Office, Washington, D. C., 1958), second edition.