

Beta-Gamma Angular Correlation Measurements on Au¹⁹⁸. I. Directional and Circular Polarization Correlation*†

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The beta-gamma directional correlation of Au¹⁹⁸ has been measured as a function of the energy of the beta-particles. The anisotropy factor $A_2(W)$ in the beta-gamma directional correlation $\mathcal{W}_{\beta\gamma}(\theta, W) = 1 + A_2(W)P_2(\cos\theta)$ is proportional to p^2/W and its value near the maximum beta energy is $A_2(2.8) = 0.029 \pm 0.001$. The beta directional gamma circular polarization correlation of Au¹⁹⁸ was measured using the backscattering of the circularly polarized gamma radiation from a radially magnetized iron disk. The correlation is of the form: $\mathcal{W}_{\beta\gamma\text{pol}}(\theta, W) = 1 + (0.45 \pm 0.07)(p/W)P_1(\cos\theta)$. The shape correction factor of the first-forbidden beta spectrum is independent of the beta energy above $W = 1.6$ within 5%.

The analysis of the data shows that the ξ -approximation for first-forbidden nonunique beta transitions represents the Au¹⁹⁸ results in a very satisfactory manner.

1. INTRODUCTION

THE decay of Au¹⁹⁸ has been the subject of numerous investigations and the decay scheme (see insert of Fig. 3) is well established.¹ The main beta transition (99%) of 0.960-Mev maximum energy which leads from the Au¹⁹⁸ ground state ($I_0=2$) to the 0.411-Mev excited state ($I_1=2$) of Hg¹⁹⁸ was for some time considered to be an allowed transition, although its $\log ft$ -value of 7.46 is characteristic of a first-forbidden transition. The statistical shape of the spectrum and the absence of a beta-gamma anisotropy ($A = 0.00 \pm 0.02$) found by several experimenters²⁻⁵ supported this conclusion. In addition, the large $\log ft$ -value ($\log ft = 11.8$) of the 0.025% beta transition to the Hg¹⁹⁸ ground state could be explained conveniently as due to a second-forbidden transition.

In this paper conclusive evidence will be presented that the 0.960-Mev beta transition is a first-forbidden transition.⁶ It will also be shown that the so-called ξ -approximation for first-forbidden beta decay⁷⁻¹⁰ represents all the experimental data of Au¹⁹⁸ in a very satisfactory manner.

2. BETA-GAMMA ANGULAR CORRELATIONS OF FIRST-FORBIDDEN BETA TRANSITIONS IN THE ξ -APPROXIMATION

Kotani and Ross⁷⁻⁹ have developed a simple approximation for first-forbidden nonunique beta transitions. They expanded the transition probability in descending powers of $\xi = \alpha Z/2R$ and retained only the leading term for any observable. R is the nuclear radius in units \hbar/mc and α is the fine structure constant. This ξ -approximation presupposes that $\xi \gg W_0$, where W_0 is the maximum energy of the beta particles. Within the framework of the ξ -approximation, the spectrum shape correction factor $C(W)$ is independent of the beta energy:

$$C(W) = |V_0|^2 + |Y_1|^2, \quad (1)$$

where V_0 and Y_1 are the following combinations of nuclear matrix elements⁹:

$$\begin{aligned} V_0 &= [b_0 + 2\xi d_0(1+\gamma)^{-1}]/a_1, \\ Y_1 &= [c_1 - 2\xi d_1 + a_1(1+\gamma)^{-1}]/a_1, \\ \gamma &= [(1 - (\alpha Z)^2)^{\frac{1}{2}}] \end{aligned} \quad (2)$$

The first-forbidden matrix elements a_1 to d_2 are (nuclear size effects are neglected):

$$\begin{aligned} a_1 &= -C_V \int \mathbf{r}, & d_0 &= C_A \int \boldsymbol{\sigma} \cdot \mathbf{r}, \\ b_0 &= C_A \int i\gamma_5, & d_1 &= C_A \int \boldsymbol{\sigma} \times \mathbf{r}, \\ c_1 &= -C_V \int i\boldsymbol{\alpha}, & d_2 &= C_A \int B_{ij}. \end{aligned} \quad (3)$$

If the beta interaction Hamiltonian is time-reversal invariant the matrix elements a_1 to d_2 are real quantities.

The beta-gamma directional correlation involving a first-forbidden beta transition $I_0 \rightarrow \beta \rightarrow I_1$ followed by a gamma transition of multipole order L , $I_1 \rightarrow \gamma \rightarrow I_2$ is represented by:

$$\mathcal{W}_{\beta\gamma}(\theta, W) = 1 + A_2(W)P_2(\cos\theta), \quad (4)$$

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† A preliminary report of this work has been presented at the Rehovoth Conference on Nuclear Structure, September, 1957. [*Proceedings of the Rehovoth Conference on Nuclear Structure* (North-Holland Publishing Company, Amsterdam 1958), p. 426.]

¹ See *Nuclear Data Sheets* (National Research Council, Washington 25, D. C.).

² R. L. Garwin, Phys. Rev. **76**, 1876 (1949).

³ S. L. Ridgway, Phys. Rev. **78**, 821 (1950).

⁴ R. Stump and S. Frankel, Phys. Rev. **79**, 243 (1950).

⁵ M. Walter, O. Huber, and W. Zünti, Helv. Phys. Acta **23**, 697 (1950).

⁶ An anisotropy in the Au¹⁹⁸ beta-gamma directional correlation, which indicated that the beta transition is first-forbidden, was first found by T. D. Novey (private communication).

⁷ T. Kotani and M. Ross, Phys. Rev. Letters **1**, 140 (1958).

⁸ T. Kotani and M. Ross, Progr. Theoret. Phys. (Kyoto) **20**, 643 (1958).

⁹ T. Kotani and M. Ross, Phys. Rev. **113**, 622 (1959).

¹⁰ I. Iben, Jr., Phys. Rev. **111**, 1240 (1958).

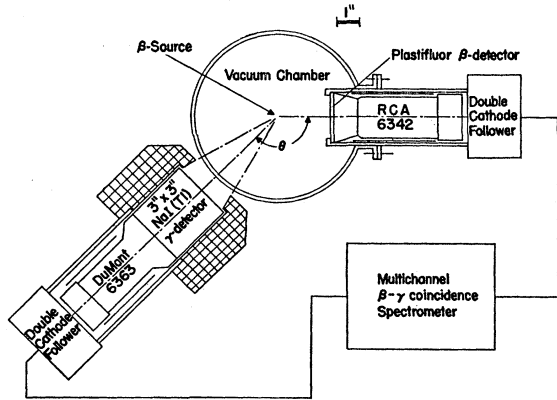


FIG. 1. The vacuum chamber and detector arrangement used in the Au^{198} beta-gamma directional correlation measurements.

where

$$A_2(W) = A_2^\beta(W) A_2^\gamma = A_2^\beta(W) F_2(LLI_2 I_1). \quad (5)$$

The F -coefficients $F_k(LLI_2 I_1)$ are tabulated by Alder et al.¹¹ In the ξ -approximation for nonunique transitions the factor $A_2^\beta(W)$ describing the beta transition is represented by:

$$A_2^\beta(W) = \lambda_2(Z, W) \frac{K(I_0 I_1)}{C(W)} \frac{p^2}{W}. \quad (6)$$

The factor $\lambda_2(Z, W)$ which contains Coulomb corrections of the order of $(\alpha Z W/p)$ is tabulated in reference 9. $K(I_0 I_1)$ is an energy-independent quantity which, if the beta interaction is time-reversal invariant, is defined by:

$$K(I_0 I_1) = (2/3)^{1/2} [g_{02}(2) V_0 d_2 - g_{11}(2) (2a_1 - d_1 V_1) - g_{12}(2) Y_1 d_2] (1/a_1). \quad (7)$$

The matrix element parameters d_2 and d_1 are given in Eq. (3) and

$$g_{L_1 L_1'}(k) = (-1)^{I_1 - I_0} W(I_1 I_1 L_1 L_1'; k I_0) (2I_1 + 1)^{1/2}. \quad (8)$$

$W(I_1 I_1 L_1 L_1'; k I_0)$ is a Racah coefficient.

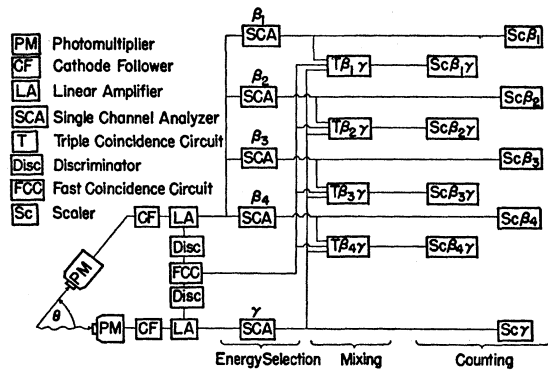


FIG. 2. Block diagram of the coincidence spectrometer.

¹¹ K. Alder, B. Stech, and A. Winther, Phys. Rev. **107**, 728 (1957).

Thus, within the validity of the ξ -approximation the anisotropy factor $A_2(W)$ of the beta gamma directional correlation is proportional to $\lambda_2 p^2/W$.

The beta directional-gamma circular polarization correlation is represented by:

$$\mathcal{W}_{\beta\gamma \text{ pol}}(\theta, W) = 1 + 3\mathcal{C} A_1(W) P_1(\cos\theta), \quad (9)$$

where

$$A_1(W) = A_1^\beta(W) A_1^\gamma = A_1^\beta(W) F_1(LLI_2 I_1). \quad (10)$$

\mathcal{C} is the helicity of the gamma radiation: $\mathcal{C} = +1$ for right circular polarization, $\mathcal{C} = -1$ for left circular polarization. In the ξ -approximation

$$A_1^\beta(W) = [2g_{01}(1) V_0 Y_1 - \sqrt{2} g_{11}(1) Y_1^2] \times (1/C(W)) (p/W). \quad (11)$$

Thus the degree of circular polarization of the gamma rays is proportional to $(p/W) = (v/c)$ as in the case of allowed beta transitions.

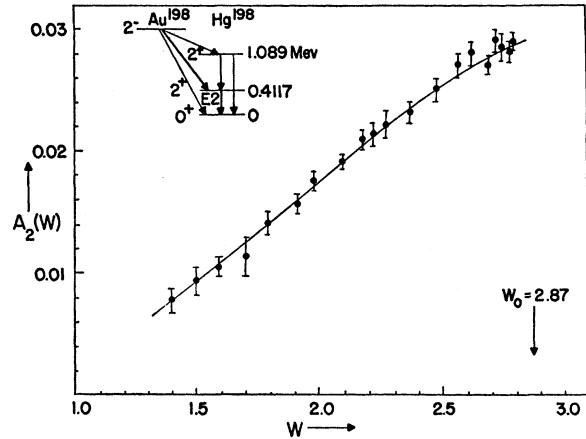


FIG. 3. Beta-gamma directional correlation of Au^{198} .

3. BETA-GAMMA DIRECTIONAL CORRELATION OF Au^{198}

The Au^{198} was obtained by exposing gold foils to the $>10^{-13}$ n/cm²/sec neutron flux in the Argonne CP5 reactor. The sources which were less than 10 $\mu\text{g}/\text{cm}^2$ thick were prepared by evaporating the radioactive gold on to a 180 $\mu\text{g}/\text{cm}^2$ Al foil backing.

The beta-gamma directional correlation measurements were performed with the vacuum chamber and counter arrangement shown in Fig. 1. A Pilot B plastic scintillator disk of $\frac{1}{8}$ inch thickness in conjunction with a Du Mont 6292 photomultiplier was used as a beta detector. The energy resolution for the 0.624-Mev conversion line of Cs^{137} was about 13%. The gamma detector was a 3-in. \times 3-in. NaI(Tl) crystal viewed by a Du Mont 6363 photomultiplier.

The coincidence electronics which was of the usual fast-slow type had four beta energy selection channels (Fig. 2) which permitted the simultaneous measurement of the Au^{198} beta-gamma correlation in four different

beta energy ranges. The pulse-height analyzer in the gamma channel was adjusted to accept the photopeak of the 0.411-Mev gamma radiation.

The beta-gamma coincidence counting rate was measured at seven different angles θ . After applying corrections for chance coincidences and background the coincidence rates were divided by the single counting rates. The data were then fitted to the function $\mathcal{W}_{\beta\gamma}'(\theta, W) = 1 + A_2'(W)P_2(\cos\theta)$ by a least squares fit. From the so determined experimental anisotropy factor $A_2'(W)$ the "true" anisotropy factor $A_2(W)$ was computed by taking into account the corrections for the finite solid angles of the detectors^{12,13} and the corrections for the backscattering of the beta particles in the beta scintillator.¹⁴ The resultant curve for $A_2(W)$ is shown in Fig. 3. According to Eqs. (5) and (6), $A_2(W)$ is proportional to $\lambda_2 p^2/W$ if the ξ -approximation is valid. Figure 4 shows a plot of $A_2(W)/(\lambda_2 p^2/W)$ versus W . Within experimental error this quantity is independent of W in agreement with the predictions of the ξ -approximation.

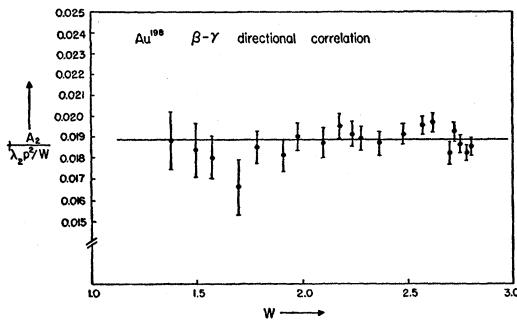


FIG. 4. The ratio $A_2/\lambda_2(p^2/W)$ of the Au¹⁹⁸ beta-gamma directional correlation versus W .

The fact that the beta-gamma directional correlation exhibits an appreciable anisotropy shows conclusively that the Au¹⁹⁸ beta decay is forbidden. Its ft -value ($\log ft = 7.6$) clearly classifies it as a first-forbidden transition.

4. CIRCULAR POLARIZATION OF THE GAMMA RADIATION FOLLOWING THE Au¹⁹⁸ BETA DECAY

Most of the circular polarization measurements on gamma rays performed so far used the forward scattering of gamma radiation on polarized electrons whose spins were almost parallel to the photon propagation direction. This method, however, is rather insensitive if it is applied to gamma rays of energies below 0.5 Mev. Since the energy of the Au¹⁹⁸ gamma radiation, 0.411 Mev, is below this critical value a different method

¹² S. Frankel, Phys. Rev. **83**, 673 (1951).

¹³ M. E. Rose, Phys. Rev. **91**, 610 (1953).

¹⁴ T. B. Novey, M. S. Freedman, F. T. Porter, and F. Wagner, Phys. Rev. **103**, 942 (1956).

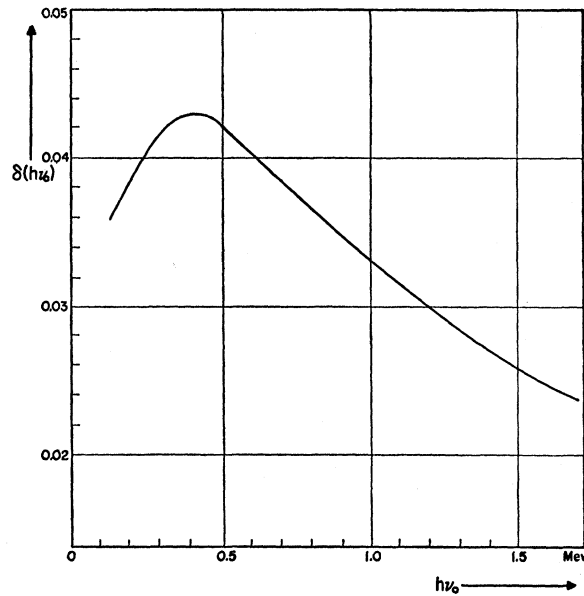


FIG. 5. Polarization efficiency $\delta(h\nu_0)$ of Beard-Rose scattering method versus gamma energy (scattering angle $\alpha = 125^\circ$, $f = 0.08$).

seems desirable. Beard and Rose¹⁵ proposed a method in which the electron spins are aligned perpendicular to the photon propagation vector. The azimuthal asymmetry of the intensity of the scattered gamma radiation is then a measure of the degree of the circular polarization of the radiation. If completely circularly polarized gamma radiation (energy $h\nu_0$) is scattered by an angle α from magnetized iron (relative number of polarized electrons = f) whose magnetization is perpendicular to the incident radiation, the relative difference δ in the intensity of the scattered radiation (energy $h\nu$) upon reversal of the magnetization is given by:

$$\delta(h\nu_0) = 2f \frac{(1 - \cos\alpha) \sin\alpha}{(h\nu_0/h\nu) + (h\nu/h\nu_0) - \sin^2\alpha} \frac{h\nu}{mc^2}. \quad (12)$$

Figure 5 shows a plot of δ versus the energy of the incident radiation for $\alpha = 125^\circ$ and for $f = 0.08$. Plural and multiple scattering effects were neglected in constructing this plot. One notices that the polarization

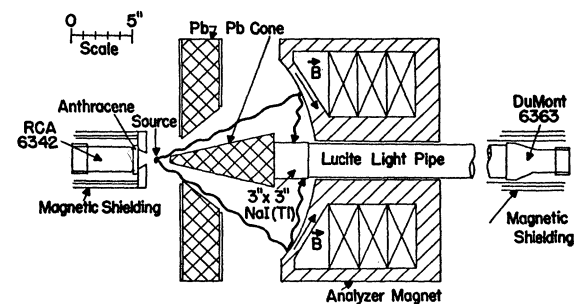


FIG. 6. Cross section of circular polarization analyzer.

¹⁵ D. B. Beard and M. E. Rose, Phys. Rev. **108**, 164 (1957).

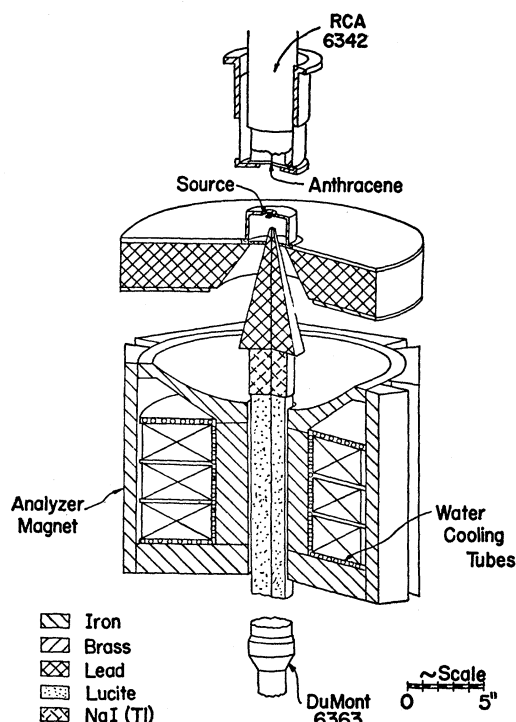
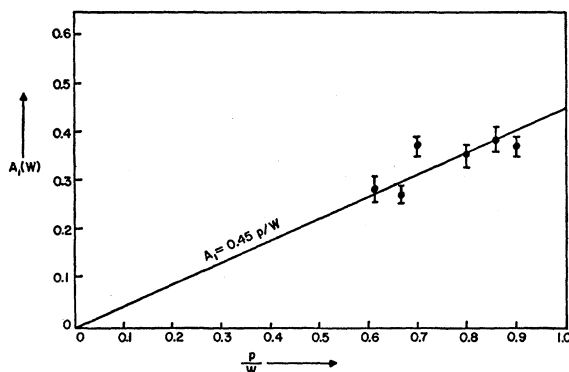


FIG. 7. Cut-away view of circular polarization analyzer magnet.

efficiency of an analyzer of this kind is particularly large for gamma energies of about 0.4 Mev.

With these considerations in mind the circular polarization correlation analyzer shown in Figs. 6 and 7 was constructed. The magnetization ($B=20\,000$ gauss) of the bowl-shaped disk from which the gamma radiation is backscattered is in a radial direction and can be reversed by reversing the current in the exciter coils. The average scattering angle of the gamma radiation is $\alpha=125^\circ$. The circular polarization efficiency of this analyzer was calibrated with the completely circularly polarized bremsstrahlung spectrum of A^{37} .¹⁶ A 50-mC source of A^{37} was used for this purpose. The

FIG. 8. Circular polarization of the 0.411-Mev gamma radiation following the beta decay of Au^{198} .

¹⁶ G. Hartwig and H. Schopper, Z. Physik 152, 314 (1958).

calibration measurements indicated a somewhat smaller polarization efficiency as expected on the basis of the theoretical calculations. This is probably a result of plural scattering in the iron. For the evaluation of the data the measured efficiency rather than the calculated one was used. As far as the absolute values of the measurements discussed below are concerned an error of about 10% should be assigned.

The circular polarization of the 0.411-Mev gamma radiation following the beta decay of Au^{198} was determined using the coincidence scintillation spectrometer described in Sec. 3. The results of the measurements, corrected for finite solid angles and for backscattering of the beta particles in the plastic scintillator are shown in Fig. 8. The data are in satisfactory agreement with $A_1(W)=\text{const}(p/W)$, as predicted by the ξ -approximation for first-forbidden nonunique beta transitions.

Taking into account the errors in the absolute calibration of the analyzer one obtains the result:

$$A_1(W) = (+0.45 \pm 0.07)(p/W). \quad (13)$$

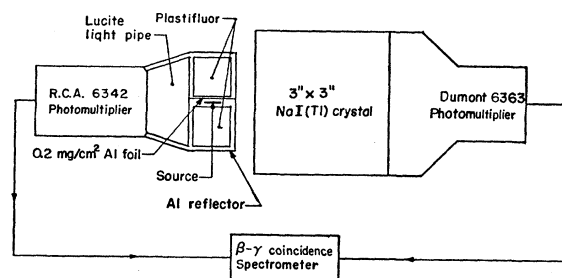


FIG. 9. Beta-gamma coincidence spectrometer.

This value is in satisfactory agreement with previous measurements.^{17,18}

5. SHAPE OF THE BETA SPECTRUM

The shape of the 0.96-Mev beta spectrum of Au^{198} was investigated by measuring coincidences between the beta particles and the following 0.411-Mev gamma ray in a split-crystal coincidence scintillation spectrometer. The experimental arrangement is shown in Fig. 9. The shape correction factor $C'(W)$ calculated from the experimental coincidence rate $N_{\beta\gamma}(W)$:

$$C'(W) = \frac{N_{\beta\gamma}(W)\Delta W}{pWF(Z,W)(W_0 - W)^2\Delta W} \quad (14)$$

is plotted in Fig. 10. As predicted by the ξ -approximation the shape factor is, within experimental errors, independent of W down to $W=1.6$ mc². The increase of the shape factor at lower energies is probably a result of the low intensity partial spectrum of 0.28-Mev

¹⁷ F. Boehm and A. H. Wapstra, Phys. Rev. 109, 456 (1958).

¹⁸ J. Berthier, P. Debrunner, W. Kündig, and B. Zählen, Helv. Phys. Acta 30, 483 (1958).

maximum energy and of Compton electrons of coincident gamma rays.

The precise measurements of Porter, Freedman, Novey, and Wagner¹⁹ also indicated a statistical shape of the Au¹⁹⁸ beta spectrum for $W > 1.6$ mc² within 2%.

6. ANALYSIS OF THE Au¹⁹⁸ RESULTS

The beta transition of Au¹⁹⁸ is nonunique ($\Delta I = 0$) first-forbidden. For Au¹⁹⁸ the parameter $\xi = \alpha Z / 2R \cong 16$ and the ξ -approximation should represent the Au¹⁹⁸ data to within about $1/\xi \cong 6\%$.

The experimentally determined shape correction factor $C'(W)$ of the Au¹⁹⁸ beta spectrum (Fig. 10) is, within limits of error (about $\pm 5\%$), independent of W (at least for $W > 1.6$) in agreement with Eq. (1).

The experimental circular polarization correlation coefficient $A_1(W)$ (Fig. 8) is, within limits of error, proportional to p/W as expected in the ξ -approximation [refer to Eqs. (10) and (11)]. For the decay scheme

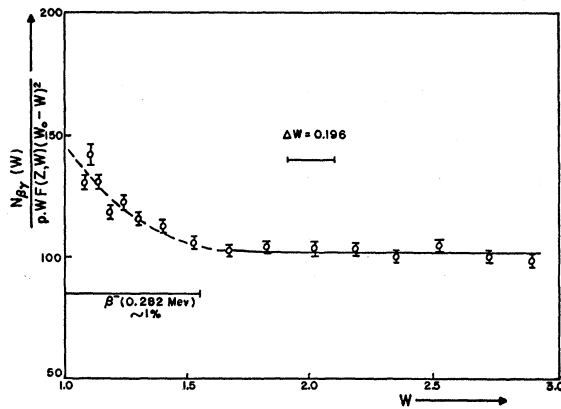


FIG. 10. Shape correction factor of Au¹⁹⁸ beta spectrum.

$2(\beta)2(\gamma, L=2)0$ the quantity $A_1(W)/(p/W)$ becomes

$$\frac{A_1(W)}{(p/W)} = \frac{0.167 - 0.816(V_0/Y_1)}{1 + (V_0/Y_1)^2}. \quad (15)$$

A plot of $A_1(W)/(p/W)$ is represented in Fig. 11 together with the experimentally determined value of $A_1(W)/(p/W)$. From this plot one extracts for the ratio V_0/Y_1 :

$$V_0/Y_1 = -1.0 \pm 0.7. \quad (16)$$

The experimental beta-gamma directional correlation factor $A_2(W)$ is, within limits of error (about $\pm 8\%$), proportional to $\lambda_2(Z, W)p^2/W$ (Fig. 4) as predicted by

¹⁹ F. T. Porter, M. S. Freedman, T. B. Novey, and F. Wagner, Phys. Rev. **103**, 921 (1956). This paper also contains a complete list of references to earlier shape measurements.

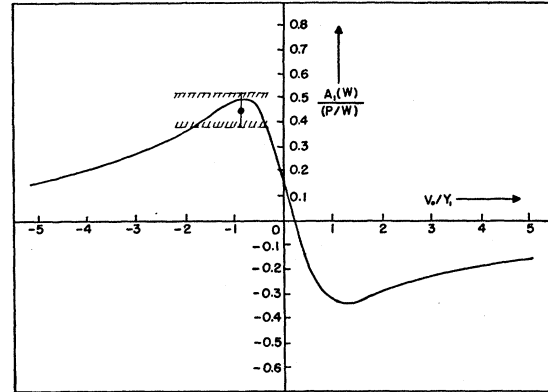


FIG. 11. The circular polarization correlation factor $A_1(W)/(p/W)$ versus the matrix element ratio V_0/Y_1 .

the ξ -approximation [Eq. (6)]. From the graph of Fig. 4 and from Eqs. (1), (5), and (6) one computes for $K(I_0I_1)/C(W)$:

$$\frac{K(I_0I_1)}{C(W)} = \frac{K(I_0I_1)}{V_0^2 + Y_1^2} = -0.032 \pm 0.002. \quad (17)$$

Making use of the result of the circular polarization correlation measurements, $V_0 \cong -Y_1$, and neglecting terms of order $1/\xi$, the following relationship between matrix element ratios is obtained [refer to Eqs. (7) and (17)]

$$-Y_1 = 5(d_1/a_1) + 9(d_2/a_1). \quad (18)$$

The relationships (16) and (18) are consistent with the ξ -approximation approach, which implies

$$|V_0| \cong |Y_1| \cong \xi(d_1/a_1) \cong \xi(d_2/a_1).$$

It may be added that in addition to the results presented in this paper measurements of the longitudinal polarization of the Au¹⁹⁸ electrons²⁰⁻²² also support the validity of the ξ -approximation.

ACKNOWLEDGMENTS

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²⁰ A. I. Alikhanov, G. P. Eliseiev, and V. A. Liubimov, Nuclear Phys. **7**, 655 (1958).

²¹ J. Heintze, Z. Physik **150**, 134 (1958).

²² N. Benczer-Koller, A. Schwarzschild, J. B. Vise, and C. S. Wu, Phys. Rev. **109**, 85 (1958).