that the events are nucleon-nucleon collisions for the purposes of the analysis. In the method used for the determination of $\gamma_c$, the mass of the primary particle is not an important factor; hence even though some of the collisions may actually be pion-nucleon collisions, the experimental results on angular, energy, and momentum distributions are not dependent on this assumption. The calculations of inelasticity are also practically independent of it.

We have searched without success for a group of interactions with special characteristics which might distinguish them as pion-nucleon collisions instead of nucleon-nucleon collisions. This negative result could mean either that there is no difference between nucleon-nucleon and pion-nucleon collisions at a given energy, that there are very few pions in the cosmic ray beam at these energies, or that the differences were too slight to be obvious in the data we have.

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# Numerical Evaluation of the Pion-Nucleon Forward Scattering Amplitude*

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The real part of the forward elastic scattering amplitude for $\pi^{\pm}$-proton scattering has been evaluated from experimental cross sections by means of dispersion relations. Recent measurements indicate two peaks in the $\pi^{-}$-proton total cross section at 590 and 870 Mev incident pion kinetic energy. Tables of the real part of the forward scattering amplitude for $\pi^{\pm}$-proton scattering are presented as a function of incident pion kinetic energy in the laboratory. The forward scattering amplitudes obtained from some recent $\pi^{\pm}$ scattering experiments are compared with the calculations. Measurement of the forward charge-exchange cross section appears to be the most suitable way of investigating the predictions of the dispersion relation at high energies. The possibility of detecting Coulomb interference at small angles is also discussed.

## I. INTRODUCTION

DISPERSION relations[1–3] have been developed which relate the real part of the forward scattering amplitudes of $\pi^{\pm}$-proton scattering with integrals over the imaginary part of the forward scattering amplitudes. Since the optical theorem relates the imaginary forward amplitude to the total cross section, the real part of the forward amplitude may be calculated from experimentally measured total cross sections. The dispersion relations were used by Sternheimer[4] to calculate the real part of the forward scattering amplitude for $\pi^{\pm}$-proton scattering up to 2 Bev using pion-proton total cross sections measured by Cool et al.[5] More recent calculations by Sternheimer appear in reference 5.

Since the work of Cool et al., extensive measurements have been made of the $\pi^{\pm}$-proton total cross sections. This work was stimulated by the discovery of a resonance in the photoproduction of $\pi^{-}$ mesons at a total center-of-mass energy (less proton mass) of 570 Mev.[6] Burrowes et al.[7] have measured $\pi^{\pm}$-proton total cross sections between 0.5 and 1.2 Bev.[8] Brisson et al.[9] have measured $\pi^{\pm}$-proton cross sections from 0.4 to 1.1 Bev with sharp energy resolution. Devlin et al.[10] have measured $\pi^{\pm}$-proton cross sections from 0.4 to 1.5 Bev. Longo et al.[11] have measured the $\pi^{+}$-proton cross sections from 1.2 to 4.0 Bev. These measurements have revealed two peaks in the $\pi^{-}$-proton cross sections at 590 Mev and 870 Mev instead of a single broad peak found by Cool et al. Figure 1 shows the $\pi^{\pm}$-proton total cross sections based on measurements referred to above.[12] The improved data and new structure warrant a new calculation of the forward scattering amplitudes based on the dispersion relations. Particular attention is given in this paper to applications of the dispersion relations above 300 Mev. Much discussion has been

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[2] M. L. Goldberger, H. Miyazawa, and R. Oehme, Phys. Rev. **99**, 986 (1955).

[3] H. L. Anderson, W. C. Davidon, and U. E. Kruse, Phys. Rev. **100**, 339 (1955).

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[6] R. R. Wilson, Phys. Rev. **110**, 1212 (1958).

[7] H. C. Burrowes, D. O. Caldwell, D. H. Frisch, D. A. Hill, D. M. Ritson, R. A. Schluter, and M. A. Wahlig, Phys. Rev. Letters **2**, 119 (1959).

[8] All energies unless otherwise specified refer to kinetic energy of the incident pion in the laboratory system.

[9] J. C. Brisson, J. Detoef, P. Falk-Vairant, L. van Rossum, G. Valladas, and L. C. L. Yuan, Phys. Rev. Letters **3**, 561 (1959).

[10] T. J. Devlin, B. C. Barish, W. N. Hess, V. Perez-Mendez, and J. Solomon, Phys. Rev. Letters **4**, 242 (1960).

[11] M. J. Longo, J. A. Helland, W. N. Hess, B. J. Moyer, and V. Perez-Mendez, Phys. Rev. Letters **3**, 569 (1959).

[12] References to the total cross sections below 300 Mev can be found in S. J. Lindenbaum, *Annual Review of Nuclear Science* (Annual Reviews, Inc., Palo Alto, 1957), Vol. 7, p. 317; also in reference 13.

given to the dispersion relations at low energies by Schnitzer and Salzman.[13]

## II. CALCULATIONS

The real parts of the forward amplitude were calculated in the laboratory system as a function of pion kinetic energy in the laboratory. The dispersion relations are given by[2]

$$D_{\pm}(k)=\tfrac{1}{2}(1\pm\omega/\mu)D_{+}(0)+\tfrac{1}{2}(1\mp\omega/\mu)D_{-}(0)+\frac{k^2}{4\pi^2}\mathrm{P}\int_{\mu}^{\infty}\frac{d\omega'}{k'}\frac{\sigma_{\pm}(\omega')}{\omega'-\omega}+\frac{k^2}{4\pi^2}\int_{\mu}^{\infty}\frac{d\omega'}{k'}\frac{\sigma_{\mp}(\omega')}{\omega'+\omega}\pm\frac{2f^2}{\mu^2}\frac{k^2}{\omega\mp\mu^2/2M}, \quad (1)$$

where $D_{\pm}(k)$ is the real part of the $\pi^{\pm}$-proton forward scattering amplitude at the pion laboratory wave number $k$, $\omega$ is the total energy of the pion in the laboratory, $\mu$ is the pion mass, $\sigma_{\pm}(\omega')$ is the total $\pi^{\pm}$-proton cross section at energy $\omega'$, $M$ is the nucleon mass. The units of $k$, $\omega$, $\mu$, and $M$ are $\mathrm{cm}^{-1}$. P means take the principal value of the integral. The coupling constant has the value $f^2=0.08$; the starting values for the amplitudes are $D_{+}(0)=-0.148\times10^{-13}$ cm and $D_{-}(0)=+0.106\times10^{-13}$ cm.[13]

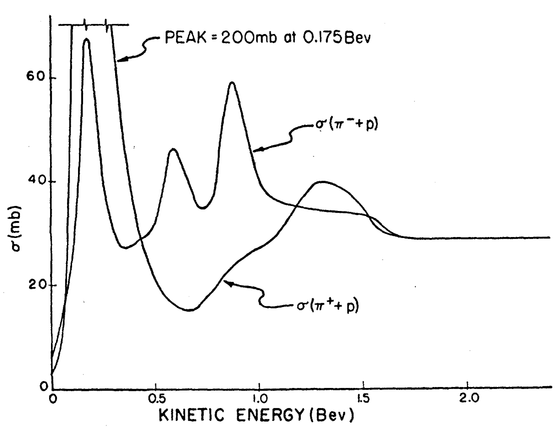


FIG. 1. Total $\pi^-$-proton and $\pi^+$-proton cross sections as a function of incident pion kinetic energy in the laboratory.

The total cross-section curve for the purpose of calculation was divided into straight line segments connecting points 10 Mev apart between 0 Mev and 1800 Mev. Beyond 1800 Mev the cross sections are assumed to be constant and equal to 29 mb[11] for both positive and negative pions. Within the framework of the straight line approximations the calculations were carried out exactly using an IBM 650 computer. Figure 2 shows the results of the calculation and Table I tabulates $D_{\pm}$ as a function of pion laboratory kinetic energy. The values of $D_{\pm}$ are expressed in the laboratory system.

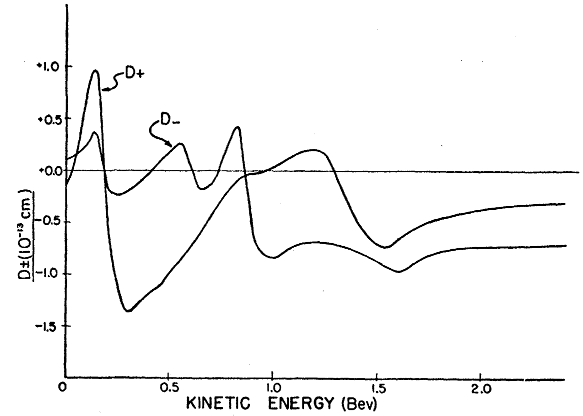


FIG. 2. Real part of forward $\pi^-$-proton and $\pi^+$-proton elastic scattering amplitudes as a function of incident pion kinetic energy in the laboratory. The values for $D_{\pm}$ are expressed in the laboratory system.

The largest source of error in $D_{\pm}$ at high energies arises from the uncertainty in the coupling constant $f^2$.

TABLE I. Calculated values of $D_+$ and $D_-$.

| Pion kinetic energy in lab (Mev) | $D_+$ ($10^{-13}$ cm) | $D_-$ ($10^{-13}$ cm) |
|---|---|---|
| 0 | −0.15 | +0.10 |
| 25 | +0.00 | +0.12 |
| 50 | +0.20 | +0.15 |
| 75 | +0.48 | +0.19 |
| 100 | +0.75 | +0.24 |
| 125 | +0.94 | +0.33 |
| 150 | +0.92 | +0.33 |
| 175 | +0.37 | +0.12 |
| 200 | −0.47 | −0.08 |
| 250 | −1.10 | −0.24 |
| 300 | −1.37 | −0.22 |
| 350 | −1.28 | −0.12 |
| 400 | −1.19 | −0.03 |
| 450 | −1.11 | +0.06 |
| 500 | −0.99 | +0.17 |
| 550 | −0.85 | +0.26 |
| 600 | −0.73 | −0.01 |
| 650 | −0.58 | −0.17 |
| 700 | −0.42 | −0.14 |
| 750 | −0.27 | +0.08 |
| 800 | −0.15 | +0.36 |
| 850 | −0.06 | +0.17 |
| 900 | −0.03 | −0.63 |
| 950 | −0.00 | −0.80 |
| 1000 | +0.03 | −0.83 |
| 1100 | +0.15 | −0.72 |
| 1200 | +0.20 | −0.69 |
| 1300 | −0.08 | −0.72 |
| 1400 | −0.47 | −0.76 |
| 1500 | −0.70 | −0.87 |
| 1600 | −0.65 | −0.96 |
| 1700 | −0.52 | −0.85 |
| 1800 | −0.45 | −0.77 |
| 1900 | −0.40 | −0.74 |
| 2000 | −0.36 | −0.73 |
| 2200 | −0.32 | −0.71 |
| 2400 | −0.29 | −0.71 |
| 2600 | −0.27 | −0.71 |

[13] H. J. Schnitzer and G. Salzman, Phys. Rev. **112**, 1802 (1958); **113**, 1153 (1959). This work gives reference to other authors who have discussed the dispersion relations at low energies.

TABLE II. Comparison of calculated and measured forward elastic scattering cross sections.

| Reaction | Kinetic energy in lab (Mev) | $[D_{\pm}]^2$ in c.m. (mb/sr) | $\frac{d\sigma^{\pm}}{d\Omega}(0)$ (mb/sr) Predicted | Experimental | Reference |
|---|---|---|---|---|---|
| | 460 | 0.04 | 2.2 | 2.3±0.5 | a |
| | 600 | 0.00 | 7.0 | 5.5±0.5 | a |
| $\pi^-+p$ | 770 | 0.14 | 6.2 | 6.5±0.7 | a |
| | 810 | 0.53 | 9.5 | 15 ±4 | b |
| | 960 | 2.00 | 14.3 | 12.5±1.5 | c |
| | 500 | 4.30 | 5.5 | 5.5±0.8 | d |
| $\pi^++p$ | 990 | 0.00 | 4.2 | 3.5±0.5 | e |
| | 1100 | 0.06 | 6.0 | >4.0 | f |

[a] R. R. Crittenden, J. H. Scandrett, W. D. Shephard, W. D. Walker, and J. Ballam, Phys. Rev. Letters **2**, 121 (1959).
[b] L. Baggett, University of California Radiation Laboratory Report UCRL-8302 (unpublished).
[c] A. R. Erwin and J. Kopp, Phys. Rev. **109**, 1364 (1958).
[d] W. J. Willis, Phys. Rev. **116**, 753 (1959).
[e] A. R. Erwin (private communication).
[f] L. O. Roellig and D. A. Glaser, Phys. Rev. **116**, 1001 (1959).

The inaccuracy in $f^2$ is about 10%.[13,14] At energies where $\omega \gg \mu$ the term of Eq. (1) which contains the coupling constant goes as $(\omega/\mu)(2f^2/\mu)$. At high energies the uncertainty in $D_{\pm}$ is $\pm 0.023(\omega/\mu)\times 10^{-13}$ cm. At 1 Bev this is an error of $\pm 0.18\times 10^{-13}$ cm. Errors due to uncertainty in the values of $D_{\pm}(0)$ are considerably smaller. Errors due to uncertainties in the total cross sections are difficult to evaluate in a systematic way. A Gaussian-like bump was deliberately added to the $\pi^+$-proton cross-section curve between 300 and 450 Mev. This bump had a peak of 6-mb and joined the total cross-section curve smoothly at the end points. The bump produced deviations in $D_+$ of about $\pm 0.1\times 10^{-13}$ cm in the perturbed region. The effect of the bump was negligible elsewhere. A similar bump with a 5-mb peak was added between 700 and 1200 Mev. This produced deviations in $D_+$ of $\pm 0.2\times 10^{-13}$ cm only in the perturbed region. Based on the above results, we conclude that errors in $D_{\pm}$ introduced by uncertainties in the total cross sections are less than $\pm 0.1\times 10^{-13}$ cm below 2 Bev.

## III. DISCUSSION

The general agreement between our result and Sternheimer's[4] is good for $D_+$. Our results for $D_-$ reflect the structure of the $\pi^-$-proton total cross-section curve. This is expected since the general appearance of the $D_-$ curve is similar to the derivative of the total cross-section curve. The locus of the forward amplitude in the complex plane has the behavior of going through a counterclockwise rotation for each peak in the total cross section. Adair[15] points out that this is qualitatively the behavior of the scattering amplitude in the region of a resonance. Thus each peak in the cross section is equivalent to a resonance or an isobaric state in the pion-nucleon system.

The $\pi^{\pm}$-proton forward elastic scattering cross section is given by

$$\frac{d\sigma_{\pm}}{d\Omega}(0)=[D_{\pm}]^2+[(k/4\pi)\sigma_{\pm}]^2. \tag{2}$$

This prediction is compared with recent results on $\pi^{\pm}$-proton forward scattering in Table II. For $\pi^-$-proton scattering above 300-Mev agreement with Eq. (2) shows only that $D_-$ is small; agreement is not sensitive to the exact value of $D_-$. The same conclusion holds true for $D_+$ above 700 Mev. The experiment of Willis does quantitatively agree with $D^+$ at 500 Mev. Experiments of a few percent accuracy in extrapolating to the forward scattering cross section are required to detect the presence of $D_{\pm}$ at energies in the Bev region.

The amount of interference between the Coulomb scattering amplitude and the nuclear amplitude is sensitive to the value of $D_{\pm}$. Above 500 Mev the Coulomb amplitude is essentially real and given at small forward angles by $|f_c|=2k/137(\Delta k)^2$ where $\Delta k$ is the momentum transfer. An attractive Coulomb potential gives a positive $f_c$. Figure 3 shows the laboratory angular dependence of the cross section near the forward direction for $\pi^-$-proton scattering at 970 Mev. At this energy $D_-=-0.83\times 10^{-13}$ cm. A curve for $D_-=0.00$ is also shown for comparison. The effect of the presence of $D^+$ is significant in the region from 2 to 3 degrees. To perform such an experiment requires angular resolution of the order of a few tenths of a degree.

The values of $D_{\pm}$ and $\sigma_{\pm}$ determine explicitly the optical model potentials for high-energy pion-nucleus interactions.[4] The real potential is proportional to the factor $\bar{D}=[ZD_++(A-Z)D_-]/A$. At high energies the sum $(D_++D_-)$ depends only slightly on $f^2$, so that the uncertainty in the prediction of the potential is small. The absorption and diffraction cross sections ($\sigma_a$ and $\sigma_d$, respectively) for elements between Be and Ca have

[14] *1958 Annual International Conference on High-Energy Physics at CERN*, edited by B. Ferretti (CERN Scientific Information Service, Geneva, 1958), p. 45.
[15] R. K. Adair, Phys. Rev. **113**, 338 (1959).

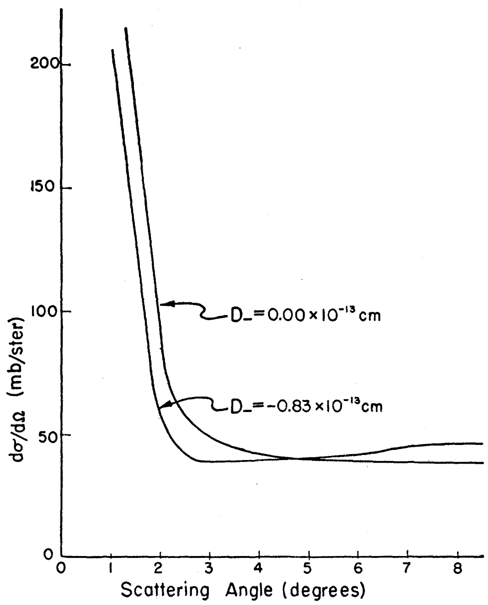


FIG. 3. $\pi^-$-proton differential elastic scattering cross section near the forward direction showing the region of Coulomb interference. The curves are plotted in the laboratory system.

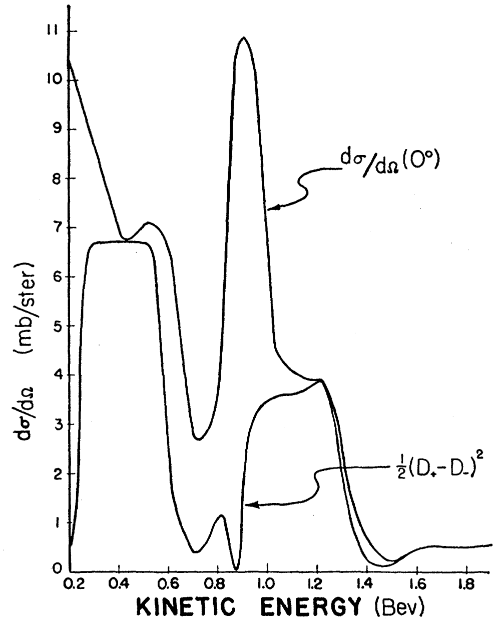


FIG. 4. Differential cross section for elastic charge-exchange scattering in the forward direction as a function of incident pion kinetic energy in the laboratory. The cross section is expressed in the laboratory system.

been measured for $\pi^-$ at 970 Mev.[16] Here the value of $\bar{D}$ is $0.41\times10^{-13}$ cm, which predicts the ratio of $\sigma_d/\sigma_a$ to be about 10% larger than if $\bar{D}$ were zero. The experimental data are consistent with a value of $\bar{D}=(1.0\pm0.4)\times10^{-13}$ cm. A careful experiment will have to measure $\sigma_d/\sigma_a$ with an accuracy of a few percent to check the dispersion relation prediction.

Cool et al.[5] point out that measurement of the elastic charge-exchange cross sections in the forward direction is sensitive to the results of the dispersion relations. Assuming charge independence this cross section is given by

$$\frac{d\sigma}{d\Omega_{\text{ch.ex.}}}(0)=\tfrac{1}{2}(D_+-D_-)^2+\tfrac{1}{2}\left(\frac{k}{4\pi}\right)^2(\sigma_+-\sigma_-)^2. \quad (3)$$

In regions where $\sigma_+\cong\sigma_-$ the cross section is dominated by the real parts of the amplitudes. Figure 4 shows the predicted forward charge-exchange cross section. The cross section depends sensitively on the value of $(D_+-D_-)$ in the energy ranges from 300 to 500 Mev, 900 to 1300 Mev, and beyond 1600 Mev. Unfortunately, the uncertainty in the difference $(D_+-D_-)$ is just twice the uncertainty in $D_+$ or $D_-$ alone since the $f^2$ terms add when $D_-$ is subtracted from $D_+$. Nevertheless, since the total cross sections are well known, there are energy regions where the forward charge-exchange cross section depends only on $(D_+-D_-)$. Measurement of these cross sections appears to be the most suitable method to detect the presence of real parts of the forward elastic scattering amplitudes at high energies.

[16] J. W. Cronin, R. Cool, and A. Abashian, Phys. Rev. **107**, 1121 (1957).

## IV. ACKNOWLEDGMENTS

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