that it is the explanation of the experiment of Fields *et al.*

As a final possibility, this paper proposes that the basic Stark effect model of reference 5 be applied to a (π^-, H_2^+) molecule. Thus, if it is assumed that such a molecule is formed, the result of Fields et al. could be accounted for with a molecular Stark effect. This would require a pion density at a proton of the same order as would be expected using a Heitler-London wave function for the pion in the Born-Oppenheimer approximation treatment of the molecule if the pion were in an $n \simeq 12$ state with $\Lambda = 0$ only. While it is expected that this approximation is quite poor for the π-mesonic molecule, it is not unreasonable to expect that the true pion wave function has a comparable density at the proton.

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Interference Phenomena in Nuclear Scattering of Neutral K Mesons*

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The scattering of neutral K mesons has been treated phenomenologically. The scattered beam, in general, contains both K_0 and $\bar{K}_0$ components having different amplitudes. These amplitudes interfere with each other in the generation of K_1 and K_2 components in the scattered beam. The relative sign of the two amplitudes may then be determined from the analysis of K_1, K_2 decays. The leptonic decay rates of the scattered beam show a dependence on ΔM, the mass difference between K_1, K_2 in such a way that the sign of ΔM can, in principle, be determined experimentally.

I. INTRODUCTION

AMONG the elementary particles, the neutral K mesons present a unique situation. They occur as two distinct kinds of particles according to their strong and weak interactions. The weakly interacting particles—the short-lived K_1 and the long-lived K_2—have been described as linear combinations of the strongly interacting K_0 and $\bar{K}_0$ particles, and vice versa.[1] Recent experiments support this description.[2] The encounter of such a mixture of particles and antiparticles shows some interesting phenomena, such as characteristic interaction in dense matter[3] and the interference between K_1 and K_2 components in the leptonic decay modes.[4]

The scattering of neutral K mesons may be explored to obtain some interesting results. Whereas a neutral K beam in dense material (Pais-Piccioni experiment) loses almost all of its $\bar{K}_0$ component,[5] such a beam being scattered by protons would contain both K_0 and $\bar{K}_0$ components having different amplitudes. In the subsequent decays, these amplitudes would interfere, and the decay ratios may be helpful in determining the relative sign of the K_0 and $\bar{K}_0$ nuclear potentials. The interference in the leptonic decay modes would also be expected to be different from that of an unscattered beam.

II. NUCLEAR SCATTERING ON PROTONS

For simplicity, we start with a K_0 beam, allowing the K_1 component to decay almost completely, and consider the scattering of K_2 mesons on protons. The attenuation of the beam due to its decay may then be neglected, because the K_2 mean life is rather large.

The wave functions for the different components are

$$\begin{aligned} \psi(K_1) &= (1/\sqrt{2})[\psi(K_0)+\psi(\bar{K}_0)], \\ \psi(K_0) &= (1/\sqrt{2})[\psi(K_1)+i\psi(K_2)], \\ \psi(K_2) &= (1/\sqrt{2}i)[\psi(K_0)-\psi(\bar{K}_0)], \\ \psi(\bar{K}_0) &= (1/\sqrt{2})[\psi(K_1)-i\psi(K_2)]. \end{aligned} \tag{1}$$

The wave function $\psi(K_2)$ is modified after scattering as

$$\psi_{\text{scat.}} = (1/\sqrt{2}i)\{[\tfrac{1}{2}(1-\eta_1)+\tfrac{1}{2}(1-\eta_0)]\psi(K_0) - [1-\bar{\eta}_1]\psi(\bar{K}_0)\}, \tag{2}$$

where $\eta_{1,0}$ is $\exp(2i\delta_{1,0})$, $\delta_{1,0}$ corresponding to the real

* This work was done under the auspices of the U. S. Atomic Energy Commission.

[1] M. Gell-Mann and A. Pais, Phys. Rev. **97**, 1387 (1955).

[2] M. Bardon, K. Lande, L. M. Lederman, and W. Chinowsky, Ann. Phys. **5**, 156 (1958). Reference to earlier works can be found in this article.

[3] A. Pais and O. Piccioni, Phys. Rev. **100**, 1487 (1955).

[4] R. G. Sachs and S. B. Treiman, Phys. Rev. **103**, 1545 (1956).

[5] A more general analysis of K_0 mesons traversing an absorber in the regeneration of K_1 and K_2 components has been made by Good in terms of forward-scattering amplitudes. [M. Good, Phys. Rev. **106**, 591 (1957).]

or complex phase shifts for $T=1$, 0 isotopic spin states for the K_0-p interaction. Similarly, $\bar{\eta}_1$ is related to the $\bar{K}_0-p$ interaction having a pure $T=1$ state.

The absorbed wave-producing hyperons is given by

$$\psi_{\text{abs}}=(i/\sqrt{2})(1-|\bar{\eta}_1|^2)^{\frac{1}{2}}\psi(\bar{K}_0). \quad (3)$$

Equation (2) can be expressed in terms of the K_1 and K_2 components using Eq. (1) as follows:

$$\psi_{\text{scat.}}=(1/2i)\{[\tfrac{1}{2}(1-\eta_1)+\tfrac{1}{2}(1-\eta_0)-(1-\bar{\eta}_1)]\psi(K_1) + i[\tfrac{1}{2}(1-\eta_1)+\tfrac{1}{2}(1-\eta_0)+(1-\bar{\eta}_1)]\psi(K_2)\}. \quad (4)$$

It may be noted from these expressions that the hyperon-producing reaction is governed by a pure isotopic-spin-state ($T=1$) interaction and that the amplitudes of K_1 and K_2 components in the scattered beam (Eq. 4) are dependent both on magnitudes and the relative sign of the K_0 and $\bar{K}_0$ amplitudes after scattering. Thus the decay of the scattered beam would show an interference between K_0, $\bar{K}_0$ nuclear fields.

III. INTERFERENCE IN THE LEPTONIC DECAY MODES

The interference in the leptonic decay modes, for example, $e^+\pi^-\nu$ and $e^-\pi^+\tilde{\nu}$ decays of neutral K mesons,[6] would occur in the scattered beam following the same mechanism as in case of a normal beam, and would be dependent on the mass difference ΔM between K_1 and K_2. However, the decay rates of different charges are found to have somewhat different dependence on ΔM.

Dropping the factor $1/2i$ in Eq. (4), which accounts for the attenuation of the original beam, we write the scattered beam as

$$\psi_{\text{scat.}}=(A+iB)\psi(K_1)+i(C+iD)\psi(K_2), \quad (5)$$

with the substitutions

$$\begin{aligned}\tfrac{1}{2}(1-\eta_1)+\tfrac{1}{2}(1-\eta_0)-(1-\bar{\eta}_1)&=A+iB,\\ \tfrac{1}{2}(1-\eta_1)+\tfrac{1}{2}(1-\eta_0)+(1-\bar{\eta}_1)&=C+iD.\end{aligned} \quad (6)$$

The time-dependent scattered wave is

$$\psi(t)=(A+iB)\exp(-\tfrac{1}{2}\lambda_1 t-i\omega_1 t)\psi(K_1) + i(C+iD)\exp(-\tfrac{1}{2}\lambda_2 t-i\omega_2 t)\psi(K_2), \quad (7)$$

where $\hbar(\omega_1-\omega_2)=c^2\Delta M$ is the mass difference between K_1 and K_2, and λ_1 and λ_2 are the respective decay constants.

Following Treiman and Sachs,[4] we can describe the decay schemes as

$$\psi(K_1)\rightarrow\alpha_1(e^+\pi^-\nu+e^-\pi^+\tilde{\nu})+\beta_1(\pi^+\pi^-)+\cdots \quad (8)$$

and

$$\psi(K_2)\rightarrow -i\alpha_2(e^+\pi^-\nu-e^-\pi^+\tilde{\nu})+\beta_2(\pi^+\pi^-\pi^0)+\cdots$$

where the α's and β's are real and specify the branching ratios of electronic decay modes to the dominant pionic modes.

By substituting schemes (8) in Eq. (7), we can find the electronic decay amplitudes of different charges. The e^+ decay rate is given by

$$\begin{aligned}R(e^+\pi^-\nu)&\\ =\alpha^2|&(A+iB)\exp(-\lambda_1 t/2)+(C+iD)\\ &\times\exp(-\lambda_2 t/2)[\cos(\Delta Mt)+i\sin(\Delta Mt)]|^2\\ =\alpha^2\{&(A^2+B^2)\exp(-\lambda_1 t)+(C^2+D^2)\\ &\times\exp(-\lambda_2 t)+2[(AC+BD)\cos(\Delta Mt)\\ &+(BC-AD)\sin(\Delta Mt)]\exp[-\tfrac{1}{2}(\lambda_1+\lambda_2)t]\}, \quad (9)\end{aligned}$$

where we have put $\alpha_1=\alpha_2=\alpha$ for simplicity. The e^- decay rate $R(e^-\pi^+\tilde{\nu})$ is obtained from Eq. (9) by changing the sign of the third term.

The leptonic (electronic) decay rates for the scattered beam differ from those for a normal beam in two respects: (a) the appearance of the sine term of ΔM dependence, and (b) the dependence of the third term of Eq. (9) on the signs and magnitudes of the amplitudes $A+iB$, and $C+iD$, in which the effect of interference of K_0 and $\bar{K}_0$ nuclear potentials is reflected. The appearance of the sine term indicates that the decay rates are also dependent on the sign of the mass difference. Thus, in principle, a determination of the sign would be possible from the rates of leptonic decays.[7]

On the assumption $\lambda_1\gg\lambda_2$, expression (9) may be simplified further to

$$R\binom{e^+}{e^-}\approx(A^2+B^2)\exp(-\lambda_1 t)+(C^2+D^2) \pm 2[(AC+BD)\cos(\Delta Mt)+(BC-AD)\sin(\Delta Mt)] \times\exp(-\lambda_1 t/2), \quad (10)$$

which is a good approximation for time intervals comparable to the mean life of K_1.

IV. NUMERICAL COMPUTATIONS

It may be of interest to obtain an idea of the effect of interference by a numerical analysis. It seems appropriate now to follow the phenomenological treatment of the scattering of K mesons by the use of a zero-range approximation.[8] Zero-energy scattering lengths are then convenient to use and are defined as usual by $k\cot\delta_T=1/A_T$, where k is the wave number. A_T may be real or complex, and the higher-order term of k^2 is neglected in the effective-range expansion. Using real scattering lengths for the K_0-p interaction $A_{1,0}=a_{1,0}$ and a complex length for the $\bar{K}_0-p$ interaction $\bar{A}_1=\bar{a}_1+i\bar{b}_1$, we relate the numbers of K_1, K_2, and hyperons H to the following cross sections derived

[6] These decay schemes are in accordance with the strangeness selection rule $\Delta S=1$ and are also supported by experiments. See reference 2.

[7] I am indebted to Dr. Myron L. Good for pointing this out to me.

[8] J. D. Jackson, D. G. Ravenhall, and H. W. Wyld, Jr., Nuovo cimento 9, 834 (1958).

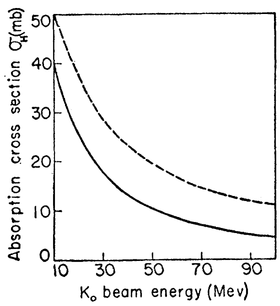

FIG. 1. Absorption cross section as a function of K_0 beam energy for solutions I (solid curve) and II (dotted curve).

from Eq. (3) and (4):

$$\sigma(K_1)=\pi\left|\tfrac{1}{2}\frac{a_1}{1-ika_1}+\tfrac{1}{2}\frac{a_0}{1-ika_0}-\frac{\bar{a}_1+i\bar{b}_1}{1+k\bar{b}_1-ik\bar{a}_1}\right|^2 =\pi|A+iB|^2$$

$$\sigma(K_2)=\pi\left|\tfrac{1}{2}\frac{a_1}{1-ika_1}+\tfrac{1}{2}\frac{a_0}{1-ika_0}+\frac{\bar{a}_1+i\bar{b}_1}{1+k\bar{b}_1-ik\bar{a}_1}\right|^2 =\pi|C+iD|^2 \tag{11}$$

$$\sigma_H=\frac{2\pi}{k}\frac{\bar{b}_1}{1+2k\bar{b}_1+k^2(\bar{a}_1{}^2+\bar{b}_1{}^2)}.$$

These expressions are in terms of cross sections, which one would observe under the hypothesis that all K_1 or K_2 decays (charged and neutral) are detectable and all absorptions are identified by themselves or from the associated pions.

The magnitudes and signs of all the scattering lengths are at present not well-known. At low energies, however, the data on K^+ and K^- scattering may be described in terms of an s-wave interaction by the use of energy-independent lengths. The K^+-nucleon data favor repulsive potentials for both the isotopic spin states. We therefore assume $a_1=-0.34$ f (1f$=10^{-13}$ cm) and $a_0=-0.20$ f. The value of a_1 is rather well-known from K^+-p scattering data; as for a_0, its magnitude seems to be smaller than that of a_1, at least at low energies.[9] As may be inferred from K^+-interaction data on emulsion nuclei, the charge-exchange scattering cross section seems to be quite energy-dependent at higher energies. At the energies considered here, within the frame-work of the effective range expansion, a weakly energy-dependent a_0 may seem to be more appropriate. However, the results would not depend much on these finer details.

Dalitz and Tuan have analysed the K^--p scattering data to determine the scattering lengths corresponding to different isotopic spin states.[10] Unfortunately, the solutions are not unique. They obtain two solutions for each isotopic spin state; those corresponding to the $T=1$ state are

$$\bar{A}_1=1.62+i0.38\text{ f} \tag{I}$$

and

$$\bar{A}_1=0.40+i0.41\text{ f.} \tag{II}$$

The positive real parts of the solutions imply a potential opposite in sign to that of the K^+-nucleon potential. Only indirect evidence of an attractive K^--nucleus potential is available at present from low energy K^- interaction on emulsion nuclei.[11] A large K^--p elastic and a small charge-exchange cross sections suggests that the potentials are of the same sign in both isotopic spin states.[12] However, the sign of the real parts of all the solutions can be changed, which would explain the K^--p scattering data equally well. If we assume the sign to be undetermined, there would then be four solutions for $\bar{A}_1$. Computations have been made to evaluate σ_{k1}, σ_{k2}, σ_H of Eq. (11) for these four solutions.

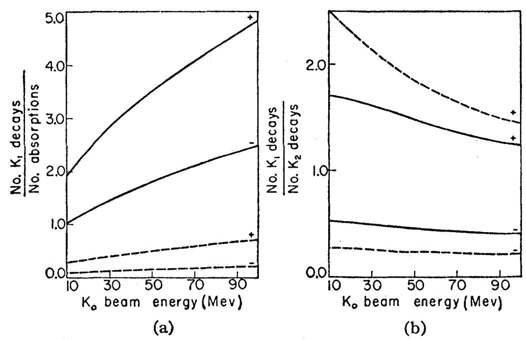

FIG. 2. (a) K_1/H ratio in the scattered beam as a function of beam energy for solutions I (solid curves) and II (dotted curves). The sign of the real part of the scattering lengths is labelled on the curves. (b) K_1/K_2 ratio for the corresponding cases of (a).

The electronic decay rates

$$R\binom{e^+}{e^-}$$

have been calculated from Eq. (10) via Eq. (11) for three values of ΔM, $\Delta M=0$, $\Delta M=+\lambda_1$, and $\Delta M=-\lambda_1$, for a K_0 beam energy of 50 Mev.

Some quantities of experimental interest are plotted in Figs. 1–4. We have restricted ourselves to the energy range of 10 to 100 Mev of the K_0 beam. The validity of the effective-range expansion at the high-energy end may be somewhat doubtful, because contribution from waves of angular momentum $l>0$ would not be unexpected.

The absorption cross section would be somewhat different (Fig. 1) for the two solutions. The ratio of the K_1 component to the absorbed component [Fig. 2(a)]

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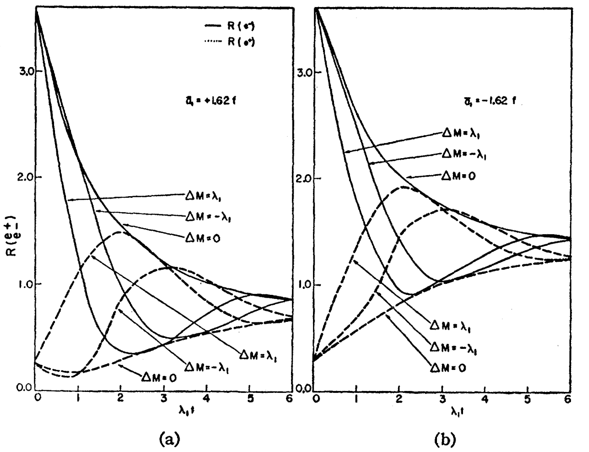

FIG. 3. (a) The electronic decay rates $R(e^+)$ (dotted curves), and $R(e^-)$ (solid curves) for three different values of ΔM, of a 50-Mev K_0 beam as a function of time. The curves correspond to solution I, with the real part having a plus sign. (b) The same as in (a) with the real part having a minus sign.

seems to be a sensitive parameter in determining the sign as well as the magnitude of the scattering length uniquely from the multiple solutions (I) and (II). The K_1/K_2 ratio is shown in Fig. 2(b) for comparison. The electronic decay rates are shown in Fig. 3(a), 3(b) (solution I) and 4(a), 4(b) (solution II). It may be noted that in some cases, the amplitude of the decay rates would be more oscillatory than that for an unscattered beam. This would enable one to obtain a greater resolution in the determination of ΔM, than that in a normal beam. The dependence of the decay rates on the sign of ΔM is also rather remarkable from these figures.

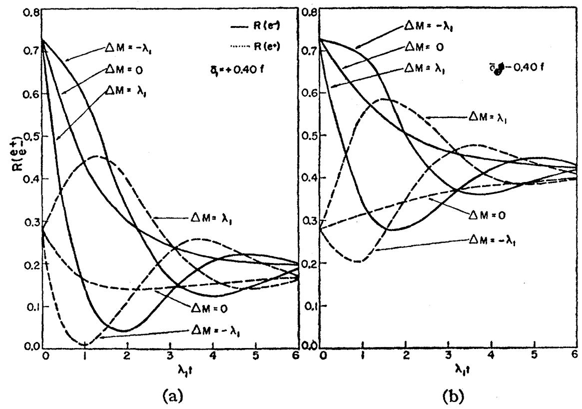

FIG. 4. (a) The same as in Fig. 3 for solution II with a positive real part. (b) The same as in (a) with a negative real part.

V. CONCLUDING REMARKS

The numerical calculations presented here are, however, subject to change with the accumulation of more accurate K^+ and K^- scattering data on nucleons. It seems reasonable that the change would not be drastic enough to obscure the effects to a great extent. Thus an experiment along this line, in spite of many technical difficulties, may yield some conclusive results.

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