Muon Capture in He^3†

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The hard-core wavefunction for a three-nucleon system is used to calculate the capture rate of the reaction $\mu^- + He^3 \rightarrow H^3$(ground state)$+\nu$. It is found to be 1.66×10^3 sec^{-1}.

THE capture rate of the reaction $\mu^- + He^3 \rightarrow H^3 + \nu$ was calculated by Fujii and Primakoff[1] in terms of the beta decay rate $H^3 \rightarrow He^3 + e^- + \bar{\nu}$. Use was made of Irving's wavefunction[2] for the three-nucleon system, but the calculated capture rate is supposed to depend on the adopted nuclear model fairly sensitively. Hence the same calculation has been repeated with a more realistic wavefunction.

The general expressions have been given already in the reference 1. For the reaction in question the problem is just reduced to find the numerical estimate of the parameter[3]

$$R = 1.04\times\left[\frac{1}{3}\int\left(\sum_{i=1}^{3}\exp(-i\mathbf{p}\cdot\mathbf{r}_i)\right)\psi^2(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3)d\mathbf{r}_1d\mathbf{r}_2d\mathbf{r}_3\right]^2,$$

where $\mathbf{p}$ is the neutrino momentum ($p=|\mathbf{p}|=5.26\times10^{12}$ cm^{-1}), ψ is the wavefunction of the three-nucleon system and $\mathbf{r}_1$, $\mathbf{r}_2$, $\mathbf{r}_3$ are the nucleon position vectors. The wave function proposed by Kikuta, Morita, and Yamada,[4,5] who took the effect of the hard core nuclear potential into account, reads

$$\psi(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3) = N\prod_{i=1}^{3}\left[e^{-\mu(\rho_i-D)} - e^{-\nu(\rho_i-D)}\right] \quad \text{for all } \rho_i \geqq D,$$

$$\psi(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3) = 0 \qquad \text{otherwise},$$

where $\boldsymbol{\varrho}$'s are the internucleon distances

$$\rho_1 = |\boldsymbol{\varrho}_1| = |\mathbf{r}_2-\mathbf{r}_3|, \quad \rho_2 = |\boldsymbol{\varrho}_2| = |\mathbf{r}_3-\mathbf{r}_1|, \quad \rho_3 = |\boldsymbol{\varrho}_3| = |\mathbf{r}_1-\mathbf{r}_2|,$$

and N is a certain normalization constant. The parameters μ and ν are to be determined by variational calculation for a chosen core radius D. It is found that

$$R = 1.04\times[\eta(p)/\eta(0)]^2,$$

$$\eta(p) = \int \exp\left[-\frac{i}{3}\mathbf{p}\cdot(\boldsymbol{\varrho}_2-\boldsymbol{\varrho}_3)\right]\psi^2(\rho_1,\rho_2,\rho_3) \times\delta(\boldsymbol{\varrho}_1+\boldsymbol{\varrho}_2+\boldsymbol{\varrho}_3)d\boldsymbol{\varrho}_1d\boldsymbol{\varrho}_2d\boldsymbol{\varrho}_3$$

$$= 8\pi^2\sum_{l=0}^{\infty}(2l+1)(-1)^l\int\int\int j_l(p\rho_2/3)j_l(p\rho_3/3) \times P_l\left(\frac{\rho_2^2+\rho_3^2-\rho_1^2}{2\rho_2\rho_3}\right)\psi^2(\rho_1,\rho_2,\rho_3)\rho_1\rho_2\rho_3d\rho_1d\rho_2d\rho_3,$$

in which j_l and P_l are the spherical Bessel and Legendre function of order l, respectively. The integration over ρ's is subject to the condition that ρ_1, ρ_2, ρ_3 form three sides of a triangle. One can see that the $l=0$ and 1 terms are dominant, since for higher l terms $|P_l|\leqq1$, and the overlap between j_l and the wavefunction is very small hence hardly makes any appreciable contribution. The completely analytic formula is obtained for the $l=0$ term by the method of integration shown in the reference 4. For the $l=1$ term the approximation $D=0$ is good, and again the integral can be performed analytically.

The constants used for the numerical estimate are[5]: $D=0.40\times10^{-13}$ cm, $\mu=0.500\times10^{13}$ cm^{-1}, $\nu=4.27\times10^{13}$ cm^{-1} for $l=0$, and $D=0$, $\mu=0.503\times10^{13}$ cm^{-1}, $\nu=\infty$ for $l=1$. The ratio $\eta(p)/\eta(0)$ is found to be 0.932, thus $R=0.903$. The predicted capture rate is $w^{(\mu)}=1.66\times10^3$ sec^{-1}, which is roughly 14% larger than the value $w^{(\mu)}=1.46\times10^3$ sec^{-1} in the reference 1. The increase is expected if the hard core wavefunction gives smaller mean square radius ($\sim1.5\times10^{-13}$ cm) than Irving's wavefunction ($\sim1.8\times10^{-13}$ cm).[6]

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1 A. Fujii and H. Primakoff, Nuovo cimento **12**, 327 (1959).
2 J. Irving, Phil. Mag. **42**, 338 (1951).
3 Reference 1, Eqs. (9), (10), and (11).
4 T. Kikuta, M. Morita, and M. Yamada, Progr. Theoret. Phys. (Kyoto) **15**, 222 (1956).
5 T. Ohmura (to be published).
6 Reference 1, Eq. (17).