

Effects of Hydrostatic Pressure on the Piezoresistance of Semiconductors: *i*-InSb, *p*-Ge, *p*-InSb, and *n*-GaSb

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A method for measuring the piezoresistance of a sample under high hydrostatic pressure by comparison with the piezoresistance of intrinsic InSb is described. The method is tested by a measurement of the piezoresistance of *p*-type germanium and *p*-type InSb up to 13 000 kg/cm². The piezoresistance of these materials is found to be independent of pressure, in good agreement with predictions based on other experiments. Measurement of the piezoresistance of *n*-GaSb as a function of pressure up to 12 000 kg/cm² confirms the correctness of the model of the conduction band which has been proposed by Sagar. Values for certain parameters of the conduction band of GaSb are deduced.

I. INTRODUCTION

MANY experiments have shown that the energy band structure of semiconductors can be significantly modified by the application of high hydrostatic pressure.^{1,2} In order to study these effects we have measured the piezoresistance of several semiconductors under hydrostatic pressure. In this paper we will describe our method for making these measurements, including the establishment of a method for measuring force in a pressure apparatus, and some results pertaining to the change of the band structure of GaSb under hydrostatic pressure.

II. PIEZORESISTANCE

All of our measurements of piezoresistance have been made on cubic crystals in arrangements of the type designated by Smith³ as "longitudinal," i.e., arrangements in which the tensile force, electric current, and the measured electric field are parallel. The piezoresistance coefficient, Π , which is measured in such an arrangement is defined as the ratio of the relative change of the longitudinal resistivity component to the applied stress. The piezoresistance effect can be described by a fourth rank tensor in a way described by Smith.³ In the case of a longitudinal effect in a cubic crystal, Π can be written in the form

$$\Pi = \left(\frac{1}{3}\right)\Pi_P + \left(\frac{2}{3}\right)\Pi_S. \quad (1)$$

Here Π_P is the logarithmic pressure derivative of the resistivity, ρ ,

$$\Pi_P = -\rho^{-1}d\rho/dP, \quad (2)$$

and can be expressed in terms of the usual coefficients by $\Pi_P = (\Pi_{11} + 2\Pi_{12})$. Π_S is a shear coefficient which is

a linear combination of the shear coefficients ($\Pi_{11} - \Pi_{12}$) and Π_{44} . The weighting of ($\Pi_{11} - \Pi_{12}$) and Π_{44} depends on the crystallographic orientation of the sample axis in a way which has been discussed by Smith³ and by Potter and McKean.⁴ On the basis of these discussions, it is easy to choose sample orientations which allow the determination of both ($\Pi_{11} - \Pi_{12}$) and Π_{44} .

Except for certain small "minor effects," the piezoresistance effect arises from the changes in the electronic band structure of the crystal which are produced by elastic strain.⁵ We will neglect these minor effects. The advantage of writing Π in the form of Eq. (1) is that the terms Π_P and Π_S arise from different features of the energy band structure. Large values of Π_P occur when there are two bands whose relative energy is changed by the application of hydrostatic pressure. On the other hand, large values of Π_S occur when there is a band of complex structure, for example, a multivalley band. In particular, we note that a large value of ($\Pi_{11} - \Pi_{12}$) is characteristic of a (100) multivalley band, and that a large value of Π_{44} is characteristic of a (111) band.^{3,6} The prototype of the latter band is *n*-type germanium.

III. EXPERIMENTAL METHODS

For the purposes of this experiment we adapted the device for measuring piezoresistance which has been described by one of the authors⁶ for use in the high-pressure apparatus previously used by the other author.⁷ The modified piezoresistance device is shown in Fig. 1. Important features of this apparatus which should be noted are as follows: The apparatus is designed to compare the piezoresistance of two semiconductors. The samples are placed in series mechanically, so that the tensile force applied to each is the same. A steady force can be applied either by passing a direct current through the electromagnet or by spring loading the armature. The force can be modulated by passing an alternating current through the electromagnet. This modulation of the force produces a modulation of

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¹ W. Paul, J. Phys. Chem. Solids **8**, 196 (1959). This reference reviews the data for the case of germanium.

² A. Sagar, Phys. Rev. **117**, 93 (1960).

³ C. S. Smith, Phys. Rev. **94**, 42 (1954).

⁴ R. F. Potter and W. J. McKean, J. Research Natl. Bur. Standards **59**, 427 (1957).

⁵ C. Herring, Bell System Tech. J. **34**, 237 (1955).

⁶ M. Pollak, Rev. Sci. Instr. **29**, 639 (1958).

⁷ R. W. Keyes, Phys. Rev. **99**, 490 (1955).

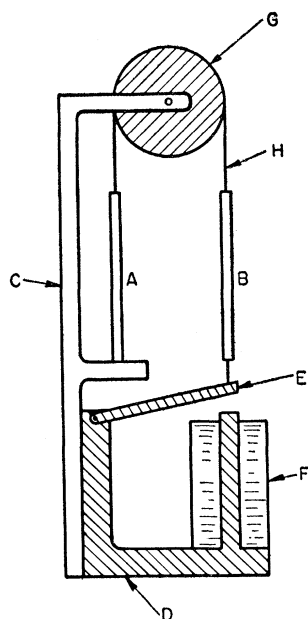


FIG. 1. Schematic drawing of the apparatus. *A* and *B* are single crystal semiconductor specimens. *C* is a rigid frame. *D* is a soft iron yoke and *E* is a soft iron armature. When a current is passed through the electromagnet *F*, a force is exerted on *E*, and the sample *B*, the wire *H*, and the sample *A* are subjected to a tensile force. The pulley *G* serves to reverse the direction of the tensile force in order to limit the height of the apparatus to that available in the high-pressure system. The electrical connections are not shown.

the resistance, which can be detected as an ac voltage on the potential probes in the usual method of measuring resistance. The resistances are determined in the ordinary way. Further details of the electrical measurements are given in the paper of Pollak.⁶

The method used in the present set of experiments differs in one respect from that described previously.⁶ In the earlier work⁶ the electromagnet was excited by a combination of dc and ac currents which produced a driving force of the ac frequency. In the present work the dc current was absent, and the driving force, being proportional to the square of current, varied with twice the ac frequency. The advantage of this modification is that voltages induced in the leads by the magnetic field have half the frequency of the effects produced by the stress, and can easily be separated by electrical filtration. In the earlier apparatus the effects of induced voltages were eliminated by maintaining a large physical separation between the magnet and the sample; this was not possible in the present experiments, however, because of the dimensional limitations imposed by the high-pressure chamber.

All of the measurements to be reported in this paper were performed at 300°K. The magnitude of the stress on the samples was about 2×10^7 dynes/cm². This stress is very small compared to the hydrostatic pressure, except for the atmospheric pressure measurement.

IV. MEASUREMENT OF THE FORCE

The force applied to the specimens for a given magnet current in the apparatus of Fig. 1 cannot be expected to remain constant when the pressure is varied. Therefore, it is necessary to measure the force under pressure in order to convert the modulation of the specimen

resistance to a piezoresistance constant. An electrical force gauge with a calibration constant which is a known function of pressure is required. Various gauges of this type have been used by other workers^{8,9}; none of these were convenient for our purpose, however. Therefore, we devised the apparatus of Fig. 1 with the intent of using one of the semiconductor specimens as a piezoresistance force gauge.

The condition that a semiconductor be a suitable force gauge is that its piezoresistance, Π , is known or can be determined as a function of pressure. One semiconductor in which Π can easily be determined as a function of pressure is intrinsic InSb. In intrinsic InSb the electrical conductivity arises almost entirely from the electrons, since the electron mobility is much larger than the hole mobility. The electron energy band, the conduction band, however, is a simple spherical band.¹⁰ Therefore, there is no large shear piezoresistance effect in this band, and we expect that Π_s , Eq. (1), will be small in *i*-InSb. This expectation has been experimentally confirmed at atmospheric pressure.^{11,12} However, the energy gap is strongly pressure dependent, and there is a large effect of hydrostatic pressure on the resistivity, which has been extensively studied.^{7,13} This model of the electrical properties accounts for the pressure dependence of the resistivity for pressures up to 12 000 kg/cm².⁴

Thus *i*-InSb can be used as a force gauge in high-pressure experiments as follows. In the course of our experiment we measure the resistivity of InSb as a function of pressure. We find Π_P by differentiating the resistivity. This knowledge of Π_P is what is required in order that *i*-InSb be a suitable force gauge. [See Eq. (1).]

Unfortunately, the shear piezoresistance constants of *i*-InSb do not entirely vanish.^{11,12} This fact introduces some error into our method of measuring the force. Since Π_P is not strongly pressure dependent and the accepted model of InSb suggests no reason to expect a large pressure dependence of Π_s , we believe that the error will not be strongly pressure dependent. Thus we have confidence in the accuracy of the important features of the pressure dependence of our results and in relative values of our piezoresistance constants. However, we estimate that errors as large as 20% may be present in the absolute values of our Π_s .

V. MATERIALS

The *i*-InSb was special high purity material purchased from the Ohio Semiconductor Company. The number of extrinsic carriers was small compared to the number of intrinsic carriers throughout the pressure range.

⁸ G. W. Tichelaar and D. Lazarus, Phys. Rev. **113**, 438 (1959).

⁹ P. Haasen and A. W. Lawson, Z. Metallk. **49**, 280 (1958).

¹⁰ B. Lax, Revs. Modern Phys. **30**, 122 (1958).

¹¹ R. F. Potter, Phys. Rev. **108**, 652 (1957).

¹² F. P. Burns and A. A. Fleischer, Phys. Rev. **107**, 1281 (1957).

¹³ D. Long, Phys. Rev. **99**, 388 (1955).

Our other samples were relatively heavily doped, mainly because our electrical measuring apparatus was designed for measurements on low resistivity material. The *p*-Ge crystal was grown in this laboratory by the Czochralski method, and contained 10^{18} donors/cm³. The *p*-InSb crystal was grown by a method previously described by Pollak,¹⁴ and contained 5×10^{18} donors/cm³. The GaSb crystals were grown and given to us by Sagar. Detailed descriptions of these crystals are contained in Sagar's paper.² For convenience, we will use Sagar's sample numbers in discussing these materials.

VI. *p*-Ge AND *p*-InSb

As a test of our method for measuring piezoresistance under high hydrostatic pressure, we have measured the piezoresistance of samples of *p*-type germanium and *p*-type InSb as functions of pressure. Both crystals were cut with the long sample axis along a (110) crystal axis. The results are plotted in Fig. 2. The Π of *p* germanium is independent of pressure up to 13 000 kg/cm², and the Π of *p*-InSb decreases by 4% in this range. The latter variation may be within our experimental uncertainty and not significant.

The results shown in Fig. 2 are not unexpected in view of the fact that the hole mobility has previously been found to be independent of pressure in these materials.^{7,15,16} The mobility and the piezoresistance depend on the same parameters of the valence band, and it is very improbable that any change in the band could affect one of these quantities without affecting the other. Also, the accepted model of the band structure of these *p*-type materials is consistent with the pressure independence of the mobility and piezoresistance, at least in moderately or heavily doped samples. In these impure samples impurity scattering limits the contribution of the light holes to the conductivity to a low value, and the electrical properties are determined by the heavy holes.

We feel that this agreement between our results for *p*-Ge and the predictions of the independently estab-

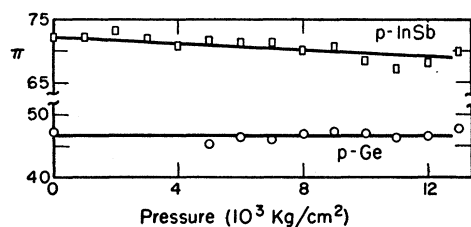


FIG. 2. The piezoresistance constant of samples of *p*-type germanium and *p*-type InSb as functions of pressure, measured by comparison with InSb in the apparatus of Fig. 1. Π is in units of 10^{-12} cm²/dyne.

¹⁴ M. Pollak, thesis, University of Pittsburgh, 1958 (unpublished).

¹⁵ P. W. Bridgman, Proc. Am. Acad. Arts Sci. **87**, 71 (1953).

¹⁶ A. C. Smith, Bull. Am. Phys. Soc. **3**, 14 (1958); thesis, Harvard University, 1958 (unpublished).

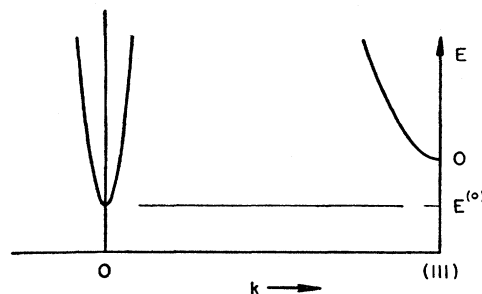


FIG. 3. Illustration of the conduction band structure in GaSb proposed by Sagar.²

lished accepted model of the germanium band structure is a confirmation of our method of measuring piezoresistance. In fact, the constancy of Π in *p*-Ge suggests that it also is a possible material for use as a force gauge in our experiments, and we have used it in this way in other experiments.¹⁷

VI. *n*-GaSb

A. Sagar's Model

Sagar has suggested a model for the conduction band structure of GaSb in a recent paper.² His model is shown in Fig. 3. According to this model two of the three piezoresistance constants discussed in Sec. III are large: (1) A nonvanishing pressure coefficient arises from the pressure dependence of the energy difference between the (111) and (000) minima; (2) A large Π_{44} is due to removal of the degeneracy among the (111) valleys by a diagonal shear, as in the well known case of germanium.^{3,5} The values of Π_P and Π_{44} naturally depend on the distribution of electrons between the (000) and (111) bands, and this distribution depends in turn on the pressure and temperature and on the total number of donors, since the effects of statistical degeneracy may be appreciable. Sagar was able to fit his data quantitatively by assigning numerical values to the various parameters of his model.²

Here we wish to note only a few facts concerning Sagar's model to which we will refer later. We will use a superscript (0) to denote properties of the (000) band and a superscript (1) to denote properties of the (111) band. We will take the zero of the energy scale to be the bottom of the (111) band, thus having $E^{(1)}=0$. In this notation the values of two of Sagar's parameters are $E^{(0)}=-0.07$ eV and $dE^{(0)}/dP=1 \times 10^{-5}$ eV/(kg/cm²).

The shear coefficient in Sagar's model has the value

$$\Pi_{44} = (\sigma^{(1)}/\sigma) \Pi_{44}^{(1)}. \quad (3)$$

Here $\sigma^{(1)}$ is the contribution of the electrons in the (111) band to the conductivity, i.e., $\sigma = \sigma^{(1)} + \sigma^{(0)}$, and $\Pi_{44}^{(1)}$ is the value which Π_{44} would have if $\sigma^{(0)}=0$.

¹⁷ M. Pollak and R. W. Keyes, Bull. Am. Phys. Soc. **4**, 185 (1959).

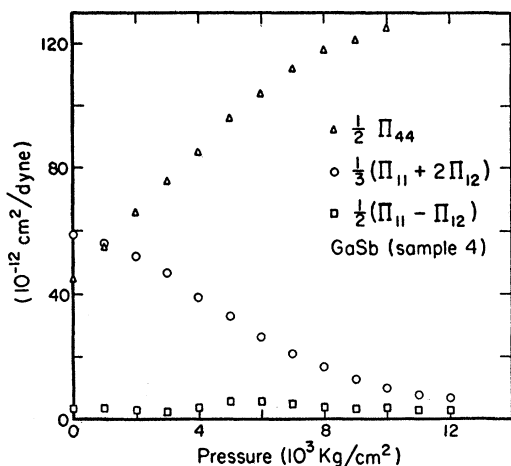


FIG. 4. The piezoresistance constants of GaSb (Sagar's material 4) as a function of hydrostatic pressure at 300°K.

A very important feature of this model, which must be continually borne in mind in interpreting experimental results, is that the effective mass in the (000) band is much smaller than the mass in the (111) band. This fact follows from Sagar's interpretation of his data and from comparison with the band structures of other crystals.¹⁰ The consequences of this difference in effective masses which are important here are that the density of states in the (000) band is small compared to the density of states in the (111) band and $\mu^{(0)}$ is five or ten times larger than $\mu^{(1)}$.

B. Measurements on GaSb

In order to subject Sagar's model to additional test and to enlarge our knowledge of the parameters of the model, we have measured the piezoresistance coefficients of *n*-type GaSb as a function of hydrostatic pressure to pressures of 12 000 kg/cm². The force on the sample was determined by using an InSb standard in the way described in the preceding sections. Our results on the material of Sagar's number 4 samples, which has a Hall coefficient of about 10 cm³/coul at 300°K and atmospheric pressure, are shown in Fig. 4 and given in Table I.

It is immediately apparent from Fig. 4 that our results are in qualitative accord with Sagar's model. At atmospheric pressure both Π_P and Π_{44} are fairly large. $\Pi_{11} - \Pi_{12}$ is small throughout the pressure range. As the pressure is increased the (000) band goes up in energy, and the number of electrons in the (000) band decreases while the number in the (111) band increases. The number of electrons in the (000) band at atmospheric pressure is a rather small fraction of the total number, and as this number is decreased toward zero the effect of pressure on the conductivity becomes small. This accounts for the decrease of Π_P with increasing pressure. The reason for the increase of Π_{44} can be seen from Eq. (3): as electrons are transferred to the (111)

band σ approaches $\sigma^{(1)}$ and Π_{44} approaches $\Pi_{44}^{(1)}$. We feel that this agreement of our results with the predictions of the model is very strong evidence for the validity of the model.

Another feature of our data is illustrated by column 6 of Table I. The product $\Pi_{44}\sigma$ is tabulated in column 6, and it is seen that this product is constant within experimental error. Equation (3) shows that constancy of $\Pi_{44}\sigma$ implies constancy of $\Pi_{44}^{(1)}\sigma^{(1)}$. This result can be understood on the basis of the following simple but approximate viewpoint: Only a small fraction of the electrons are in the (000) band. The transfer of these electrons to the (111) band will not change $\sigma^{(1)}$ or $\Pi_{44}^{(1)}$ appreciably, and, therefore, these quantities and their product are pressure independent. The few electrons in the (000) band account for a significant fraction of the conductivity, however, since the mobility of electrons in the (000) band is about an order of magnitude higher than the mobility in the (111) band.

Sagar attempted to determine the parameters of his model by fitting curves to his data on the pressure dependence of the conductivity and the Hall constant. Our measurement of the conductivity as a function of pressure agrees well with that of Sagar. In view of the simplicity of the relation between Π_{44} and σ , it is apparent that no improvement in the determination of the parameters of the model could be obtained by supplementing the curve fitting criteria with the piezoresistance data. Therefore, we will analyze our data from a different point of view, based on the fact that at the highest pressures at which we obtain data the contribution of the (000) band to the conductivity is small. Consequently, the properties of the sample are determined primarily by the electrons in the (111) band, and analysis of the data at the high-pressure end of our measurements will give values for the properties of this band. Even at our highest pressures, the facts that Π_P does not vanish and that Π_{44} is pressure dependent show that the effect of the (000) band is not entirely negligible. We will attempt to extrapolate out the remaining effects due to the (000) band, and thus determine the properties of electrons in the (111) band.

TABLE I. Piezoresistance constants of GaSb (sample 4) as a function of pressure.

Pressure 10 ³ kg/cm ²	σ (ohm-cm) ⁻¹	Π_{44} 10 ⁻¹² cm ² /dyne	$(\Pi_{11} - \Pi_{12})$ 10 ⁻¹² cm ² /dyne	$\frac{1}{2}\Pi_P$ 10 ⁻¹² cm ² /dyne	$\sigma\Pi_{44}$ 10 ⁻² , cm/dyne-ohm
0	250	91	4	59	24.0
1	218	111	4	55.5	24.4
2	185.4	131	3	50	24.5
3	160.7	151	2	45	24.4
4	141.2	171	4	39	24.3
5	126.6	192	6	32.5	24.5
6	115.7	208	6	26	24.3
7	108.2	224	5	21	24.5
8	102.4	237	4	17	24.4
9	97.8	243	3	13	23.9
10	95.2	253	3	10	24.2
11	92.6	...	3	8	...
12	90.7	...	3	7	...
(111) band ^a	83	278	...	0	23.0

^a Determined by extrapolation, as described in the text.

In making this analysis and, especially, in identifying the results with the properties of the (111) band at atmospheric pressure, it is assumed that the parameters of the band are independent of pressure. Support for this assumption is afforded by the example of germanium, in which experimental work has shown that the pressure dependence of the parameters of the (111) band is weak.¹ The pressure dependence of the elastic constants is also neglected; this may amount to one or two percent in our pressure range. We believe that in spite of the various errors which are introduced by our extrapolation procedure and our experimental method, the values of the parameters which we obtain are to be preferred to the values of Sagar. This is because Sagar assumed that the ratio of the mobilities of the electrons in the two bands was independent of pressure. This is probably a rather poor assumption, since most of the final states for scattering of an electron in the (000) band are in the (111) band. Thus the density of final states will be a strong function of pressure, and it is to be expected that $\mu^{(0)}$ will decrease rapidly with increasing pressure.

There is no obviously unique method for extrapolating the data to the condition in which all of the electrons are in the (111) band. Our method is as follows: The pressure dependence of Π_P , Π_{44} , σ , and the Hall constant are related, since they all arise from the same physical mechanism. We know the limiting value of Π_P , namely, zero, according to our model. Therefore we will extrapolate the other parameters to the limit of no electrons in the (000) band by plotting them against Π_P and extrapolating to the value for $\Pi_P=0$. The extrapolations for σ and for Π_{44} are shown in Fig. 5, and the extrapolated values are given in Table I. In

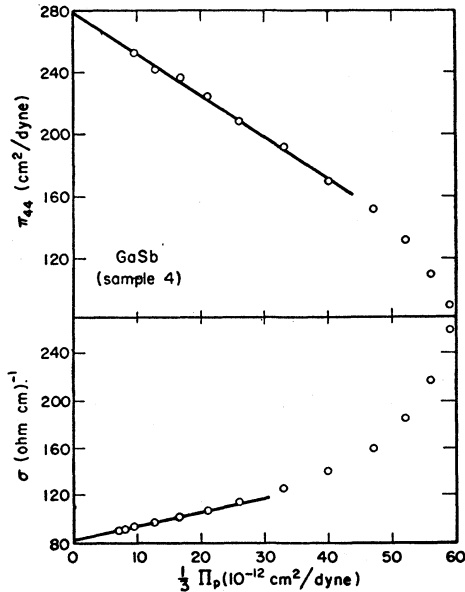


FIG. 5. The extrapolation of Π_{44} and σ to the case in which all electrons are in the (111) band. Details are given in the text.

TABLE II. Piezoresistance constants of GaSb (sample 9) as a function of pressure.

Pressure 10^8 kg/cm ²	σ (ohm cm) ⁻¹ / dyne	Π_{44} 10^{12} cm ² / dyne	$\frac{1}{3}\Pi_P$ 10^{-12} cm ² / dyne	$\sigma\Pi_{44}$ 10^{-9} cm/ dyne-ohm
0	830	100	39	83
1	744	108	37	80
2	667	119	34	79
3	604	129	32	78
4	551	138	30	76
5	504	147	28	74
6	467	154	24	72
7	438	165	20	72
8	414	172	17	71
9	396	173	13	68
10	382	177	11	68
11	370	178	9	66
12	362	182	7	66
(111) band ^a	301	195	0	59

^a Determined by extrapolation, as described in the text.

accord with the above discussion, we identify these extrapolated values with $\sigma^{(1)}$ and $\Pi_{44}^{(1)}$. The product $\sigma^{(1)}\Pi_{44}^{(1)}$ should be compared with the measured products as an index of the accuracy of our extrapolation. The discrepancy is about 5%.

The important energy band parameter which determines $\Pi_{44}^{(1)}$ is Ξ_u , the shear deformation potential constant of the (111) band. An accurate determination of Ξ_u requires that the anisotropy of the conductivity tensor of a valley be known. However, if the anisotropy is large, the calculated value of Ξ_u is not very sensitive to the anisotropy. If we assume that the anisotropy is very large and use the atmospheric pressure elastic constants we calculate $\Xi_u=18$ ev. If the anisotropy is the same as that of the (111) valleys of germanium, the calculated Ξ_u is raised to 21 ev. These values should be compared to that of germanium, $\Xi_u=17$ ev.^{18,19}

If we knew the number of carriers in our sample we could calculate the mobility in the (111) band. Unfortunately, Sagar's Hall effect data for this sample do not extend to high enough pressures to allow the application of our extrapolation method to the Hall constant. However, if we accept Sagar's figure for the carrier concentration, $N=1.4\times 10^{18}$ cm⁻³, we find that $\mu^{(1)}=370$ cm²/v sec. The value obtained by Sagar² by his method of analysis was 570 cm²/v sec.

We have also measured the large piezoresistance constants of Sagar's impure material 9, which has a Hall constant of 2.7 cm³/coul at 300°K and atmospheric pressure, as a function of pressure. The results are tabulated in Table II and plotted in Fig. 6. They are also in qualitative agreement with the predictions of the model. However, there are significant quantitative differences between these results and those obtained with material 4. The differences are attributable to two effects²: (1) statistical degeneracy is important in both

¹⁸ C. Herring and E. Vogt, Phys. Rev. **101**, 944 (1956).

¹⁹ C. Herring, T. H. Geballe, and J. E. Kunzler, Bell System Tech. J. **38**, 657 (1959).

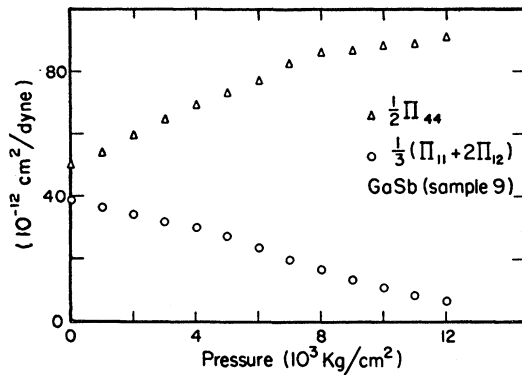


FIG. 6. The large piezoresistance constants of GaSb (Sagar's material 9) as a function of hydrostatic pressure at 300°K.

bands in material 9 because of the large number of electrons; (2) the mobilities are smaller in material 9 than in material 4 because of the larger impurity content of the former.

One difference between the results on the two materials is that in the impure material the product $\sigma\Pi_{44}$ is not constant, but decreases by about 20% as the pressure is raised from 0 to 12 000 kg/cm². A possible explanation of this effect might be based on the fact that $\Pi_{44}^{(1)}$ would be enhanced by strong band-band scattering in a way similar to the way in which it is enhanced by intervalley scattering.⁷ Then, with increasing pressure the density of final states for scattering from the (111) band to the (000) band decreases, and the band-band scattering contribution to $\Pi_{44}^{(1)}$ decreases. Thus the product $\sigma^{(1)}\Pi_{44}^{(1)}$ might decrease. A higher band-band scattering rate in the impure sample is not implausible, since it has been shown that various impurities induce intervalley scattering in germanium.^{20,21} However, we have not been able to account quantitatively for the pressure variation of $\sigma\Pi_{44}$ in material 9, and therefore, do not claim that we understand it.

We have applied the extrapolation procedure described above in connection with the interpretation of the data on material 4 to the data of Table II, and also to Sagar's data on the Hall effect of material 9.² The extrapolated values of σ and Π_{44} , i.e., $\sigma^{(1)}$ and $\Pi_{44}^{(1)}$, are given in Table II. The extrapolated value of the Hall constant is $R^{(1)}=0.83$ coul/cm³.

The value of $\Pi_{44}^{(1)}$ in the impure material is only 0.70 times $\Pi_{44}^{(1)}$ of material 4. We attribute this lowering of $\Pi_{44}^{(1)}$ to the effect of degeneracy on the shear piezoresistance coefficient, an effect which Pollak²² has shown to be important in *n*-type germanium. The theory of the piezoresistance coefficients of a degenerate multi-

valley semiconductor has been given by Keyes,²³ and the effective mass of the (111) band of GaSb can be estimated by using this theory in the form in which Pollak found that it applied to *n*-type germanium. Pollak's interpretation of both the piezoresistance and the temperature dependence of the mobility in degenerate *n*-type Ge requires that the momentum relaxation time be a constant, independent of electron energy.²² Making this assumption for GaSb, we find from the lowering of $\Pi_{44}^{(1)}$ that the Fermi level is $0.6kT$ above the bottom of the (111) band. The extrapolated Hall constant gives 7.5×10^{18} cm⁻³ for the carrier concentration in the band, and thus we find for the density of states effective mass $m^{(1)}/m_0=0.4$. For comparison, we note that in germanium $m^*/m_0=0.56$.¹⁰

We have also applied our extrapolation method to Sagar's data on the conductivity and Hall effect in his material 1. We find $R_H^{(1)}=32$ coul/cm³ and $\sigma^{(1)}=10.5$ (ohm-cm)⁻¹. Further, the analysis of the conductivity data for the two nondegenerate materials, number 1 and number 4, shows that at high pressures ($P>7000$ kg/cm²) $\sigma^{(0)}/\sigma^{(1)}=1.70 \exp(-2.44 \times 10^{-4}P)$. The significance of these numbers is not clear because of the expected pressure variation of $\mu^{(0)}$, which has already been mentioned. However, the close coincidence of the high-pressure values of $\sigma^{(0)}/\sigma^{(1)}$ for the two materials serves as a check on our method of extrapolation, since the derived values of $\sigma^{(0)}$ are very sensitive to the extrapolated value $\sigma^{(1)}$ at high pressures, where the difference between σ and $\sigma^{(1)}$ is small.

C. Summary of GaSb Results

Our measurements on *n*-type GaSb are in excellent qualitative agreement with the model of the band structure proposed by Sagar.² We have estimated certain parameters of the (111) band of GaSb from our data, and present these estimates in Table III. The corresponding values for the conduction band of germanium are also presented for comparison. It should be borne in mind that the accuracy with which the parameters are known for germanium is considerably greater than that of our measurements.

The values for the electron mobility in the (111) band of GaSb given in Table III were calculated from the extrapolated Hall constants and conductivities.

TABLE III. Estimated values of certain parameters of the (111) band of GaSb, and a comparison with the conduction band of Ge.

Quantity	GaSb	Ge
m (density of states mass)	$0.4m_0$	$0.56m_0$
μ ($n=2 \times 10^{17}$)	340 cm ² /v sec	1600 cm ² /v sec
μ ($n=1.4 \times 10^{18}$)	370 cm ² /v sec	750 cm ² /v sec ^a
μ ($n=7.5 \times 10^{18}$)	250 cm ² /v sec	440 cm ² /v sec ^a
E_g	20 ev	17 ev

^a Based on R. Bowers, Phys. Rev. **108**, 683 (1957) and unpublished measurements of the authors.

²³ R. W. Keyes, J. Electronics **2**, 279 (1956).

²⁰ G. Weinreich and H. G. White, Phys. Rev. **106**, 1104 (1957); G. Weinreich, T. M. Sanders, Jr., and H. G. White, Phys. Rev. **114**, 33 (1959).

²¹ R. W. Keyes, J. Phys. Chem. Solids **3**, 102 (1957).

²² M. Pollak, Phys. Rev. **111**, 798 (1958).

These values suggest that the concentration of scattering centers does not decrease with decreasing electron concentration. This result is at least partly due to the fact, stated by Sagar,² that the materials also contain an appreciable concentration of acceptor centers.

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Electronic Structure of Tin Investigated by Ultrasonic Attenuation*

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The magnetic field dependence of the ultrasonic attenuation has been measured in very pure tin single crystals. Oscillations were found that can be explained as a result of resonant conditions between the electron orbit diameter and the periodic field set up by the sound wave. These oscillations yield information about the Fermi-momentum, and the general features of a possible electron distribution in tin are suggested.

MEASUREMENTS of the ultrasonic attenuation as function of magnetic field in metal single crystals where the electron mean free path (l) is comparable to the acoustical wavelength (λ), suggest that this method is a rather successful one to investigate gross properties of the Fermi-surface.^{1,2} Here are reported some measurements made in very pure tin single crystals. Since tin has a tetragonal crystal structure, four sets of experiments were done. Longitudinal waves were propagated along the tetragonal axis, the $[100]$ axis, and the $[110]$ axis as well as along an arbitrary angle in the plane perpendicular to the tetragonal axis. The magnetic field was always in the plane normal to the direction of propagation, and the field dependence of the attenuation was measured for different directions of the field.

The field dependence of the attenuation was found to be very anisotropic. In most of the cases oscillations were found that can be explained as a result of resonant conditions between the electron orbit diameter ($2r$) and the periodic fields set up by the sound wave. The oscillations were most pronounced when the propagation vector was along the $[100]$ axis and the field along the tetragonal axis. Figure 1 shows this case for the frequency 45.5 Mc/sec. The data are plotted on a reciprocal scale since the oscillations should be periodic in H^{-1} . Besides the main period two more are likely to be present, one of which is causing the interference with the second maximum of the main period. By observing the movements of the peaks when the field is rotated, the variation of

the average momenta as function of field direction can be traced. Due to the obviously complex electron distribution in tin, the relationship between the momenta deduced this way and the actual Fermi-surface is not likely to be a simple one. However, Kjeldaas and Holstein,³ assuming a spherical Fermi-surface, find the period to depend upon the extremal orbit momentum only. If this also holds true, at least to a first approximation, for more complicated surfaces, the momentum determining the oscillations will be a Fermi-momentum perpendicular to both the magnetic field and direction of propagation. The momentum (k_{\perp}) given by the main period in Fig. 1 will according to this be along the $[010]$ axis, and, estimated from the relation $2r/\lambda = 2\hbar k_{\perp}/eH\lambda$, it will be equal to $0.47 \times 10^8 \text{ cm}^{-1}$. However, any lattice vector of the reciprocal lattice space can be added to it. When the field is rotated away from the tetragonal axis

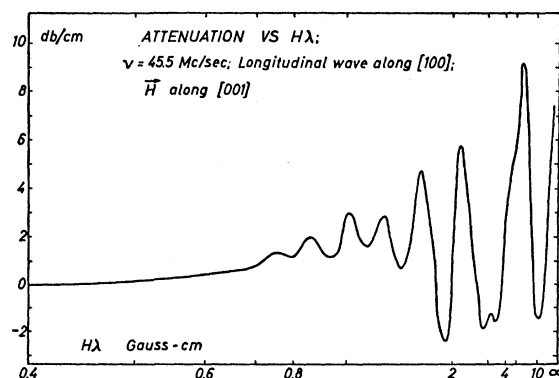


Fig. 1. Attenuation vs magnetic field times wavelength plotted on a reciprocal scale for a longitudinal wave with the propagation vector along the $[100]$ axis and the field along the tetragonal axis.

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