These values suggest that the concentration of scattering centers does not decrease with decreasing electron concentration. This result is at least partly due to the fact, stated by Sagar,[2] that the materials also contain an appreciable concentration of acceptor centers.

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Electronic Structure of Tin Investigated by Ultrasonic Attenuation*

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The magnetic field dependence of the ultrasonic attenuation has been measured in very pure tin single crystals. Oscillations were found that can be explained as a result of resonant conditions between the electron orbit diameter and the periodic field set up by the sound wave. These oscillations yield information about the Fermi-momentum, and the general features of a possible electron distribution in tin are suggested.

MEASUREMENTS of the ultrasonic attenuation as function of magnetic field in metal single crystals where the electron mean free path (l) is comparable to the acoustical wavelength (λ), suggest that this method is a rather successful one to investigate gross properties of the Fermi-surface.[1,2] Here are reported some measurements made in very pure tin single crystals. Since tin has a tetragonal crystal structure, four sets of experiments were done. Longitudinal waves were propagated along the tetragonal axis, the [100] axis, and the [110] axis as well as along an arbitrary angle in the plane perpendicular to the tetragonal axis. The magnetic field was always in the plane normal to the direction of propagation, and the field dependence of the attenuation was measured for different directions of the field.

The field dependence of the attenuation was found to be very anisotropic. In most of the cases oscillations were found that can be explained as a result of resonant conditions between the electron orbit diameter ($2r$) and the periodic fields set up by the sound wave. The oscillations were most pronounced when the propagation vector was along the [100] axis and the field along the tetragonal axis. Figure 1 shows this case for the frequency 45.5 Mc/sec. The data are plotted on a reciprocal scale since the oscillations should be periodic in H^{-1}. Besides the main period two more are likely to be present, one of which is causing the interference with the second maximum of the main period. By observing the movements of the peaks when the field is rotated, the variation of the average momenta as function of field direction can be traced. Due to the obviously complex electron distribution in tin, the relationship between the momenta deduced this way and the actual Fermi-surface is not likely to be a simple one. However, Kjeldaas and Holstein,[3] assuming a spherical Fermi-surface, find the period to depend upon the extremal orbit momentum only. If this also holds ture, at least to a first approximation, for more complicated surfaces, the momentum determining the oscillations will be a Fermi-momentum perpendicular to both the magnetic field and direction of propagation. The momentum (k_1) given by the main period in Fig. 1 will according to this be along the [010] axis, and, estimated from the relation $2r/\lambda = 2\hbar k_1/eH\lambda$, it will be equal to 0.47×10^8 cm^{-1}. However, any lattice vector of the reciprocal lattice space can be added to it. When the field is rotated away from the tetragonal axis

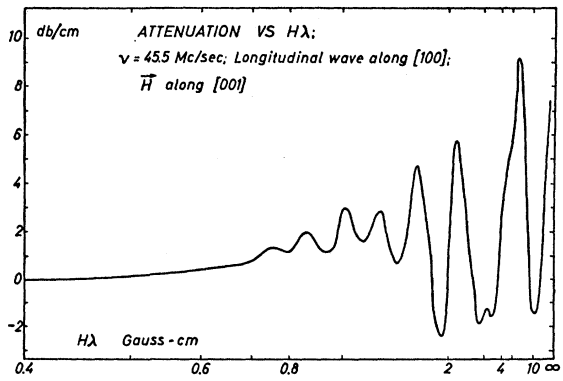

FIG. 1. Attenuation vs magnetic field times wavelength plotted on a reciprocal scale for a longitudinal wave with the propagation vector along the [100] axis and the field along the tetragonal axis.

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† Present address.

[1] R. W. Morse and J. D. Gavenda, Phys. Rev. Letters 2, 250 (1959); and J. D. Gavenda, Ph.D. thesis, Brown University, 1959 (unpublished).

[2] T. Olsen and R. W. Morse, Bull. Am. Phys. Soc. 4, 167 (1959).

[3] T. Kjeldaas and T. Holstein, Phys. Rev. Letters 2, 340 (1959).

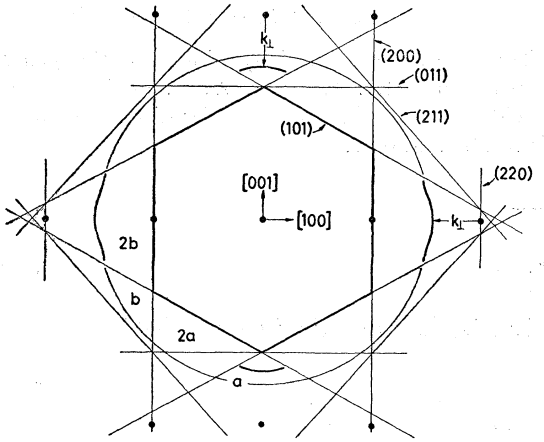

FIG. 2. Cut through the [100] axis and [001] axis in **k** space showing the intersecting planes of energy discontinuities. ● denotes a lattice point in the reciprocal lattice space. The distance between lattice points in the [100] direction is 1.08×10^8 cm^{-1}. Observed momenta $k_\perp$ have been plotted close to a sphere containing four electrons per atom.

within $\pm35°$ the picture remains approximately the same as in Fig. 1 although the main period shifts toward greater values of $k_\perp$. A similar period can also be observed when the field is within $\pm20°$ of the [010] axis. No simple period has been observed when the field has a direction between these two limits.

Several different zone structures have been proposed in the literature[4,5] none of which seem to fit experimental data. Among the planes with small indices only the following groups have a structure factor that does not vanish: {101}, {112}, {200}, {220}, and {211}. Here $\{n_1n_2n_3\}$ means the group where the third place is either n_3 or $-n_3$ only. These planes should therefore represent planes of energy discontinuities in **k** space. Figure 2 shows a cut through the [100] axis and [001] axis and most of the above-mentioned planes. The first zone bounded by planes of energy discontinuities contains 1.81 electrons per atom. The second zone, defined by the volumes having one plane in common with the first zone, can be shown to contain 1.41 electrons per atom. The surface of a sphere containing the four electrons of tin will have a radius of 1.64×10^8 cm^{-1} as shown by the circle in Fig. 2. From this sphere it is reasonable to plot the two branches of the main momentum as indicated in Fig. 2, and it is known from the structure factors for the {101} planes and {200} planes that the energy discontinuities across the latter group should be greater. It is therefore reasonable to suggest an electron distribution in tin having the following general features: A completely filled first zone, a filled second zone except for the outer parts of the volumes marked 2b in Fig. 2, and some overlap into higher zones mainly in the volumes marked a and b. If the Fermi-surface in volume 2b can be approximated as a plane perpendicular to the [100] axis near the $k_\perp$ plot, this volume will contain close to one electron per atom. This together with the first zone and volume 2a leaves an overlap into higher zones of 0.8 electrons per atom.

Such an electron distribution will be in general agreement with several other experimental investigations. The direction of motion of the electrons are concentrated around the tetragonal axis and the plane perpendicular to it as Pippard[6] suggested from his measurements of the anomalous skin effect. Similar measurements[7,8] give a number of free electrons per atom close to one. In agreement with this the free electrons in this model will consist of the overlap into higher zones and part of the electrons in volume 2b.

Furthermore, from the ultrasonic measurements made on the superconducting energy gap made by Morse, Gavenda, and the author[9] the total electronic attenuation (α) for zero field can be found and can be expressed as $\alpha=A\omega$ when the electron mean free path is greater than the wavelength. Here ω is the angular frequency and A is a constant proportional to the number of free electrons and their effective mass. This experiment selects mainly the electrons on a ring through the origin perpendicular to the direction of propagation, and the observed ratio $A_{100}/A_{110}=3$ where the indices denote propagation along [100] and [110], respectively. This result is in agreement with the proposed model of the Fermi-surface on the assumption that the anisotropy is mainly in the number of free electrons, because in the latter case there will be no surfaces corresponding to the surface in volume 2b.

Like the oscillations in the attenuation also the attenuation at high fields (6500 gauss) is highly anisotropic. These data and results from transverse wave measurements together with the above mentioned data will be treated in greater detail in a subsequent paper.

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[4] N. F. Mott and H. Jones, *The Theory of the Properties of Metals and Alloys* (Oxford University Press, New York, 1936).

[5] R. G. Chambers, Can. J. Phys. **34**, 1395 (1956). It should be noted that the zone proposed by Chambers contains one rather than two electrons per atom.

[6] A. B. Pippard, Proc. Roy. Soc. (London) **A203**, 98 (1950).

[7] R. G. Chambers, Proc. Roy. Soc. (London) **A215**, 418 (1952).

[8] E. Fawcett, Proc. Roy. Soc. (London) **A232**, 519 (1955). Dr. Fawcett has drawn my attention to another point of agreement between his results and the above measurements. It follows from his results that the direction of motion of the electrons in the plane perpendicular to [001] is concentrated about [100] but not about [110].

[9] R. W. Morse, T. Olsen, and J. D. Gavenda, Phys. Rev. Letters **3**, 15 (1959).