density, the K matrix is given by the scattering limit:

$$(k_ik_j|K|k_ik_j) \underset{\rho\to 0}{=} -(4\pi/M\Omega)k_{ij}^{-1}\delta_0(k_{ij}), \qquad (9)$$

with k_{ij} the relative momentum. The sum over k_i, k_j in Eq. (6) now can be replaced by an integral over the relative momentum, with the result

$$\frac{E}{N}=\frac{3p_F^2}{10M}-\frac{8}{\pi M}\int^{k_F} dk\, k\delta_0(k)\left(1-\frac{3}{2}\frac{k}{k_F}+\frac{1}{2}\frac{k^3}{k_F^3}\right). \qquad (10)$$

The s-state phase shift is given by the effective range formula

$$k\cot\delta_0 = -(1/a)+\tfrac{1}{2}r_0k^2, \qquad (11)$$

with

$$a=-23.6\times 10^{-13}\text{ cm.}$$
$$r_0= 2.65\times 10^{-13}\text{ cm.} \qquad (12)$$

The energy determined by Eq. (10) is given in Fig. 5 as a function of r_0. Again we see that there is no minimum and, as expected, that the low-density formula gives considerably too much interaction energy even for densities as low as $r_0=3\times 10^{-13}$ cm. This is readily understood since the large singlet scattering phase shifts are markedly reduced by the perturbation of the interaction arising through the effects of the exclusion principle.

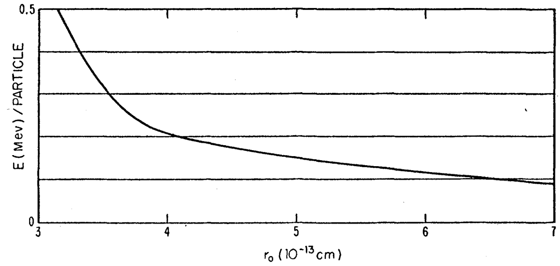

FIG. 5. Energy at very low density as given by scattering phase shifts.

We conclude that a neutron gas is not bound at any density and also that there is no relative minimum in the energy as a function of density. A constrained neutron gas would, however, show superfluidity.

PHYSICAL REVIEW VOLUME 118, NUMBER 4 MAY 15, 1960

K^- Absorption and $\pi-\Sigma$ Phase Shifts*

RICHARD H. CAPPS
Laboratory of Nuclear Studies, Cornell University, Ithaca, New York
(Received December 14, 1959)

The relations between $\bar{K}+N\rightarrow\pi+Y$ absorption amplitudes and pion-hyperon scattering amplitudes that are implied by the unitarity of the scattering matrix are considered. It has been shown by Kawarabayashi that if Λ production and the K^0-K^- mass difference are neglected, the zero kinetic energy K^--p absorption data of the Berkeley hydrogen bubble chamber group imply that at least one of the angular momentum $\frac{1}{2}$ pion-hyperon scattering amplitudes is much larger than are any of the $j=\frac{1}{2}$ pion-nucleon amplitudes at a corresponding energy. It is demonstrated that the conclusion of Kawarabayashi remains valid if one includes the effects of Λ production and the K^0-K^- mass difference.

IT is generally assumed that the interactions of π mesons with nucleons, Σ and Λ particles are primarily responsible for the binding of Λ's in nuclear matter. If this assumption is correct the $\pi\Sigma\Lambda$ interaction is among the strongest of all particle interactions, so that an understanding of πY interactions is essential to understanding the strange particles. Unfortunately, direct πY scattering experiments cannot be done. However, as has been noted by many people, some information concerning the πY scattering amplitudes may be obtainable from analyzing the results of $\bar{K}+N\rightarrow\pi+Y$ absorption experiments.

In this paper we are concerned with the phase difference between isotopic spin one and zero $\pi-\Sigma$ scattering amplitudes that may be indicated by the absorption data. We make the usual isotopic spin assignments and follow Day, Snow, and Sucher[1] in assuming that the K^-'s stopped in liquid hydrogen are nearly all absorbed from S orbitals. For definiteness we assume that the $KN\Lambda$ and $KN\Sigma$ interactions have odd intrinsic parity, so that the $\pi\Lambda$ and $\pi\Sigma$ states produced by stopped $\bar{K}$ particles are also S states. (The consequences of the opposite parity assumption are discussed later.) We assume that the branching ratios for the different final states produced by stopped K^-'s in hydrogen are those given by the Berkeley bubble chamber group,[2] i.e., $\Sigma^-+\pi^+$ (45%), $\Sigma^++\pi^-$ (21%), $\Sigma^0+\pi^0$ (27%), and $\Lambda^0+\pi^0$ (7%). These data

* Supported by the joint program of the Office of Naval Research and the U. S. Atomic Energy Commission.

[1] T. B. Day, G. A. Snow and J. Sucher, Phys. Rev. Letters **3**, 61 (1959).

[2] L. W. Alvarez, Proceedings of the 1959 International Conference on physics of High-Energy Particles at Kiev, July, 1959 (to be published).

implies that the production amplitudes for the different isotopic spin states satisfy the relations,

$$|T_{0K\Sigma}|^2/|T_{1K\Sigma}|^2=6.7, \quad |T_{1K\Lambda}|^2/|T_{1K\Sigma}|^2=0.58, \quad |\theta_{1K\Sigma}-\theta_{0K\Sigma}|=62°, \quad (1)$$

where the first subscript denotes the isotopic spin, and $\theta_{1K\Sigma}$ and $\theta_{0K\Sigma}$ are the phases of the amplitudes $T_{1K\Sigma}$ and $T_{0K\Sigma}$. Unfortunately, there is considerable uncertainty in these values (especially in the value for $\theta_{1K\Sigma}-\theta_{0K\Sigma}$) arising from the experimental difficulty in distinguishing between $\Sigma^0+\pi^0$ and $\Lambda^0+\pi^0$ events. We neglect the infrequent $\Lambda+2\pi$ production processes.

Only the two channels $\bar{K}N$ and $\pi\Sigma$ are present for isotopic spin zero. In this case the unitarity of the scattering matrix implies that the phase $\theta_{0K\Sigma}$ is the sum of the real parts of the S-wave elastic scattering phase shifts (δ_{0K} and $\delta_{0\Sigma}$) for the $I=0$ $\bar{K}N$ and $\pi\Sigma$ states.[3,4] At the threshold for $\bar{K}N$ scattering, the phase shift δ_{0K} must be zero, so that $2\delta_{0\Sigma}=2\theta_{0K\Sigma}$. (The factors of two are included in this equation because the unitary condition relates the phase angles only to within an additive factor of π, i.e., $\theta_{0K\Sigma}$ is equal to either $\delta_{0\Sigma}$ or $\delta_{0\Sigma}+\pi$.) A similar conclusion holds for the $I=1$ phase shifts, *provided that it is legitimate to neglect the* $\pi\Lambda$ *channel.* In this approximation,

$$2|\delta_{1\Sigma}-\delta_{0\Sigma}|=2|\theta_{1K\Sigma}-\theta_{0K\Sigma}|=124°. \quad (2)$$

Since the pion-nucleon S-wave scattering phase shifts at a corresponding energy (130-Mev lab pion kinetic energy) are about $\delta_{\frac{1}{2}}\approx 9°$, and $\delta_{\frac{3}{2}}\approx -8°$, it is seen from Eq. (2) that at least one of the $I=0$ and $I=1$ π-Σ phase shifts must be much larger than either of the corresponding pi-nucleon phases. This conclusion has been made previously by Kawarabayashi,[5] who points out that the large phase difference $\theta_{1K\Sigma}-\theta_{0K\Sigma}$ is the principal reason that the experimental data does not satisfy the global symmetry inequaltity of Amati and Vitale.[6] If one assumes strict global symmetry (i.e., equality of corresponding pion-baryon scattering amplitudes as well as coupling constants), the difference between the $I=1$ and $I=0$ $\pi\Sigma$ scattering amplitudes is related to the πN amplitudes by the equation $T_{1\Sigma}-T_{0\Sigma}=\frac{1}{3}(T_{\frac{3}{2}N}-T_{\frac{1}{2}N})$. Hence, in this model $\delta_{1\Sigma}-\delta_{0\Sigma}$ is quite small ($\sim -6°$).

The purpose of this paper is to show that the above result of a large phase difference $\delta_{1\Sigma}-\delta_{0\Sigma}$ remains valid if one includes the effects of the $\pi\Lambda$ $I=1$ channel. We apply the many channel unitarity condition previously presented by the author[4] to the case of three open channels. The unitarity condition on the phase $\theta_{1K\Sigma}$ is

$$\cos[2(\delta_{1\Sigma}+\delta_{1K}-\theta_{1K\Sigma})]=\frac{1-\frac{1}{2}(R_\Sigma+R_K+R_{\Sigma\Lambda}R_{K\Lambda}/R_{K\Sigma})}{(1-R_\Sigma)^{\frac{1}{2}}(1-R_K)^{\frac{1}{2}}},$$

where $R_\Sigma=R_{K\Sigma}+R_{\Sigma\Lambda}$, $R_K=R_{K\Sigma}+R_{K\Lambda}$, and $R_{\Sigma\Lambda}$, $R_{K\Lambda}$, and $R_{K\Sigma}$ are the transition rates between the isotopic spin one S-wave $\Sigma\pi$, $\Lambda\pi$, and $\bar{K}N$ states. All rates are normalized so that $R_{ij}=1$ is the maximum rate consistent with unitarity for an inelastic process (i.e., the S-wave cross section from state i to state j is given by $\sigma_{ij}=\pi\lambda_i^2R_{ij}$). Since we are considering the threshold energy for $\bar{K}N$ scattering, we may set $\delta_{1K}=0$, $R_K=0$, and $R_\Sigma=R_{\Sigma\Lambda}$. The equation then becomes,

$$\cos[2(\delta_{1\Sigma}-\theta_{1K\Sigma})]=\frac{1-\frac{1}{2}R_{\Sigma\Lambda}(1+x)}{(1-R_{\Sigma\Lambda})^{\frac{1}{2}}}, \quad x=\frac{R_{K\Lambda}}{R_{K\Sigma}}. \quad (3)$$

The ratio x is equal simply to the production ratio $|T_{1K\Lambda}|^2/|T_{1K\Sigma}|^2$ of Eq. (1), since the proportionality between R_{KY} and $|T_{1KY}|^2$ is independent of the mass of the πY state. On the other hand, $R_{\Sigma\Lambda}$ is unknown. Hence we examine the expression in Eq. (3) as a function of $R_{\Sigma\Lambda}$ for fixed x. It is seen that if $x<1$, the phase angle $\delta_{1\Sigma}$ is necessarily in the range,

$$\cos[2(\delta_{1\Sigma}-\theta_{1K\Sigma})]\geqq(1-x^2)^{\frac{1}{2}}. \quad (4)$$

Furthermore for $x<1$ the condition $\cos[2(\delta_{1\Sigma}-\theta_{1K\Sigma})]\leqq 1$ implies that $R_{\Sigma\Lambda}$ satisfies the inequality $R_{\Sigma\Lambda}\leqq 4x/(x+1)^2$. If we take $x=0.58$, as given by Eq. (1), the condition on $R_{\Sigma\Lambda}$ is not very restrictive, but Eq. (4) reduces to the significant relation $2|\delta_{1\Sigma}-\theta_{1K\Sigma}|<36°$. Hence the inclusion of the $\pi\Lambda$ channel leads to the following modification of Eq. (2):

$$88°<2|\delta_{1\Sigma}-\delta_{0\Sigma}|<160°. \quad (5)$$

One may still conclude that at least one of the $\pi\Sigma$ scattering phase shifts is large.

The inequalities of Eqs. (4) and (5) were derived without making any assumption concerning the magnitude of $R_{\Sigma\Lambda}$. However, if the global symmetry model is approximately correct, $R_{\Sigma\Lambda}$ is not extremely large, and the inequalities can be strengthened. In order to estimate a reasonable limit for $R_{\Sigma\Lambda}$, we observe that for S-wave channels at low energy, R_{ij} is proportional to p_ip_j, where p_i denotes the momentum in the center-of-mass system of a particle in the state i. The product $p_\Sigma p_\Lambda$ at $\bar{K}N$ threshold is equal to the corresponding product p_N^2 for pion-nucleon scattering at a pion lab energy of about 170 Mev. At this energy the rates for isotopic spin $\frac{1}{2}$ and $\frac{3}{2}$ S-wave pion-nucleon scattering (defined in terms of the corresponding cross sections by $\sigma_j=\pi\lambda_N^2R_j$) are both approximately equal to 0.12. Hence we may safely say that either $R_{\Sigma\Lambda}<0.2$ or the $\pi\Lambda\leftrightarrows\pi\Sigma$ reaction is much stronger than πN scattering. However, if $R_{\Sigma\Lambda}<0.2$ and $x\leqq 0.58$,

[3] K. M. Watson, Phys. Rev. 95, 228 (1954).
[4] Richard H. Capps, Phys. Rev. Letters 2, 475 (1959).
[5] Ken Kawarabayashi, Progr. Theoret. Phys. (Kyoto) 22, 451 (1959).
[6] D. Amati and B. Vitale, Nuovo cimento 19, 895 (1958). See also M. H. Ross and G. L. Shaw, Phys. Rev. 115, 1773 (1959).

it may be seen from Eq. (3) that $2|\delta_{1\Sigma}-\theta_{1K\Sigma}|<20°$, so that the inequality of Eq. (5) may be strengthened.

If the $KN\Sigma$ and $KN\Lambda$ interactions have even parity, the relevant $\pi\Sigma$ and $\pi\Lambda$ states are the $P_{\frac{1}{2}}$ states. The conclusions reached above concerning the possible values of the phase shifts [Eqs. (3) and (5)] are valid in this case too. Since the $P_{\frac{1}{2}}$ pion-nucleon scattering phase shifts at energies around 130 Mev are even smaller than the S phase shifts, the difference between the $\pi\Sigma$ and πN scattering would be even more striking in this even parity case.

The mass difference between the K^-p pair and the $\bar{K}^0n$ pair is large enough ($\sim$5.3 Mev) to lead to appreciable corrections in the charge independence model used here. Since the experimental ratio $|T_{1K\Sigma}|^2/|T_{0K\Sigma}|^2$ is small, we must worry particularly about corrections to the amplitude $T_{1K\Sigma}$. We will estimate the possible effects of the mass difference on the phase difference $\delta_{1\Sigma}-\theta_{1K\Sigma}$, again assuming odd $KN\Sigma$ and $KN\Lambda$ intrinsic parities. We ignore the mass differences between the different $\pi\Sigma$ states, since near the $\bar{K}N$ threshold energy, the relative differences in center-of-mass momenta of the $\pi^+\Sigma^-$, $\pi^0\Sigma^0$, and $\pi^-\Sigma^+$ states are small.

The K^-p—$\bar{K}^0n$ mass difference leads to transitions between the $I=0$ and $I=1$ amplitudes, but does not lead to transitions between the $\bar{K}N$ and $I=2$ $\pi\Sigma$ states.[7] Hence, there are four relevant open S-wave channels of total charge zero at the K^-p threshold, which we choose to be the K^-p, $\pi^0\Lambda$, and the $\pi\Sigma$ states of isotopic spin 1 and 0. These states are denoted with the respective subscripts, K, Λ, 1Σ, and 0Σ. The four-channel form of the unitarity condition of reference 4 may be used to write the following inequality for the phase shift $\delta_{1\Sigma}$:

$$(1-R_{1\Sigma,\Lambda}-R_{1\Sigma,0\Sigma})^{\frac{1}{2}}\cos[2(\delta_{1\Sigma}-\theta_{K,1\Sigma})] \geqq 1-\tfrac{1}{2}[R_{1\Sigma,\Lambda}(1+x)+R_{1\Sigma,0\Sigma}(1+y)] -(xyR_{1\Sigma,\Lambda}R_{1\Sigma,0\Sigma})^{\frac{1}{2}}, \quad (6)$$

where

$$x=R_{K,\Lambda}/R_{K,1\Sigma} \quad \text{and} \quad y=R_{K,0\Sigma}/R_{K,1\Sigma}.$$

Although we are now using the K^-p state as a basic state, rather than the $I=0$ and $I=1$ $\bar{K}N$ states used earlier, the different amplitudes defined in these two ways are proportional. Hence, the experimental data of Eq. (1) implies the same proportional relations as used earlier, i.e.,

$$x=0.58, \quad y=6.7, \quad |\theta_{K,1\Sigma}-\theta_{K,0\Sigma}|=62°. \quad (7)$$

If charge independence were strictly valid, the transition rate $R_{1\Sigma,0\Sigma}$ between the $I=0$ and $I=1$ $\pi\Sigma$ channels would be zero. In this case the equality sign of Eq. (6) would apply, and the equation would reduce to Eq. (3).

The principal contribution to the isotopic spin mixing rate $R_{1\Sigma,0\Sigma}$ is expected to come from scattering of the $\pi\Sigma$ pair through K^-p and $\bar{K}^0n$ intermediate states and results from the difference in phase space between these two states. This contribution to $R_{1\Sigma,0\Sigma}$ can be calculated approximately in terms of the threshold cross sections for the relevant $\bar{K}+N\to\Sigma+\pi$ processes.[8] If we take these cross sections to be given by the analysis made by Dalitz and Tuan[9] of the $\bar{K}N$ data, the resulting rate is $R_{1\Sigma,0\Sigma}=0.028$. This corresponds to an S-wave cross section of about 1.1 mb, and may not be small compared to the scattering in pure isotopic spin states. Nevertheless our central point concerning the phase difference $\delta_{1\Sigma}-\delta_{0\Sigma}$ remains valid. To see this we make the generous assumption that $R_{1\Sigma,0\Sigma}$ is no greater than 0.05 and again assume that $R_{1\Sigma,\Lambda}$ is less than 0.2. Equations (6) and (7) may then be used to show that $2|\delta_{1\Sigma}-\theta_{K,1\Sigma}|<60°$. It should be pointed out that the equality sign in Eq. (6) holds only if the phases of the quantities $V_{K,\Lambda}V_{1\Sigma,\Lambda}^*$ and $V_{K,0\Sigma}V_{1\Sigma,0\Sigma}^*$ (where V_{ij} is defined in reference 4) are equal. Such an equality is unlikely, so that one expects the quantity $2|\delta_{1\Sigma}-\theta_{K,1\Sigma}|$ to be smaller than the maximum allowed by the unitarity condition.

In a similar manner one may estimate the effect of the K^-p—$\bar{K}^0n$ mass difference on the $I=0$ phase difference $\delta_{0\Sigma}-\theta_{0K\Sigma}$. In this case it is the inverse of the quantity $y=R_{K,0\Sigma}/R_{K,1\Sigma}$ that enters into the relation for $\cos[2(\delta_{0\Sigma}-\theta_{K,0\Sigma})]$. Since y^{-1} is small, the mass difference effect is much smaller than in the $I=1$ case.

It is concluded that if the experimental $\bar{K}N$ absorption phase difference is larger than $\sim$45° (as is presently believed), at least one of the $j=\frac{1}{2}$ pion-hyperon scattering amplitudes is much larger at the $\bar{K}N$ threshold energy than are any of the $j=\frac{1}{2}$ pion-nucleon amplitudes at a corresponding energy. Such a large πY amplitude would imply either that the global symmetry assumption concerning the πNN, $\pi\Lambda\Sigma$, and $\pi\Sigma\Sigma$ coupling constants is wrong, or that the presence of the $\bar{K}N$ channel leads to large modifications in the πY amplitudes at the $\bar{K}N$ threshold energy.

---

[7] For a discussion and calculation of the mass difference effects in $\bar{K}N$ absorption and scattering, see R. H. Dalitz and S. F. Tuan (to be published).

[8] The calculation may be done in the following manner. One first calculates the rate $R_{1\Sigma,0\Sigma}$ at an energy slightly above the $\bar{K}^0n$ threshold, using only real intermediate $\bar{K}^0n$ and K^-p states. One then analytically continues the result down in energy to the K^-p threshold, assuming that the momenta in the K^-p and $\bar{K}^0n$ states are the only factors that vary appreciably in this region. This method is equivalent to that used in reference 7.

[9] R. H. Dalitz and S. F. Tuan, Phys. Rev. Letters **2**, 425 (1959).