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Inelastic Scattering of High-Energy Protons Exciting a Collective Level of Nucleus

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An analysis is made of the angular distribution and polarization of 185-Mev protons inelastically scattered by carbon. A rough calculation using the distorted wave Born approximation shows, even though the quantitative agreement is poor, that the 4.4-Mev level of C^{12} may be interpreted as a collective state.

I. INTRODUCTION

IN analyzing the polarization of high-energy nucleons elastically scattered by nuclei, it is customary to use an optical model potential which has a spin-orbit interaction term proportional to the gradient of the spin independent part. Such a potential may be written as

$$V=V_C\rho(\mathbf{r})+V_S(\hbar/\mu c)^2\frac{1}{r}\frac{d\rho}{dr}\boldsymbol{\sigma}\cdot\mathbf{l}, \qquad (1)$$

where $\hbar\mathbf{l}$ is the nuclear angular momentum operator and ρ contains the spatial dependence of the potential. The purpose of this paper is to extend the use of this potential to the treatment of inelastic scattering in terms of the nuclear collective model.

Ruderman,[1] and independently Maris,[2] proved that the treatment of the inelastic scattering in terms of the collective model in the Born approximation gives the same explicit expressions for both elastic and inelastic polarization. But the agreement of the Born approximation calculation with the experimental results is known to be only qualitative. The disagreement becomes significantly large in the neighborhood of the diffraction minimum. An improvement over the Born approximation is achieved by taking into account the distortion of the wave functions. Since the low excited states, as well as the ground states, of many nuclei are well described by the independent particle model, many authors have studied the inelastic scattering in terms of this model.[3] On the other hand, the collective model has been equally successful in bringing out certain characteristics of nuclei and there have also been attempts to interpret the inelastic scattering along this line.[4,5] In reference 5 (further referred to as I) the 96-Mev proton data of Strauch and Titus[6] were analysed. Since no polarization effect was measured in this experiment, the analysis was made, for the sake of simplicity, without including the spin-orbit interaction term of (1). In order to bring out the collective nature of the nucleus, the nuclear boundary was taken as a spheroid

$$r=R[1+\beta Y_2{}^0(\Omega')], \qquad (2)$$

where Ω' is measured from a principal axis of the spheroid which, in turn, makes an angle Ω_0 in the space-fixed coordinate system. In this paper we follow the method of I but include the spin-orbit term of (1).

The nuclear wave functions are simply $Y_0{}^0(\Omega_0)$ and $Y_2{}^m(\Omega_0)$ for a $0+$ state and a $2+$ state, respectively. The nucleon wave functions are constructed by taking the distortion of the waves into account. For high-energy incident nucleons the fractional change of the wave number is negligible and hence the incident beam

[1] M. Ruderman, Phys. Rev. **98**, 267 (1955).
[2] Th. A. J. Maris, Nuclear Phys. **3**, 213 (1957).
[3] P. Benoist, C. Marty, and Ph. Meyer, Physica **22**, 1173 (1956); C. A. Levinson and M. K. Banerjee, Ann. Phys. **2**, 471 (1957); H. A. Bethe, Ann. Phys. **3**, 190 (1958); E. J. Squires, Nuclear Phys. **6**, 504 (1958).
[4] H. S. Köhler, Nuclear Phys. **9**, 49 (1958/59).
[5] K. Nishimura, Nuclear Phys. **7**, 425 (1958).
[6] K. Strauch and F. Titus, Phys. Rev. **103**, 200 (1956).

and the scattered beam are written as

$$\exp\left[i\mathbf{k}_0\cdot\mathbf{r}-\frac{i}{\hbar v}\int_0^\infty V(\mathbf{r}-\hat{k}_0 s)ds\right], \tag{3a}$$

and

$$\exp\left[i\mathbf{k}_F\cdot\mathbf{r}+\frac{i}{\hbar v}\int_0^\infty V(\mathbf{r}+\hat{k}_F s)ds\right], \tag{3b}$$

where $\hbar\mathbf{k}_0$ and $\hbar\mathbf{k}_F$ are the nucleon momenta before and after the scattering and v is the speed of the scattered nucleon; $\hat{k}\equiv\mathbf{k}/k$.

II. CALCULATION

The nucleon-nucleus potential is (1) where the well parameters are complex,

$$\begin{aligned} V_C&=-(V_{CR}+iV_{CI}),\\ V_S&=V_{SR}+iV_{SI}. \end{aligned} \tag{4}$$

As for ρ, a spheroidal Woods-Saxon potential would naturally be more realistic but in order to simplify the calculation we use the spheroidal square well, i.e., $\rho=1$ inside the boundary (2) and $\rho=0$ outside.

The nuclear wave functions are simply spherical harmonics and the nucleon wave functions are obtained by substituting (1) into (3). However, a direct substitution of (1) into (3) would lead us to expressions containing spin operators in the exponentials. We can avoid such expressions by first noticing

$$V_S\ll\mu c^2, \tag{5}$$

for a high-energy incident beam[7] and then neglecting the distortion of the waves due to the spin dependent part of the potential. This method of simplification was already used in the calculation of elastic polarization[8] and yielded a satisfactory result even though, as pointed out by Köhler,[4] it is in general not correct to omit the distortion due to the smaller part of the scattering potential. In addition, we shall assume $\beta\ll1$ so that we can approximately write the nucleon wave functions before and after scattering:

$$\begin{aligned} \psi_0&=\exp\left[i\mathbf{k}_0\cdot\mathbf{r}-\frac{iV_C}{\hbar v}\int_0^\infty\rho'(\mathbf{r}-\hat{k}_0 s)ds\right],\\ \psi_F&=\exp\left[i\mathbf{k}_F\cdot\mathbf{r}+\frac{iV_C}{\hbar v}\int_0^\infty\rho'(\mathbf{r}+\hat{k}_F s)ds\right], \end{aligned} \tag{6}$$

where $\rho'(\mathbf{r})$ is now not a spheroidal but a spherical square well, i.e., $\rho'(\mathbf{r})=1$ for $r<R$ and $\rho'(\mathbf{r})=0$ for $r>R$. Then the integrals in (6) are easily reduced;

$$\int_0^\infty\rho'(\mathbf{r}-\hat{k}_0 s)ds=\mathbf{r}\cdot\hat{k}_0+[(\mathbf{r}\cdot\hat{k}_0)^2+R^2-r^2]^{\frac{1}{2}},$$

$$\int_0^\infty\rho'(\mathbf{r}+\hat{k}_F s)ds=-\mathbf{r}\cdot\hat{k}_F+[(\mathbf{r}\cdot\hat{k}_F)^2+R^2-r^2]^{\frac{1}{2}}.$$

The transition amplitude from a ground state (0+) to an excited state (2+) is obtained from matrix elements of the form

$$\begin{aligned} I_m&=\frac{1}{(4\pi)^{\frac{1}{2}}R^3\beta}\int Y_2^{m*}(\Omega_0)\psi_F^*(\mathbf{r})VY_0^0(\Omega_0)\psi_0(\mathbf{r})d\Omega_0 d\mathbf{r}\\ &=A_m+B_m\boldsymbol{\sigma}\cdot\hat{n}, \end{aligned} \tag{7}$$

where A_m and B_m are complex numbers and $\hat{n}$ is a unit vector normal to the scattering plane. The differential cross section and the polarization are

$$d\sigma/d\Omega\propto\sum(|A_m|^2+|B_m|^2), \tag{8}$$

and

$$P=\frac{2\sum(A_{mR}B_{mR}+A_{mI}B_{mI})}{\sum(|A_m|^2+|B_m|^2)}, \tag{9}$$

where the summations are from $m=-2$ to $+2$ and the second subscripts R and I on A and B mean the real and imaginary parts of A_m and B_m.

An approximate calculation is done as in I. First, by assuming $\beta\ll1$, we rewrite the integral of (7) as

$$\int d\Omega_0 d\mathbf{r}\to\int_0^R r^2dr\int d\Omega d\Omega_0+R^3\beta\int_{\text{at }r=R}Y_2(\Omega')d\Omega d\Omega_0.$$

The first term on the right-hand side vanishes due to the orthogonality of the spherical harmonics. Secondly, we observe that $RV_C/\hbar v$ is satisfactorily smaller than 1 and expand the second terms in the exponential (6) in power series. We keep the first two terms of the expansion when they are multiplied by the smaller term of the potential. The result is:

$$A_0=-V_Cj_2(KR)-\frac{iV_C^2KR}{\sqrt{5}\hbar vk}G_1(KR)-\frac{2iV_C^2R}{\sqrt{5}\hbar v}G_2(KR),$$

$$B_0=\left(\frac{\hbar}{\mu c}\right)^2k^2V_S\sin\Theta\left\{ij_2(KR)-\frac{2iV_C}{\hbar vk}j_2(KR)-\frac{V_CKR}{\sqrt{5}\hbar vk}G_1(KR)-\frac{2V_CR}{\sqrt{5}\hbar v}G_2(KR)\right\},$$

$$A_{\pm1}=0,$$

$$B_{\pm1}=i\left(\frac{\hbar}{\mu c}\right)^2\frac{k^2V_CV_S\sin\Theta}{\sqrt{5\pi}\hbar vk}G_3(KR),$$

$$A_{\pm2}=-(2iV_C^2R/\sqrt{5}\hbar v)G_4(KR).$$

$$B_{\pm2}=-2(\hbar/\mu c)^2(k^2V_CV_SR\sin\Theta/\sqrt{5}\hbar v)G_4(KR),$$

[7] W. B. Riesenfeld and K. M. Watson, Phys. Rev. **102**, 1157 (1956).

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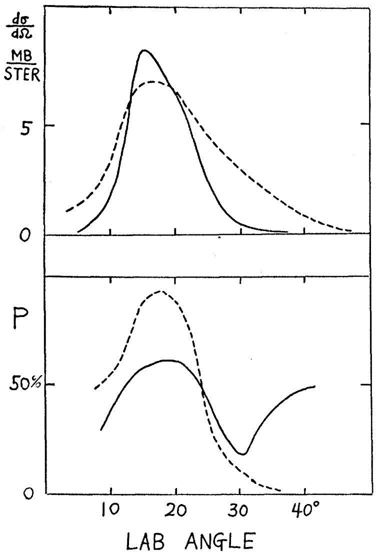

FIG. 1. The calculated (solid curves) and the observed (dashed curves) angular distribution and polarization of 185-Mev protons inelastically scattered by carbon.

where Θ is the scattering angle, $\hbar K$ is the momentum transfer and the G's are the functions defined as

$$G_1(\xi)=(i/\sqrt{5})[2j_1(\xi)-3j_3(\xi)],$$

$$G_2(\xi)=\sum_{l=0}^{\infty}(2l+1)^{\frac{1}{2}}i^l j_l(\xi)\int Y_l^0(\Omega)Y_2^0(\Omega)|\sin\theta\cos\varphi|d\Omega,$$

$$G_3(\xi)=\sum_{l=0}^{\infty}(2l+1)i^l j_l(\xi)\int_{-1}^{1}P_2^1(\omega)P_l(\omega)d\omega,$$

$$G_4(\xi)=\sum_{l=0}^{\infty}(2l+1)^{\frac{1}{2}}i^l j_l(\xi)\int Y_l^0(\Omega)Y_2^{-2}(\Omega)|\sin\theta\cos\varphi|d\Omega.$$

These integrals are easily evaluated and the G functions are expressed as series involving spherical Bessel functions. These series are quickly converging and easily evaluated.

III. DISCUSSION

Since the improvement over the Born approximation was one of the aims of the present work, we shall first compare our results with what we would obtain if the Born approximation were used. By the Born approximation, we would obtain

$$A_0=-V_C j_2(KR),$$
$$B_0=i(\hbar/\mu c)^2 k^2 V_S \sin\Theta j_2(KR),$$

and all the other A's and B's are identically zero. Then the differential cross section is zero at $j_2(KR)=0$ and this gives diffraction minima, or by measuring the position of the first diffraction minimum one would be able to deduce R.

On the other hand, the expression (9) takes the form of zero over zero at a diffraction minimum and the polarization does not show a dip which one observes experimentally.[9]

Now in the present treatment of the problem, the correction terms to the Born approximation are of considerable magnitude at $j_2(KR)=0$ and hence R cannot be determined so readily. But because of these correction terms, A's and B's do not become zero at the same scattering angle any more. Since I_0 is much greater than the other I's in the vicinity of the first diffraction minimum, we shall examine the expression

$$\frac{2(A_{0R}B_{0R}+A_{0I}B_{0I})}{|A_0|^2+|B_0|^2} \tag{10}$$

in order to study how a dip can appear in the present calculation. With a reasonable choice of well parameters, we find that B_{0I} becomes zero first, then A_{0I}, B_{0R}, and A_{0R} in this order as we increase the scattering angle. Then between the two angles at which A_{0I} and B_{0I} become zero, respectively, and also between those for A_{0R} and B_{0R}, the two terms of the numerator of (10) have opposite signs and the result is a dip.

In Fig. 1, the observed and the calculated differential cross sections and polarization are shown for 185-Mev protons exciting the 4.4-Mev level of C^{12}. The theoretical curves are calculated with $V_{CR}=1$ Mev, $V_{CI}=10$ Mev, $V_{SR}=0.5$ Mev, $V_{SI}=-1$ Mev, and $r_0=R/A^{\frac{1}{3}}=1.3\times10^{-13}$ cm. Since the aim of this paper is not to obtain the best fit to the experimental result, no extensive search for the best set of the well parameters was carried out. It is found that the differential cross-section curve is less sensitively dependent upon the well parameters than the polarization curve.

Although the quantitative agreement is still poor one may conclude along the line of reference 5 that the 4.4-Mev level of C^{12} can be regarded either as a collective state or as an individual particle excitation.

[9] P. Hillman, A. Johansson, and H. Tyrén, Nuclear Phys. 4, 648 (1957).