obtain the result that 4.5±0.9% of the annihilations produce $K\bar{K}$ pairs.

In view of the uncertainties in scanning, and recognizing that some of the tentatively identified K mesons are probably valid, we feel that a best estimate is that 4.0±1.0% of all annihilations produce $K\bar{K}$ pairs.

In Fig. 16 we present the kinetic energy spectrum of all the K mesons observed.

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High-Energy Neutron Beam of 45% Polarization*

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A beam of polarized neutrons has been produced by allowing the 164-Mev internal proton beam of the Harvard synchrocyclotron to strike a beryllium target. The neutrons produced in the forward direction are then polarized by scattering from carbon at 15°. When neutrons of energy greater than 110 Mev are selected by the detection process, an average beam energy of 124-Mev results. An intensity of 2.9×10^5 neutrons/in.^{2} min through 2 in. by 6-in. collimator has been obtained, with a polarization 0.447±0.020. The shielding techniques are also discussed.

INTRODUCTION

PREVIOUSLY, high-energy polarized neutrons have been produced[1-3] by allowing the internal unpolarized proton beam of a cyclotron to strike a target and by collimating neutrons which are emitted at about 26° from (p,n) reactions in the target. In this way beam polarizations ranging from 0.08 at 95 Mev to 0.16 at 350 Mev have been obtained. Such a low polarizing efficiency limits the ultimate accuracy of a double scattering experiment. Observed left-right asymmetries in the second scattering can never exceed the polarizing efficiency. However, errors in geometrical alignment at the analyzer contribute an absolute error to the asymmetry measurement. The result is, that when the data are expressed as the asymmetries resulting from a fully polarized beam, the alignment errors are amplified. This type of error, rather than statistical uncertainty, has limited experiments to date.

The work of Harding[2] suggested that it is possible to obtain a less intense beam of higher polarization by scattering an unpolarized neutron beam from a carbon polarizer, and such a beam has been produced at the Harvard cyclotron.

* Supported by the joint program of the Office of Naval Research and the U. S. Atomic Energy Commission.
† National Science Foundation Cooperative Fellow.
[1] G. H. Stafford, C. Whitehead, and P. Hillman, Nuovo cimento **5**, 1589 (1957).
[2] R. S. Harding, Phys. Rev. **111**, 1164 (1958).
[3] R. T. Siegel, A. J. Hartzler, and W. A. Love, Phys. Rev. **101**, 838 (1956).

BEAM PRODUCTION

A plan view of the arrangement is given in Fig. 1. A beryllium target (A) 2-in. high, 1-in. wide and 20 Mev along the beam is placed at $41\frac{1}{2}$-inch radius in the vacuum tank of the machine, where it intercepts the un-regenerated beam of maximum energy 164 Mev. Neutrons produced at zero degrees travel 57 inches and strike a carbon polarizer (B) 2 in. wide by 6 in. high by 10 in. along the scattered beam, situated just outside the cyclotron tank. Protons are prevented from reaching the polarizer by the magnetic field and shielding at (C) and (D). A beam pipe 2 in. by 6 in. pierces 7 ft of iron and lead shielding and accepts neutrons scattered at 15° by the polarizer. An additional 4 ft of collimator 4 in. wide serves as an antiscattering slit near target (E). The height of the beam pipe was made small enough so that the target (E) could not view neutrons scattered by the pole tips of the cyclotron magnet. The 15° scattering angle provides a reasonably high beam polarization without allowing excessive dilution by inelastic scattering.

A large amount of shielding is necessary to reduce the neutron flux in the experimental area to a value low enough for neutron detection. The shielding described here has proven adequate for measuring recoil protons from a liquid hydrogen target and for measuring neutrons scattered by a carbon analyzer. Additional shielding is planned for measuring neutrons scattered from hydrogen. The amount of shielding required was calculated by assuming that a neutron which undergoes an inelastic interaction is lost. A linear absorption

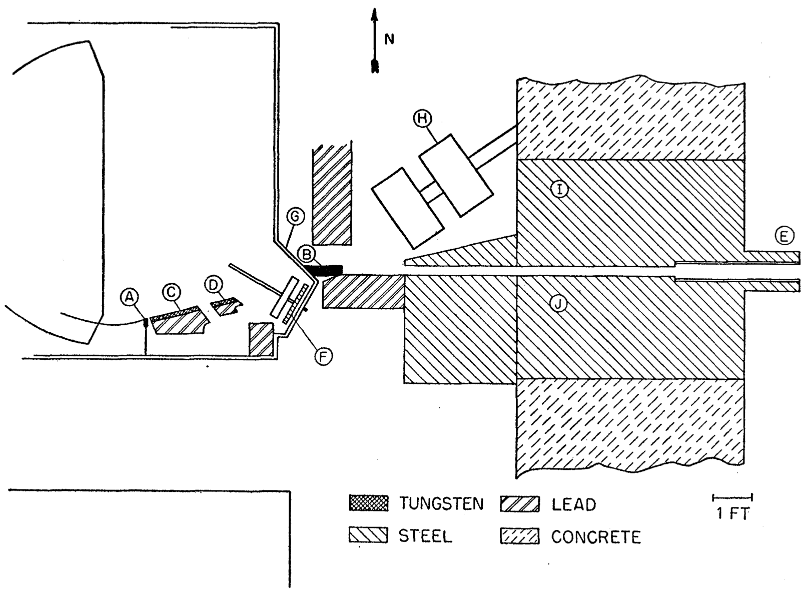

FIG. 1. Plan view of targets and shielding for the polarized neutron beam. (A) Beryllium target; (B) carbon polarizer; (C) movable lead shield faced with tungsten; (D) fixed lead shield faced with tungsten; (E) analyzer; (F) polarized proton regenerator; (G) Lucite window; (H) polarized proton quadrupole magnets; (I), (J) iron shielding in vault wall.

coefficient, μ was calculated from the inelastic cross section.[4] To determine whether a proposed portion of the shielding was sufficient, the following procedure was employed.

(1) By consulting a scale drawing, a portion of the shielding which might scatter neutrons from the beryllium target into the counting area was located.

(2) An arbitrary small volume of this potential scatterer is selected, and, using solid angles and elastic scattering cross sections, the flux of neutrons in the counting area due to a unit flux leaving the beryllium target was calculated, assuming that there no shielding existed.

(3) This result was then multiplied by e^{-a}, where $a=\int \mu dx$ for the path through all shielding from beryllium to scattering volume to counting area.

(4) These steps were repeated for adjacent volume elements, and the result is added to the background flux already obtained. It was found in practice that there were usually one or two volume elements for which the shielding was much less effective than for others, and that only these elements had to be considered.

(5) Finally, shielding was added to the design as necessary to reduce the background flux to the desired value.

The goal of the shielding installation was to reduce the flux at the target area to 0.1% of the beam flux. Sufficient shielding was placed on the north side of the beam pipe to protect against scattering of the primary neutron beam as it interacts with the existing quadrupole magnets (H) and the cyclotron vault in the region (I) of Fig. 1. It was then necessary to place shielding at (D) and (C) to reduce scattering by the walls of the beam pipe and by the shielding in region (I). Next, shielding had to be placed in region (J) to attenuate the neutrons coming directly from the beryllium target as well as those scattered by the shielding at (C) and (D). A vertical section through the beam pipe is shown in Fig. 2. The shielding was extended above and below the median plane to provide a long attenuating path for neutrons scattered in region (K). The coils of the cyclotron magnet also protect against scattering in the vertical plane.

The amount of shielding which could be placed at (C) and (D) was limited because clearance had to be maintained for the ion source probe and for the polarized proton regenerator. Calculation indicated that

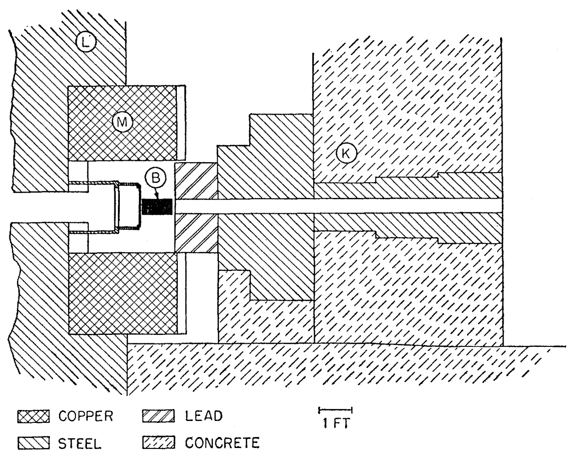

FIG. 2. Vertical section through the polarized neutron beam pipe. (B) polarizer; (K) vault wall; (L) pole piece; (M) magnet coil.

[4] R. G. P. Voss and R. Wilson, Proc. Roy. Soc. (London) **A236**, 41 (1956).

filling the available space with lead would provide insufficient attenuation, and it was decided to face this shielding with sintered tungsten[5] on the small radius side. The tungsten bars were 1 in. square and from 2 in. to 8 in. long. Their density was 17.9 g/cm^3, giving an attenuation coefficient $\mu = 0.226$ in.$^{-1}$, compared to $\mu = 0.139$ in.$^{-1}$ for lead. The tungsten bars were placed in the bottom of a mold which was then filled with molten lead. When the mold was removed, the solidified lead held the tungsten in place at one edge of the block. Shielding (D) is left in place at all times. Section (C) interfered with the polarized proton beam, and was placed on a track. A shaft extending from the shielding through a window in the wall of the vacuum tank allows the shielding to be positioned or retracted without opening the tank. A similar shaft controls the beryllium target, so that changeover to the polarized neutron beam can be accomplished in ten minutes without a tank opening.

The bulk of the shielding consists of iron bricks 2 in. by 4 in. by 12 in. piled as shown in Figs. 1 and 2.

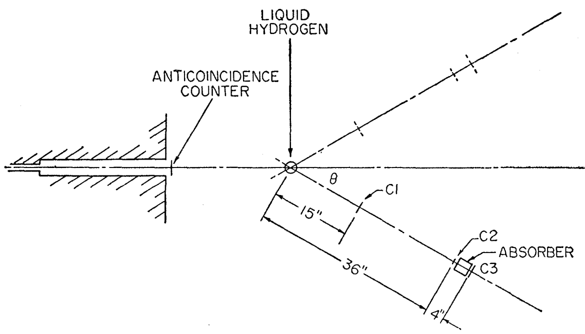

FIG. 3. Recoil proton telescope for measuring beam energy distribution.

Where the magnetic field is appreciable, lead bricks have been used in place of iron.

Measurements with a neutron counter[6] of the neutrons with energy greater than 100 Mev indicate that 6 feet from the wall the background flux is 0.003 times as intense as the beam at the analyzer. Closer to the wall, the ratio is less, because the protection of the shielding in the vertical plane is more effective. Protection equivalent to an additional three feet of iron should be sufficient to allow measurement of neutrons scattered from a 2-inch diameter hydrogen target.

BEAM CHARACTERISTICS

The flux and energy spectrum of the beam have been measured by observing recoil protons at 30° in the laboratory system when a 2-inch diameter liquid hydrogen target is placed in the beam. Differential range measurements were made by varying the thickness of

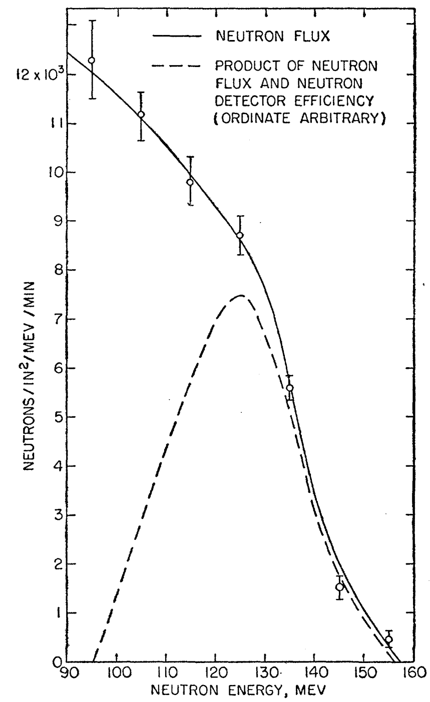

FIG. 4. Energy spectrum of polarized neutron beam and effective spectrum when using neutron detector.

the polyethylene absorber in the proton telescope (Fig. 3). The spectrum is given approximately by the following equation:

$$n(E) = \left(\frac{R(E+\Delta E) - R(E-\Delta E)}{2\Delta E}\right) \frac{A}{\sigma(E,\theta)\Omega v \rho L_0}, \quad (1)$$

where $R(E)$ = counting rate observed with neutron cutoff energy E, A = atomic mass number of target, ρ = target density, Ω = solid angle subtended by defining counter at target, v = target volume, $\sigma(E,\theta)$ = laboratory n-p differential cross section, L_0 = Avogadro's number, and $n(E)$ = neutron flux. Values of the cross section at various energies were obtained from the center-of-mass cross sections given by Hess.[7] The energy spectrum is given in Fig. 4. For a minimum energy of 110 Mev, the flux is 2.9×10^5 neutrons/in.^{2} min through a 2 in. by 6 in. opening. The internal cyclotron beam is about 1 microampere. The lack of a peak[8] in the energy distribution is attributed to the presence of lower energy neutrons which have undergone an inelastic interaction.

The scanning device shown in Fig. 5 was used to verify that the transverse beam distribution is symmetric and that its centroid coincides with the optical axis of the beam collimator. A threefold coincidence between counters A, B, and C is interpreted as a recoil proton from the interaction of a neutron with the

[5] Sintered tungsten was obtained from Sylvania Electric Company, Towanda, Pennsylvania.
[6] D. Miller and R. Hobbie, Rev. Sci. Instr. (to be published).
[7] W. N. Hess, Revs. Modern Phys. **30**, 368 (1958).
[8] T. C. Randle, J. M. Cassels, T. G. Pickavance, and A. E. Taylor, Phil. Mag. **44**, 425 (1953).

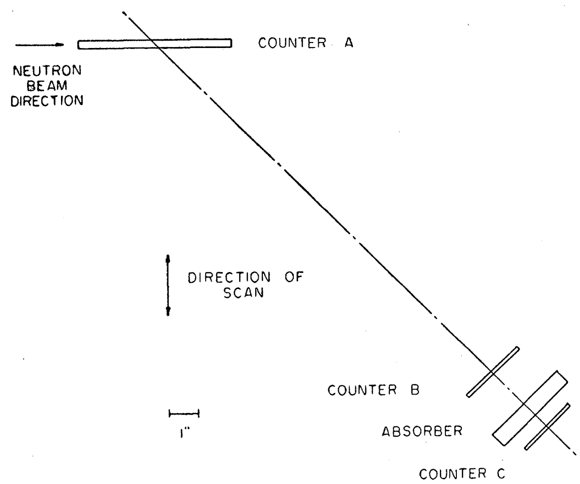

FIG. 5. Arrangement of counters for determining beam profile.

hydrogen in plastic scintillator A, whose dimensions are 5 in. by 6 in. by $\frac{1}{4}$ in. transverse to the beam. Polyethylene absorber placed between counters B and C restricts the neutrons to energies above 110 Mev. The entire telescope is swept through the beam on the bed of a small milling machine in $\frac{1}{4}$ inch steps. The dimensions are such that counters B and C never enter the direct beam. A typical scan is shown in Fig. 6. The centroid of this distribution is computed by numerical integration, and is found to lie within 0.01 in. of the optical center line, both at the analyzer and at the position of the counters used for asymmetry measurements.

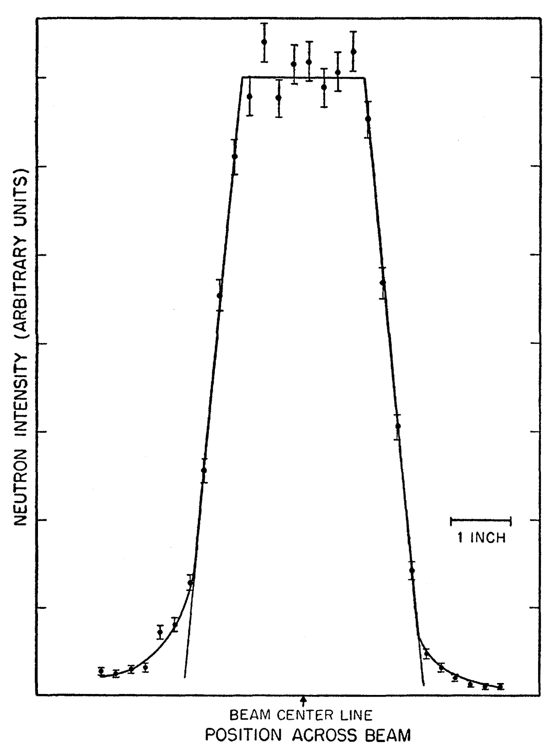

FIG. 6. Typical beam profile.

The beam polarization has been measured by double scattering from carbon. The neutron counter used has been described elsewhere[6]; its cutoff was such that the average beam energy was 124 Mev. The counter was mounted on a pivoted arm which can be swung from one side of the beam to the other. The pivot was aligned with the centroid of the beam distribution, while the zero degree position of the arm was determined by sweeping the neutron counter through the beam in 1 degree steps. Zero degrees was identified with the centroid of the distribution thus obtained. Additional shielding was provided by the blocks shown in Fig. 7. Monitoring was done with a BF_3 proportional counter located outside the cyclotron vault on the south side. This type of monitor was unaffected by the removal of the polarizer.

A direct subtraction of the background cannot be used to isolate the scattering from the carbon analyzer, since the background is attenuated in passing through the analyzer. If the superscript i refers to the counting rate observed with the analyzer in place, o refers to the rate with the analyzer removed, and a refers to that portion of the rate i which is due to the scattering from the analyzer, then the counting rates observed with a neutron counter on the north and south may be expressed as follows:

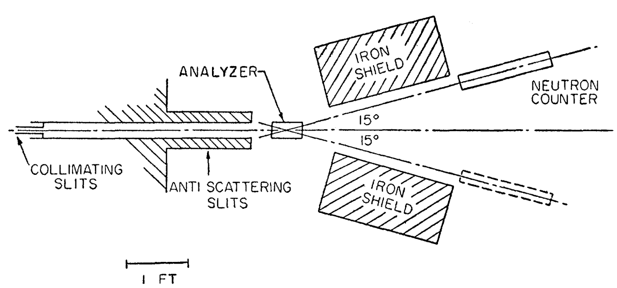

FIG. 7. Analyzer geometry for carbon double scattering.

$$N^i = N^a + \beta_N N^0,$$
$$S^i = S^a + \beta_S S^0. \quad (2)$$

Here β_N and β_S are the fractions of the measured backgrounds which reach each counter when the analyzer is installed. The desired asymmetry is then given by

$$e = \frac{N^a - S^a}{N^a + S^a}. \quad (3)$$

Although these transmission coefficients were not determined accurately it is possible to place limits on them which in turn set limits on the asymmetry of the analyzer scattering.

The transmission coefficient would have its minimum value if all the background neutrons were to pass

through the analyzer when it is in place. Calculations based on the inelastic cross section for neutrons scattered by carbon[4] show that, for the analyzer used, $0.8 \leqslant \beta \leqslant 1.00$. In order to restrict even further the range of values which β might assume, it is necessary to consider possible sources of the background. The background may be divided into two components: (1) neutrons from general leakage through the shielding; (2) neutrons which have travelled down the beam pipe and scattered from the collimating or antiscattering slits (Fig. 7). The first type of background is not attenuated by the analyzer. Only a portion of the second type will be attenuated, but a lower limit on β can be established by assuming that all of these neutrons are attenuated by the analyzer.

The fact that the intensity of the second type of background depends on the number of neutrons scattered into the beam pipe by the polarizer, while the first background component is independent of changes of the polarizer, enables the two types of background to be separated. Some neutrons will be scattered into the beam pipe by such things as the up-stream end of the collimator and the cyclotron tank walls when the polarizer is removed, so that it does not suffice to simply measure the background without the polarizer. If the flux of neutrons in the beam pipe were zero, then the neutrons measured would all be of type 1, and the counting rate would be the same with and without the analyzer. Assuming that the neutron flux in the beam pipe is a linear function of the number of scattering centers in the polarizer (i.e., neglecting self absorption) a plot of counting rate vs polarizer volume may be extrapolated to the point where the analyzer in rate equals the analyzer out rate. This rate is then the type 1 background rate. The remainder of the background is assumed to be attenuated by the analyzer. Since leak through is not the same on north and south, this determination is made separately for the two sides. The lower limits on the transmission coefficients now become $\beta_N \geqslant 0.92$, $\beta_S \geqslant 0.94$.

The neutron beam consists of neutrons scattered by elements other than carbon. Therefore it is not correct to apply directly the relationship $e = P^2$, for it is valid only when the two scatterings are identical. By making asymmetry measurements with the polarizer in place and removed, it is possible to calculate an asymmetry due to double scattering from carbon. From this asymmetry, e_c, the carbon analyzing efficiency is obtained as $P_c^2 = e_c$. The beam polarization; P_b, is then calculated

TABLE I. Values of carbon polarization and beam polarization obtained by assuming the limiting values on the transmission coefficients. Errors reflect counting statistics only.

	$\beta_n = 0.92$; $\beta_s = 1.00$ $\gamma = 0.657$	$\gamma = 1$	$\beta_n = 1.00$; $\beta_s = 0.94$ $\gamma = 0.657$	$\gamma = 1$
P_{carbon} (15°)	0.464 ± 0.010	0.476 ± 0.010	0.469 ± 0.010	0.480 ± 0.010
P_{beam}	0.438 ± 0.010	0.426 ± 0.010	0.426 ± 0.010	0.417 ± 0.010

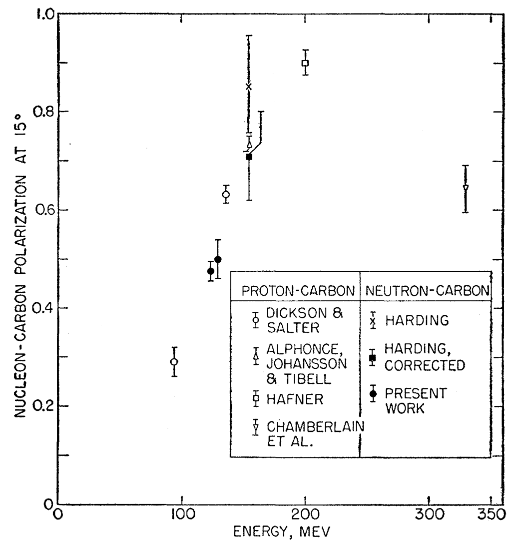

FIG. 8. Plot of polarization vs energy for nucleons scattered at 15° from carbon.

by using the equation

$$P_{\text{beam}} = e_{\text{beam}}/P_c. \tag{4}$$

The inelastic component of the beam has been estimated to cause an error of less than 0.01 in the beam polarization.

As in the case of the analyzer background subtractions, the attenuation of background in the polarizer must be considered. Only a fraction, γ, of the neutrons scattered into the beam pipe when the polarizer is removed, are transmitted through the polarizer when it is in place. Limits are placed on γ by assuming that all or none of these neutrons pass through the polarizer. A calculation based on the inelastic cross section[4] gives the lower limit. The result is $0.657 \leqslant \gamma \leqslant 1$. The results of this calculation, for the various limits on the transmission coefficients, are given in Table I. The resulting beam polarization, taken as the average of these four possibilities and corrected by 0.02 ± 0.02 for energy spread and inelastic contribution is

$$P_b = 0.447 \pm 0.034.$$

The error in P_b reflects the uncertainty in the transmission coefficients, alignment errors, energy spread, multiple scattering and inelastic contributions as well as counting statistics.

CONCLUSION

A polarized neutron beam has been produced by double scattering from carbon. A fivefold increase in beam polarization has been realized at the expense of a reduction in beam intensity. This method should be particularly effective at higher energies, where a greater beam polarization could be realized. Calculations based on inelastic cross sections have proven quite satisfactory for designing neutron shielding.

TABLE II. Values of carbon polarization and beam polarization obtained for different average beam energies. Standard deviations include all known sources of error.

Energy, Mev	P_{carbon} (15°)	P_{beam}
124±1	0.474±0.020	0.447±0.034
128±1	0.502±0.040	0.498±0.051

The polarization is a rapidly increasing function of energy. Measurements were made with different cutoff energies in the neutron counter, and the results are displayed in Table II. As a check on the method, it is interesting to compare the value of P_c obtained with the p-carbon[9–12] and n-carbon[2] polarizations measured at other laboratories. Figure 8 shows this comparison. All errors have been increased to include the uncertainty in beam polarization. The n-carbon result of Harding has been shown as originally reported, and as corrected for a higher beam polarization.[13]

It is also clear that neutron shielding can be designed with accuracy. Calculations based on inelastic cross sections have been shown to be adequate.

ACKNOWLEDGMENTS

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[9] J. M. Dickson and D. C. Salter, Nuovo cimento, 6, 235 (1957).
[10] R. Alphonce, A. Johansson, and G. Tibell, Nuclear Phys. 3, 185 (1957).
[11] E. M. Hafner, Phys. Rev. 111, 297 (1958).
[12] O. Chamberlain, E. Segrè, R. D. Tripp, C. Wiegand and T. Ypsilantis, Phys. Rev. 102, 1659 (1956).
[13] Richard Wilson, Phys. Rev. 114, 260 (1959).

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Electromagnetic Waves in Gravitational Fields

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The scattering of plane electromagnetic waves by the gravitational field of an isolated physical system is studied. On the level of the geometrical optics approximation the general theory of light rays is formulated. In particular, the generalized formula for the Einstein deflection of light rays is obtained. On the level of the vectorial optics the problem of polarization is examined in detail. The formula obtained, describing a rotation of the plane of polarization due to the presence of the gravitational field, admits a direct geometrical interpretation. The theory is applied to the rotating body and a system of point masses. The physical results established concerning the asymptotic behavior of the electromagnetic waves are independent of the coordinate system used in the computations.

1. INTRODUCTION

THE aim of this paper is to investigate the scattering of electromagnetic plane waves due to the gravitational field of a general isolated physical system, e.g., to the field of a rotating body or the field of a system of masses carrying out the motion according to Newton's laws (a double star, for instance). Generally, by "gravitational field of an isolated system" we understand the metric tensor $g_{\alpha\beta}$ in an arbitrary coordinate system, this metric being induced by matter in motion which during the motion is concentrated in a somewhat finite 3-region Ω of the spatial coordinates x^a. We assume the deviations of these quantities from Galilean values $\eta_{\alpha\beta}$ ($\eta_{00}=1$, $\eta_{0a}=0$, $\eta_{ab}=-\delta_{ab}$) defined as $\Delta g_{\alpha\beta}=g_{\alpha\beta}-\eta_{\alpha\beta}$ to vanish at infinity together with their derivatives at least as $O(r^{-1})$, $r=(x^ax^a)^{\frac{1}{2}}$.

We will also assume that the influence of the electromagnetic field on the metric field can be neglected. When the intensities of the waves scattered by the g field are small, the last assumption is certainly physically correct. We should like to mention here that the scattering of electromagnetic waves by the field of a rotating body has been recently examined independently by Skrotskii,[1] and Balazs.[2] In the center of interest of this paper, however, is the behavior of electromagnetic waves in the presence of a gravitational field induced by an isolated system when this field is as general as possible, consistently with it being physically reasonable. The solution of our general scattering problem, how-

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[1] G. B. Skrotskii, Doklady Akad. Nauk S.S.S.R. 114, 73 (1957) [translation: Soviet Phys.-Doklady 2, 226 (1957)].
[2] N. L. Balazs, Phys. Rev. 110, 236 (1958).