THE

PHYSICAL REVIEW

A journal of experimental and theoretical physics established by E. L. Nichols in 1893

SECOND SERIES, VOL. 118, NO. 6 JUNE 15, 1960

Ionic Conductivity of Highly Ionized Plasmas*

M. SAKUNTALA,† A. VON ENGEL,‡ AND R. G. FOWLER
Physics Department, University of Oklahoma, Norman, Oklahoma
(Received March 30, 1959; revised manuscript received March 17, 1960)

When a cloud of highly ionized gas ejected by a plasma shock tube is made to travel across a constant magnetic field, an electromotive force is produced in the plasma in a direction normal to both the field and the plasma path. Using two probes facing one another this electromotive force has been measured with an oscillograph. Its maximum value was found to be proportional to the field and the probe separation. By taking the maximum probe potential for different values of the external resistance between the probes, the lowest value of the "resistivity of the plasma" as measured by a current entering and leaving it was obtained. The resistivity has been shown to be independent of the magnetic field, the collecting area, the separation and surface state of the probes. All experiments were made in hydrogen at a gas pressure between 0.5 and 5 mm Hg with a nearly critically damped current pulse of order 10^4 amperes lasting for about 6–8 μsec and fields <2000 gauss.

The plasma resistivity between the probes was found to be of the order 1 ohm cm at a gas pressure of a few mm of Hg. This is about 100 times larger than the electronic resistivity of a fully ionized gas for currents circulating internally. The degree of ionization is thought to be high enough for interaction between charged particles to predominate. The measured values of the plasma resistivity agree with the results obtained from theory based on ionic conduction in the plasma which is here the necessary prerequisite for maintaining the electric neutrality of the moving plasma.

From the measured probe voltage the flow velocity of the plasma was derived. Its variation with gas pressure agrees with shock wave theory.

1. INTRODUCTION

THE electric conductivity of highly ionized plasmas which has been measured hitherto is either that controlled by the motion of the plasma electrons only, for current loops completed inside the plasma, or that for currents flowing in a plasma which is drawn from an electron emitting cathode. Although the former problem has been extensively treated theoretically[1–5] as well as experimentally,[6–10] there is still general disagreement over whether theory and observations agree or not.

In this paper another aspect of plasma conductivity has been studied and a simple method of measuring it was used which has been developed recently.[11] The results do not bear directly upon the fundamental question of agreement cited, but may well assist in understanding those situations which arise when currents are drawn to nonemissive electrodes.

The experiment is as follows: A highly ionized plasma produced in a shock tube moves down the tube into a region across which a uniform magnetic field is applied. At a certain point in the field region, two cold electrostatic probes facing one another are inserted into the plasma. From the potential difference between the probes with various external resistances the internal resistance and hence the resistivity of the plasma can be derived as well as its flow velocity. The values of the resistivity obtained here are of importance to the

* Supported in part by the National Science Foundation and the U. S. Office of Naval Research, through the University of Oklahoma Research Institute.
† Present address: Banaras Hindu University, Varanasi 5, India.
‡ Present address: Oxford University, Clarendon Laboratory, Oxford, England.
[1] D. E. Osterbrock, Phys. Rev. **87**, 468 (1952).
[2] T. G. Cowling, Proc. Roy. Soc. (London) **183**, 453 (1945).
[3] L. Spitzer, Jr., *Physics of Fully Ionized Gases* (Interscience Publishers, New York, 1956).
[4] H. Margenau, Phys. Rev. **109**, 6 (1958).
[5] L. Oster, Z. Astrophys. **42**, 228 (1957).
[6] E. L. Resler, S. C. Lin, and A. Kantrowitz, J. Appl. Phys. **23**, 1390 (1952).
[7] S. C. Lin, E. L. Resler, and A. Kantrowitz, J. Appl. Phys. **26**, 95 (1955).
[8] H. Petscheck and S. Byron, Ann. Phys. **1**, 270 (1957).
[9] A. C. Kolb, Phys. Rev. **107**, 345 (1957).
[10] A. L. Besbatschenko et al., *Electromagnetic Phenomena in Cosmical Physics*, edited by B. Lehnert (Cambridge University Press, New York, 1958), p. 464.
[11] M. Sakuntala, B. E. Clotfelter, W. Edwards, and R. G. Fowler, J. Appl. Phys. **30**, 1669 (1959).

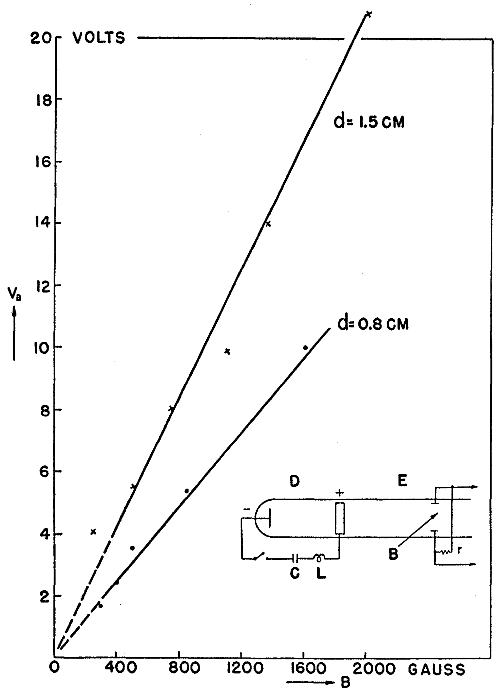

FIG. 1. Probe potential V_B as a function of the magnetic induction B in the plasma for two values of the probe separation d. Probes situated 6 cm from the end (anode) of the discharge tube D. E=expansion tube. V_c=2kv, i_{max}=6700 amp, p=1.2 mm Hg of hydrogen. C=14.5 μf, $L=4\times10^{-7}$ h, circuit resonance frequency $\nu=6\times10^4$ sec^{-1}. Insert: Experimental arrangement; tube diameter: 1.5 cm, probe diameter: 0.5 cm, length of tube D: 18 cm.

problem of electric generators converting thermal energy of flames into electric energy. In what follows we present absolute measurements of this plasma resistivity and of the flow velocity in hydrogen at various gas pressures together with a discussion of the theoretical aspects of the problem. It should be understood that these measurements have been taken in general in a highly but not fully ionized gas and evidence will be adduced here to show to what degree the gas has approached full ionization.

If a magnetic field B acts perpendicular to an electric field, theory of a partially ionized gas shows that the conductivity of the gas is a complex quantity due to the gyration of ions and electrons about the magnetic field lines interrupted by scattering collisions with molecules of the neutral gas. The net conductivity (σ) can be expressed as the sum of the direct (σ') and the transverse (σ'') conductivity which are related to the conductivity σ_0 in zero magnetic field by

$$\sigma=\sigma'+i\sigma''=\sigma_0/(1-i\omega\tau), \tag{1}$$

where $\omega=eB/mc$ is the cyclotron frequency and τ the mean collision time. Hence for sufficiently small fields B such that $\omega\tau\ll1$, σ becomes independent of B and equal to the direct current conductivity σ_0. Equation (1) holds rigorously when interaction between charged particles is negligible and collisions between charges and neutral atoms predominate. It holds approximately for a fully ionized gas when the magnetic field has no effect on the distribution function which represents the contribution to the current by electrons of various (total) velocities. In some of our measurements B was of the order 10^2 gauss; thus the cyclotron frequency ω_e for electrons was of the order 10^9 sec^{-1}. Their collision time in hydrogen at a density equivalent to $p=1$ mm Hg and an average energy of ≈1 ev is $\tau_e=1/Qv_e\approx10^{-10}$ sec [12] (Q being the total cross section and v_e the electron velocity) and hence $\omega_e\tau_e<1$. In this case, since $\omega\propto1/m$ and $\tau_i<\tau_e$, $\omega_i\tau_i\ll1$ for positive ions.

If on the other hand the gas is fully ionized and a very strong magnetic field applied (so that apparently $\omega\tau\gg1$ where τ is now the electron-ion collision time) the distribution function of the current is changed by the magnetic field in the sense that at large B electrons of lower velocity contribute a larger fraction to the current than at $B=0$. However, theory shows that the plasma resistivity ($\rho=1/\sigma$) for electrons in strong magnetic fields is only 2 to 3 times that in zero fields.[3] Thus with the moderate fields used the resistivity of a sufficiently highly ionized gas is not likely to vary much with B, a conclusion later born out by experiment. This would be even more reasonable, were the current due in part to positive ions. As to the applicability of the theory, the degree of ionization seems not to be a very critical factor provided it exceeds several percent[13] (see Sec. 6).

The measurements reported below have been carried out at a point 6 cm from the outer edge of the anode of the shock tube, i.e., in a region of zero electric field for $B=0$. Shock speeds, continuum radiation intensity, color temperature[14] and Stark broadening[15] suggest an equilibrium (plasma) temperature at $p=1$ mm Hg of about 7000°K and an ion concentration of up to 30% of the initial particle concentration ($\approx10^{16}$ ions/cm^3).

2. APPARATUS AND EXPERIMENTAL METHOD

The insert in Fig. 1 shows the arrangement used.[11] It consists of a discharge tube D filled with hydrogen in which the plasma is generated and the expansion tube E with the probes. As the conducting plasma is swept through the uniform magnetic field an emf with respect to the laboratory frame is induced in the plasma between the probes whose maximum value is of magnitude

$$V_B=Bud\times10^{-8}(\text{volts}), \tag{2}$$

where B is the magnetic induction in gauss, u the maximum flow velocity in cm/sec and d the separation between the probes in cm. The emf's so obtained are quite considerable. With $B=10^2$ gauss, $u=10^6$ cm/sec and $d=1$ cm, the potential difference between the

[12] A. von Engel, *Ionized Gases* (Clarendon Press, Oxford, 1955).
[13] W. P. Allis, *Handbuch der Physik*, edited by S. Flügge (Springer-Verlag, Berlin, 1956), Vol. 21, p. 442.
[14] R. G. Fowler et. al., Phys. Rev. 88, 137 (1952).
[15] R. G. Fowler et al., Phys. Rev. 87, 966 (1952).

probes is 1 volt. (Ionization by collision even for the largest induced fields is negligible.) Equation (2) holds only if no charge flows in the probe circuit. If, however, a charge flows through an external resistance r connected to the probes, the voltage V_r between the probes will be smaller than V_B. This reduction can have three causes: the plasma resistance, the reaction which the current has on the applied magnetic field and possibly sheath voltages at the probes (not of the conventional kind since an electric field is induced in the plasma). The last two effects would depend on the magnitude of the current. Since our measurements have shown that the plasma resistance is constant with respect to changes in probe current, it follows that these effects are quantitatively insignificant and it is thus permissible to derive the plasma resistance R_p from V_B, V_r, and r.

3. CHARGE SEPARATION AND PROBE CURRENTS

When a magnetic field acts on a highly ionized plasma moving across it, an emf is induced in the gas. It is not convenient to treat this emf as being due to the normal Hall effect because the moving plasma carries very nearly zero net space charge, thus making the component of electric current parallel to the velocity vector of plasma flow zero (the difference between the positive and negative charge densities which balances the induced emf is here $<10^{-6}$ part of either charge density). Ions and electrons of the plasma, subjected to the induced electric field, are mainly driven towards the probes and glass wall, i.e., in a direction perpendicular to the plasma flow and the magnetic field. In this way a potential difference between the two probes is set up.

The picture is relatively simple when the probes and the wall are isolated. However, when a current flows through an external resistance connected to the probes, the magnetic field of the probe current deflects the charges so that some of the ions and electrons acquire a component of velocity relative to the moving plasma and opposite to the direction of the plasma flow. It will be shown later that this current is determined by the drift velocity of the positive ions moving to one probe since electrons can only reach the other probe at the same rate.

The force acting on an ion is eBv_d, the electric field acting in the direction $(-u)$ is thus Bv_d and the reverse velocity component

$$v_{rev}=\mu^+Bv_d, \tag{3}$$

where μ^+ is the ion mobility. The current between the probes is

$$i=eNv_dA, \tag{4}$$

where N is the charge concentration and A the active area of the probe surface. Eliminating v_d from (3) and (4) we obtain,

$$v_{rev}=\mu^+Bi/eNA. \tag{5}$$

Assuming that the current in the probe does not exceed $i=10^{-2}$ amp, one obtains for $B\leq 10^3$ gauss and a charge concentration of $N\leq 10^{16}/\text{cm}^3$ a reverse velocity component of $v_{rev}\leq 10$ cm/sec and 30 times more for free electrons. Neither of these represent a degree of momentum transport comparable with that which is resident in the flowing plasma.

Two other points require consideration: the redistribution time for the space charges in the plasma when it enters the magnetic field and the time constant for establishing the current in the probe circuit when the plasma passes the probes. Assume that at a certain point the magnetic field changes discontinuously from zero to its final value. When the plasma with the net space charge zero passes this point, an electric field is induced in it given by the flow velocity which displaces ions and electrons to opposite sides of the walls of the tube until the surface density of charge has reached a value which is equal to the induced electric field. Assuming linear geometry, one obtains from Laplace's equation, the known ion concentration in the plasma ($\geq 10^{15}/\text{cm}^3$) and the ion mobility ($\geq 10^5$ esu), a redistribution time of the order $\tau=10^{-12}$ sec which is small compared with the time the plasma takes to move from the edge of the magnetic field to the probes (4×10^{-6} sec). The other question is whether the probe current has sufficient time to reach its final value when the plasma passes the probes. If the motion of the positive ions determines the rate of rise of the current whereby the ion mass has to be accelerated by the electric field, the time constant for the rise of the current is $\tau_i=\mu^+/(e/m_i)$. For hydrogen ions one obtains τ_i of the order $=10^{-9}$ sec. Since the flow velocity of the plasma is of the order 10^6 cm/sec, this means that the current is established in a time interval corresponding to a distance travelled by the plasma of the order $=10^{-3}$ cm which is small compared with the smallest diameter of the probe used (0.1 cm).

4. PARASITIC VOLTAGES INDUCED IN THE MEASURING CIRCUIT

At first the oscillograms of the open circuit probe voltage[11] were found to differ erratically. This was traced to voltages, induced by the large fluctuating discharge currents, in the cathode follower circuits to which the two probes were connected. The disturbances were caused by the magnetic field of the current loop formed by the discharge path which runs radially from the center of the tube to the ring anode. Because of the large currents the discharge is constricted. During each current pulse the anode spot takes up a different position on the ring anode thereby inducing varying potentials in the probe circuit. By arranging symmetrical leads to the cathode follower, this disturbance was removed.

In earlier measurements[11] of the plasma resistance each probe was connected through a resistor to the ground. The results were unsatisfactory because of large parasitic pulses induced in this circuit. By using a symmetrical probe circuit with two equal resistances

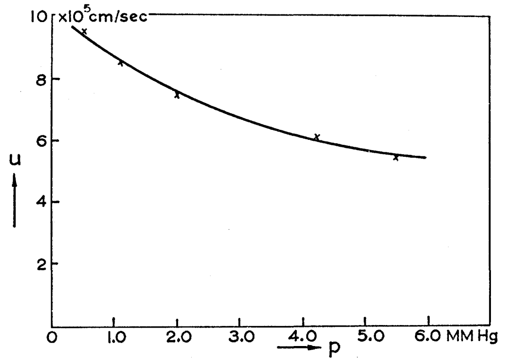

FIG. 2. Plasma flow velocity u in hydrogen as a function of the ambient gas pressure p. ($B=500$ gauss, $i_{max}=6700$ amp, $V_c=2$kv.)

joining the two probes (one on each side of the expansion tube), and omitting any ground connection, traces free of parasitic voltages were obtained.

5. MEASUREMENTS AND DISCUSSION OF THE RESULTS

(a) General Observations

The potential difference between the probes was measured as a function of the various discharge parameters. From Eq. (1) it follows that the maximum emf V_B (open circuit voltage) is proportional to the magnetic induction B and the probe separation d, provided the plasma velocity u is constant. Figure 1 shows that this is true. The data refer to a gas pressure of $p=1.2$ mm Hg, a condenser potential $V_c=2$ kv and a maximum discharge current $i_{max}=6700$ amp. Since the current pulse was constant u must be constant.

It is noteworthy that the absolute values of u are in agreement with the earlier measurements based on mirrorgrams.[16] In Fig. 2 the variation of u with the gas pressure p is shown for a constant current pulse. The fall of u with increasing pressure is in agreement with shock tube theory which assumes that a given mass of a gas which is proportional to p receives a certain fraction of the energy stored in the capacitor to acquire an energy of flow corresponding to a velocity u.[17] For the two curves in Fig. 1 and from (2) the plasma flow velocity was found to be $u=7\times10^5$ and 7.5×10^5 cm/sec, respectively.

The plasma resistance R_p between the probes was found by measuring V_B and V_r for various values of the external resistor r while B, V_c and p were kept constant. The resistors were of the carbon film type and, selected in pairs, ranged between 1000 and 20 ohms. The values of R_p obtained for the various external resistances r were the same within $\pm5\%$ and displayed no systematic variations.

Figure 3 shows that R_p decreases first as i_{max} is increased and then seems to become constant. The rapid change of R_p with rising i_{max} may be thought to be caused by the variation of the degree of ionization of the plasma whereas the slow decrease of R_p at higher currents may be due to a slow rise of the plasma temperature. In another series of experiments R_p was measured as a function of the gas pressure p (in hydrogen) between 0.5 and 5.0 mm Hg, while V_c and B were kept constant. Figure 4 shows this for two values of the probe separation; the interpretation of it is given below.

(b) Effect of Probe Separation

Experiments in which the probe separation was changed confronted us with a new problem. When the separation was 1.5 cm, the probes were in line with the walls of the glass tube; with 0.8 cm separation they were situated in the central region of the plasma. At first probe potentials V_B were observed to be independent of the separation and equal to the value obtained at 1.5 cm. This curious result suggests that since the stems of the probes were originally without insulating shield, the probes acquired the highest potential difference to which they were exposed in the moving ionized gas and this potential appeared along stems of the probes.

In order to test this explanation the stems of the probes were painted with Glyptal and baked in an oven for about six hours at a temperature of 120°C. After testing the stems for perfect insulation the probes were

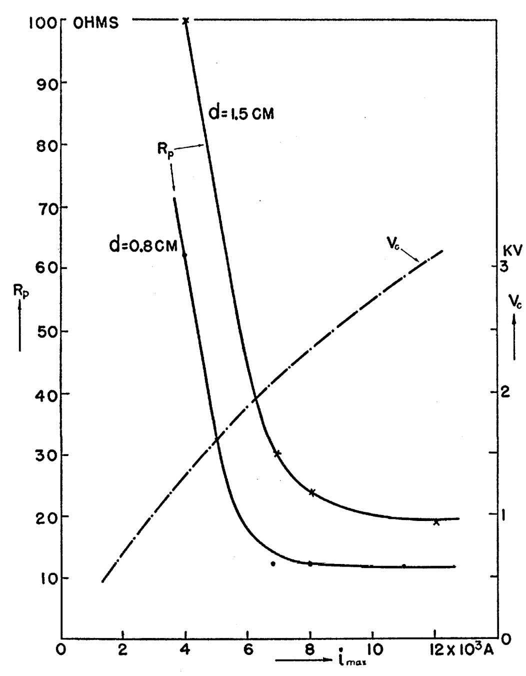

FIG. 3. Discharge current amplitude i_{max} as function of the condenser potential V_c and plasma resistance R_p between the probes as a function of i_{max} for two values of the probe separation d. ($B=500$ gauss, $p=1.1$ mm Hg.)

[16] R. G. Fowler, J. S. Goldstein, and B. E. Clotfelter, Phys. Rev. 82, 879 (1951).

[17] R. G. Fowler, W. R. Atkinson, W. D. Compton, and R. J. Lee, Phys. Rev. 88, 137 (1952).

inserted into the expansion chamber at a separation of 0.8 cm. The results are given in Figs. 1, 3, and 4. They show that with the stems insulated the voltage V_B is proportional to the probe separation. If the probe distance was reduced to less than 0.5 cm erratic results were obtained which we believed to be due to aerodynamic effects of the probes on the flow of the gas.

(c) Effect of Probe Contamination

In order to test whether a contamination of the probe surface affects the probe potential, observations were made before and after the probes were cleaned by sputtering. No difference could be detected.

(d) Configuration of Current Density Between the Probes

From Fig. 4 we find that the R_p, the resistance of the plasma between the probes, at $p=1$ mm Hg and $i_{\max}=6700$ amp, is 24 ohms for $d=1.5$ cm and 13.5 ohms for $d=0.8$ cm. If the probe current in the plasma is uniformly distributed over an area of cross section equal to that of a single probe surface (0.2 cm^2) this corresponds to a plasma resistivity $\rho=3.2$ ohm cm and $\rho=3.4$ ohm cm for $d=1.5$ and 0.8 cm, respectively. The constancy of ρ with d means that the charges from the plasma are drawn to the inner surfaces of the probes only, i.e., the lines of flow of current are parallel to one another and normal to the probe surface. Since ρ has been derived from the maximum values of V_r it represents the lowest value at any time at 6 cm from the anode of the discharge tube.

When the two probes are at a separation of 1.5 cm (Fig. 5, insert) and situated in the holes in the glass tube, the flow of charges is directed essentially towards the surfaces facing one another. When the probes are at 0.8 cm apart (where it was found that R_p has changed in the same ratio as the distance) the collecting area of the probes must again be equal to one single surface though the front and the back surfaces are now exposed to the plasma. This contradiction is resolved by remembering that the rear sides of the probes cannot

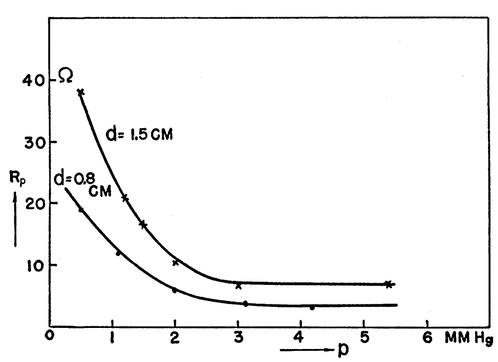

FIG. 4. Plasma resistance R_p as a function of the ambient gas pressure p for two probe separations d. ($B=500$ gauss, $V_c=2$kv, $i_{\max}=6700$ amp.) R_p found from $R_p=r(V_B/V_r-1)$.

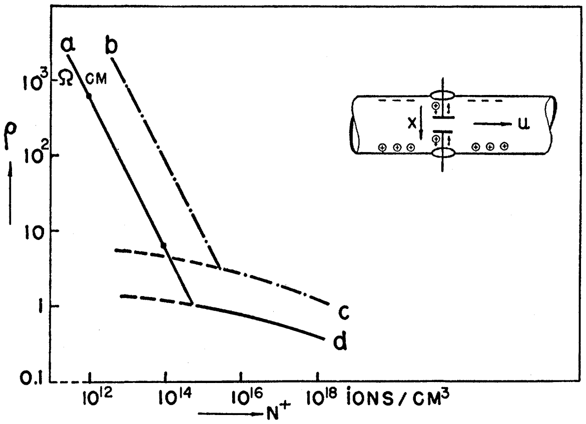

FIG. 5. Calculated ionic resistivity ρ of the plasma as a function of the ion concentration N^+. Curve a: $\mu^+=10^4$ cm^2/sec v, 300°K, $p=1$ mm Hg. Curve b: mobility at 7000°K (assumed proportional to gas density). Curve c: mobility of fully ionized hydrogen gas at 7000°K calculated from Eq. (10). Curve d: as for c, gas at 10^4°K. Insert: Electric field X in the gas and contribution of charges to the probe current.

take part in the charge "collection": the induced electric field essentially separates the plasma charges until it is balanced by space charges. No further flow of electrons or ions from the gas to the rear of a probe can occur.

(e) Effect of Probe Surface Area

The conclusions were also confirmed by measuring the plasma resistance with probes (of copper) and of different surface area. When the diameter of the disk of the probe was 0.12, 0.25, and 0.5 cm, R_p was found to be 300, 95, and 24 ohms at $p=1$ mm Hg, 6700 amp and $d=1.5$ cm. The resistance is thus inversely proportional to the collecting area. With 0.12 cm probes the collecting area is of course slightly larger than the face because the probe surface cannot be regarded any longer as a part of an infinite plane.

6. COMPARISON BETWEEN THEORY AND EXPERIMENT

When the experimental values of ρ are compared with the theoretical values of the plasma resistivity of a fully ionized gas (associated with the motion of electrons and derived[3] under the assumption that the electrons make distant encounters with positive ions), it is found that at $\approx 10^4$°K [Eq. (11)] in hydrogen ρ is of the order 10^{-2} ohm cm, i.e., more than 100 times smaller than the values measured here. This difference is due to the peculiar mode of charge transport from the plasma into the probe circuit. If in an ionized gas containing equal number of positive and negative charges the number of electrons flowing to one probe were larger than the number of ions to the other probe, a positive space charge would be set up in the gas until the electric field which developed between the charges and the probe prevented further separation. Thus

fewer electrons enter the probe until charge neutrality is re-established. The maximum number of charges reaching a probe per unit time cannot exceed that given by the drift velocity of the positive ions. Their low mobility is the cause of the high value of the "ionic resistivity" of the plasma.

Actually the mobility is not that of ions in a feebly ionized stationary gas because in a highly ionized gas ion-ion collisions are predominant and the collision frequency between the constituents of the moving plasma and the ions and electrons moving towards probes is somewhat enhanced.

A simple calculation will explain the foregoing. Assume a gas at a temperature high enough to be strongly ionized. An approximate value of ρ can be derived from Langevin's mobility theory[12]:

$$\rho = 1/eN^+\mu^+, \tag{6}$$

where N^+ is the ion concentration and μ^+ is the ion mobility. Then, if λ^+ is the mean free path and $\bar{v}$ the mean velocity of the ions,

$$\mu^+ \approx \frac{3}{4}\frac{e}{m^+}\frac{\lambda^+}{\bar{v}} = \frac{e}{m^+}\frac{0.75}{N^+q^+(3kT/m^+)^{\frac{1}{2}}}. \tag{7}$$

If, on the average, scattering of ions by ions occurs through an angle $\pi/2$, the maximum ion potential energy $e^2/p_0 = m^+v^2 = 3kT$ where p_0 is the collision parameter (least distance of approach).[3] The collision cross section is

$$q^+ = p_0{}^2\pi = (e^2/3kT)^2\pi. \tag{8}$$

From (6), (7), and (8) the ionic resistivity of the plasma is,

$$\rho \approx \frac{4}{3\sqrt{3}}\frac{(m^+)^{\frac{1}{2}}e^2}{(kT)^{\frac{3}{2}}}. \tag{9}$$

Equation (9) gives the well-known dependence on T and shows that to a first approximation ρ is independent of N^+ and the gas pressure. A more rigorous treatment[3] includes a factor $\ln\Lambda$ which is probably between 1 and 10 and is known for electron-ion interaction only. Its inclusion in (9) increases the approximate value of ρ. The term $\ln\Lambda$ accounts for the fact that because of the shielding of the electric field of the ions by electrons the cross section for distant encounters decreases more rapidly than would be the case if the effect of the electrons would be neglected.

In this way the ionic resistivity of fully ionized hydrogen (in ohm cm) is

$$\rho = \frac{4}{3}\left(\frac{m^+}{3}\right)^{\frac{1}{2}}\frac{e^2}{(kT)^{\frac{3}{2}}}\ln\Lambda,$$

where

$$\Lambda = \frac{3}{2e^3}\frac{(kT)^{\frac{3}{2}}}{(\pi N^+)^{\frac{1}{2}}}. \tag{10}$$

For $T = 10^4$°K, $\ln\Lambda = 6$, $N^+ = N^- = 10^{15}/\text{cm}^3$, one obtains $\rho = 0.7$ ohm cm. On the other hand at larger gas pressures the measured values of ρ (Fig. 6) seems to approach 1 ohm cm. The agreement with theory is satisfactory particularly in view of the fact that T is probably $< 10^4$°K. Equation (10) is only an approximation as the finer details of interaction between an ion and the other charges of the plasma have not been considered by the theory.

Another approach is to calculate first the normal electronic resistivity ρ_e of the fully ionized plasma.[3] With $N^- = 10^{15}/\text{cm}^3$ we find for hydrogen

$$\rho_e = 2.4 \times 10^4/T^{\frac{3}{2}}, \tag{11}$$

yielding, for $T = 10^4$°K, $\rho_e \approx 2.4 \times 10^{-2}$ ohm cm. Since the numerical constant in Eq. (11) is proportional to $(m^+)^{\frac{1}{2}}$, the corresponding ionic resistivity is $45\rho_e$ or $\rho \approx 1$ ohm cm.

The ionic character of the resistivity measured here can be tested by applying the relation $\rho \propto (m^+)^{\frac{1}{2}}$ to observation in various gases. Probe measurements by Lin et al. in argon[7] gave values of ρ about 10^3 times larger than expected from the theory of fully ionized gases. (They thought that this was due either to a cool boundary layer or absence of a temperature equilibrium in the gas.) Since an argon ion has 40 times the mass of H^+, the resistivity ρ in argon at the same T should be more than 6 times our value observed in hydrogen, as indeed is the case. Moreover, exploratory probe measurements in our laboratory, comparing relative values ρ in H with those in He and A, have shown that ρ is the higher the larger the ion mass.

The transition from a partially to a fully ionized gas is given in Fig. 5 which shows the calculated variation of ρ with N^+. From earlier work with shock tubes the plasma temperature is known to be ≈ 7000°K at pressures of order 1 mm Hg[14,18]; the corresponding curves for a fully ionized gas are represented by the two lines marked c and d. For low ion density, the elementary theory of ion mobility gives the two

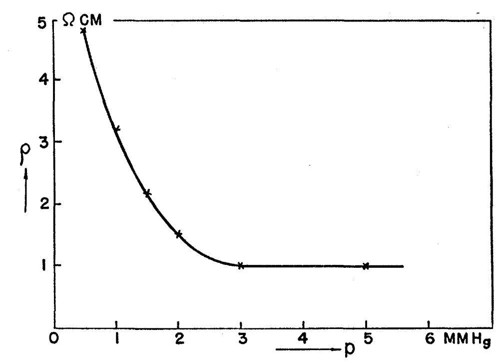

FIG. 6. Observed plasma resistivity ρ as a function of the gas pressure p in hydrogen (assuming uniform current flow between the probes, see 5d). ($B = 500$ gauss, $V_c = 2$kv, $i_{max} = 6700$ amp.)

[18] R. G. Fowler and W. R. Atkinson, Phys. Rev. 113, 1268 (1959).

straight lines marked a and b which set the boundary for gas temperatures between 3×10^2 and 7×10^{3}°K. Comparison with Fig. 6 shows that the lowest measured values of ρ lie near the theoretical limits.

Another way of obtaining a theoretical value of ρ is to determine the collision cross section for the ions and using Langevin's mobility equation. Oster[5] has calculated the ion cross section in fully ionized hydrogen, viz.,

$$q^+ = \left(\frac{\pi^3}{96}\right)^{\frac{1}{2}} \left(\frac{1}{\gamma} - 1\right) \frac{e^4 \ln(d/b_0)}{(kT)^2}, \qquad (12)$$

where the average distance between two ions $d = (4\pi N^+/3)^{-\frac{1}{3}}$; $b_0 = e^2/3kT$ is the average collision parameter, and $\gamma = 0.58$ a transport coefficient. The ion mobility

$$\mu^+ = \frac{3}{4} \frac{e}{m^+} \frac{1}{\bar{v} N^+ q^+}. \qquad (13)$$

From (12) and (13) we find

$$\rho = \left(\frac{\pi^3}{18}\right)^{\frac{1}{2}} \left(\frac{1}{\gamma} - 1\right) \frac{(m^+)^{\frac{1}{2}} e^2 \ln(d/b_0)}{(kT)^{\frac{3}{2}}}. \qquad (14)$$

With $T = 10^4$°K and $N^+ = 10^{15}/\text{cm}^3$, we obtain $\rho = 0.56$ ohm cm, which agrees fairly well with the theoretical results found above.

It has been said before that a correction has to be applied to the variation of ρ with p which is caused by collisions between the ions traveling to the probes and the particles in the moving plasma. If u is the flow velocity of the plasma and v the random velocity of the ions, the time τ between two successive collisions will be reduced by a factor $(1+u^2/v^2)^{-\frac{1}{2}}$, which increases as p rises. Thus μ^+ should increase and ρ decrease with increasing p. It seems, however, that the measured decrease of ρ with increasing pressure (Fig. 6) is mainly due to an increase of the concentration of positive ions in the plasma at the point of observation, though other factors (e.g., radial distributions) may contribute.

The large plasma resistivity measured requires perhaps some comment. When the plasma moves across zero magnetic field, a thin sheath of charge is set up between it and the wall, the latter being about 1 volt negative with respect to the plasma. When the plasma crosses the magnetic field, the induced emf in the plasma drives the charges to the isolated probes. The movement of charges stops when the electric field in the gas becomes zero, i.e., when the field due to the charges on the probes annuls the induced field. When the probes are connected to a resistance, and a current develops in the circuit, the charges of the probes are drained off so that the field in the gas becomes finite because the induced field remains the same, but the field due to the charges on the probes becomes smaller. This effective field in the gas would drive electrons faster to one probe than ions to the other, were it not for a very small positive space charge which essentially retards the drift motion of the electrons and thus keeps the velocity of both charges moving to the probes equal and constant. This explains why the positive ions are here the controlling factor and why the effective plasma resistance appears to be ionic in this experiment.

From the decay of the probe voltage pulse along the shock tube or of the luminosity of the plasma, the process leading to neutralization of charges in the moving plasma can be inferred. The "life" of the moving plasma so estimated is of the order 10^{-4} to 10^{-5} sec at 1 mm Hg. The corresponding neutralization time τ is determined either by electron-ion recombination, or ion-ion recombination (if a sufficient number of negative hydrogen ions are present) or by "diffusion" to the wall of the tube where the charges are neutralized. The first process gives τ of the order 10^{-2} sec (recombination coefficient $a_e = 10^{-12}$ cm^3/sec). For the second process we obtain τ of the order 10^{-5} sec ($a_i = 10^{-9}$ cm^3/sec). However, Fowler and Atkinson[18] found that although a considerable portion of the plasma's total radiation is emitted by electron attachment to hydrogen atoms, in spite of the large emission rate the equilibrium concentration of H^- is much too low to affect the electron recombination rate. The third process invokes ambipolar diffusion loss with a coefficient $D_a \approx D^+ \approx (kT_e/e)\mu^+ \approx 10^4$ cm^2/sec; with a tube radius $r = \frac{3}{4}$ cm we find $\tau = 3\times10^{-5}$ sec. It follows that at these gas pressures charge recombination on the wall of the tube is more likely to account for the decay of the moving plasma than volume losses of charge.

ACKNOWLEDGMENTS

The authors wish to thank Dr. P. F. Little for helpful discussions, Dr. O. Theimer for bringing to our notice Oster's treatment, and R. N. Franklin for comments.