

# Magnetic Susceptibility of Metallic Europium

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The susceptibility of europium is measured from 1.3 to 300°K in fields up to 12 000 oersteds. This metal is found not to be ferromagnetic, but to have at low temperatures a very high paramagnetic susceptibility, about forty times higher than for the free ion or hydrated salts. Variation of susceptibility with field strength is observed below 100°K. The saturation curvature at low temperatures is very large, and practically independent of temperature. The susceptibility at high temperatures is consistent with a divalent model. The magnetic behavior at low temperatures is hard to interpret on the basis of a divalent ion and can be more readily explained with a trivalent one for which the theory is developed. Metallurgical evidence, however, indicates that metallic europium is divalent even at low temperatures. On the other hand, it is generally believed that europium is trivalent in  $\text{EuIr}_2$ , but if one uses the conventional molecular field theory the reported ferromagnetism of  $\text{EuIr}_2$  cannot be ascribed to  $\text{Eu}^{+++}$  with a value of the exchange field consistent with that in  $\text{GdIr}_2$ .

## PART I. EXPERIMENT

IN the periodic table, europium is followed by the ferromagnetic elements<sup>1</sup> gadolinium through erbium, and it is therefore of interest to know if europium is ferromagnetic also. Previous measurements<sup>2,3</sup> on europium have been carried to the temperature of liquid nitrogen, and La Blanchetais and Trombe there observed that with decreasing temperature the susceptibility increased and became field-dependent. Our measurements, carried to 1.3°K, also show a dependence of susceptibility on field strength below 100°K, but hysteresis could not be detected even at the lowest temperatures.

### Material

Metallic europium was obtained from Research Chemicals, and was said by them to contain 0.5% oxygen; spectroscopic analysis showed a trace of tantalum and no other impurities. About 0.5 g of material was cut from a larger piece under kerosene, weighed under kerosene, placed in a small aluminum capsule, and transferred to the cryostat in which it was always in an atmosphere of helium. After measurement the specimen was found to have a bright surface.

### Measurements

Measurements were made with the pendulum magnetometer previously described.<sup>4</sup> Between 75° and 4.2°K

temperatures were measured with a carbon resistance thermometer calibrated at the boiling points of He, H<sub>2</sub>, and N, and at the triple points of H and N. Above 75° a calibrated copper-constantan couple was used. Both resistor and couple were in contact with the aluminum capsule.

Magnetization per mole,  $\sigma_m$ , was measured at field strengths of 4, 8, and 12 kilo-oersteds as dependent on temperature, and results are shown in Fig. 1. Magnetization curves,  $\sigma_m$  vs  $H$ , for temperatures of 1.3, 4.2, and several higher temperatures are shown in Fig. 2; below 100°K the variation of molar susceptibility  $\chi_m$ , with field strength is apparent.

The unusual results in the temperature range of 90 to 100°K correspond qualitatively to those of La Blanchetais and Trombe<sup>2</sup> and bear a close resemblance to the maxima observed in a number of rare earth metals<sup>1</sup> heavier than gadolinium; they are ascribed to antiferromagnetic ordering, and neutron diffraction experiments on erbium by Koehler and Wollan<sup>5</sup> appear to support this point of view.

Measurements of  $\sigma_m$  vs  $H$  at low temperatures showed that these curves are completely reversible, within the accuracy of our measurements; we found no evidence of hysteresis. Figure 3 shows the  $\sigma_m$  vs  $H$  curve taken with increasing and decreasing fields, and for comparison similar data for holmium which shows hysteresis in accord with its known ferromagnetism.<sup>1</sup>

A plot of  $1/\chi$  vs temperature from 100 to 300°K gives  $\mu_{\text{eff}} = 8.3$ ; this is to be compared to the values 7.1 and 8.2 obtained, respectively, by La Blanchetais and Trombe and by Klemm and Bommer.

## PART II. THEORY

The magnetic behavior of europium is curious and unusual. It is not ferromagnetic, but it has a suscepti-

<sup>5</sup> W. Koehler and E. O. Wollan, Phys. Rev. 97, 1177 (1955).

\* Part of the work was performed while a summer guest at the Bell Telephone Laboratories, Murray Hill, New Jersey.

<sup>1</sup> Spedding, Legvold, Daane, and Jennings, *Progress in Low-Temperature Physics*, edited by C. J. Gorter, (North-Holland Publishing Company, Amsterdam, 1957), p. 368.

<sup>2</sup> C. Henry La Blanchetais and F. Trombe, Compt. rend. 243, 707 (1956).

<sup>3</sup> W. Klemm and H. Bommer, Z. anorg. allgem. Chem. 231, 138 (1937).

<sup>4</sup> R. M. Bozorth, H. J. Williams, and D. E. Walsh, Phys. Rev. 103, 572 (1956).

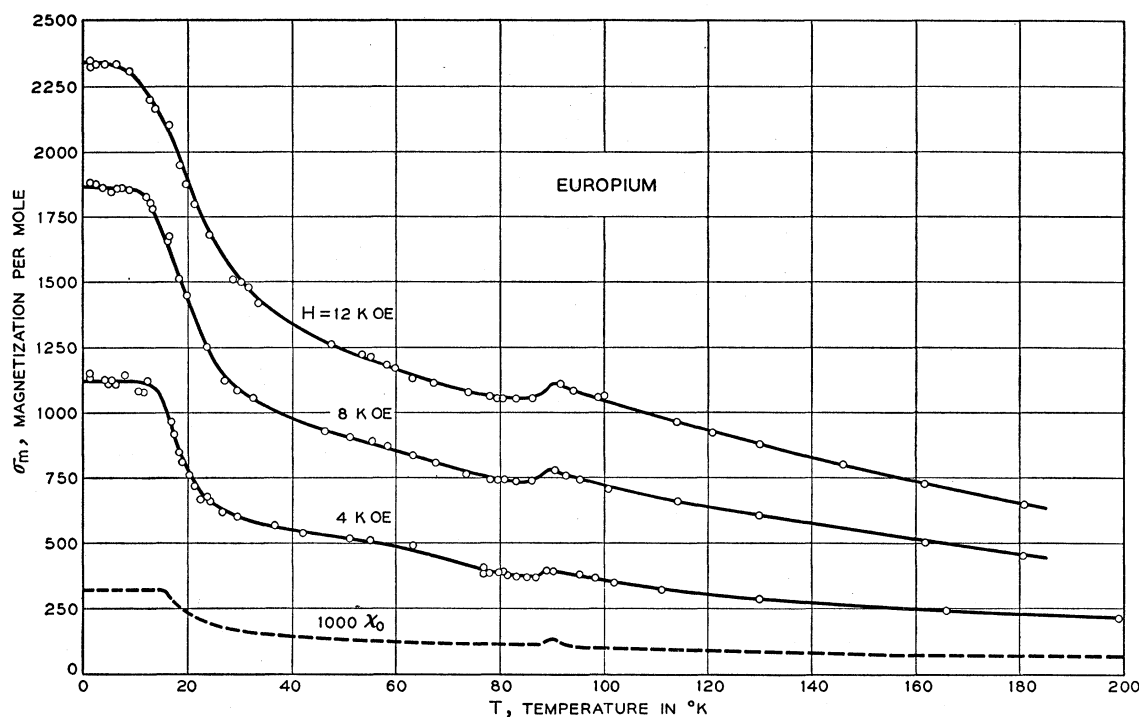


FIG. 1. Magnetization vs temperature for several field strengths.

bility at low temperature about forty times larger than for the ideal gaseous state, or hydrated europic salts. This high value of the susceptibility is practically independent of temperature in the helium region. At the same time, in this region, there is a huge saturation curvature, considerably more than for the celebrated case of  $\text{Gd}_2(\text{SO}_4)_3 \cdot 8\text{H}_2\text{O}$  at 4°K, and about the same as the latter at 2°K. The most striking fact is that this curvature is independent of temperature near  $T=0$ , being the same within the experimental error at 1.3 and 4.2°K (see Fig. 2). On the other hand, in hydrated gadolinium sulphate, the curvature increases enor-

mously as the temperature is lowered, corresponding to the fact that the susceptibility is approximately a function only of  $H/T$ .

No one atom model can possibly fit the experimental data. Since the susceptibility is independent of temperature in the vicinity of  $T=0$ , it is natural to assume, at least to get an idea of orders of magnitude, that we have a nondiagonal element  $g\beta H$  of Zeeman energy connecting two states separated by an interval  $\Gamma$  large compared to  $kT$ . (A diagonal matrix element would give a  $1/T$  term in the susceptibility.) The magnetic moment per atom of the lowest doublet component is then  $Hg^2\beta^2/(H^2g^2\beta^2 + \frac{1}{4}\Gamma^2)^{1/2}$ . To obtain a susceptibility of 0.3 per gram atom in weak fields, and 40% reduction in the moment because of partial saturation at 12 000 gauss, requires  $g=0.5$ , and  $\Gamma/k=0.6^\circ$ . The interval  $\Gamma$  is, however, then much smaller than  $kT$ . It is consequently necessary to correct for the influence of the upper state, and the moment becomes drastically dependent on temperature. Formulas qualitatively different from those with a one-atom model are thus indicated.

At high temperatures, the susceptibility of Eu is represented empirically by a formula of the form

$$\chi = N\mu_{\text{eff}}^2\beta^2/3kT, \quad (1)$$

with  $\mu_{\text{eff}}=8.3$ . This is a susceptibility about ten times as large as for the free  $\text{Eu}^{++}$  ion. There are of course, corrections for exchange interactions in the solid state, but it is highly unlikely that these corrections would change the moment by a factor 10 at room temperatures or higher. Thus, a trivalent model fails to represent the

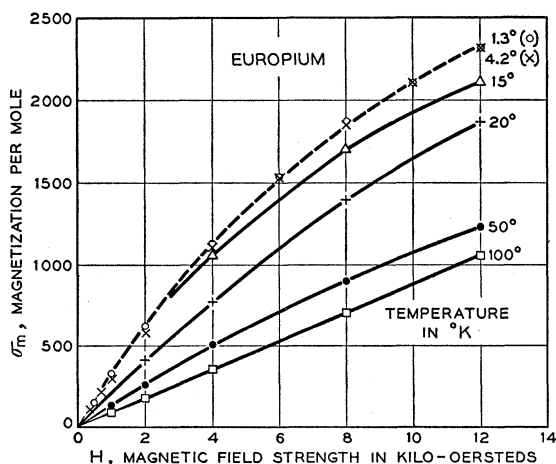


FIG. 2. Magnetization curves for several temperatures.

susceptibility of metallic Eu. This is in contrast to the rare earths beginning with Gd, as de Gennes<sup>6</sup> has shown that their magnetic behavior can be explained fairly well on the assumption that the magnetic ions are trivalent, with the three other electrons in nonmagnetic conduction bands.

Instead the magnetic behavior of Eu at high temperatures is more nearly approximated with a divalent model. The magnetic ingredient is then a  $\text{Eu}^{++}$  ion, which like  $\text{Gd}^{+++}$ , is in an  $S$  state. If exchange interaction is neglected, the theoretical effective Bohr magneton number is then  $\mu_{\text{eff}} = (7 \times 9)^{1/2} = 7.9$ . Furthermore, it is generally believed by metallurgists<sup>1</sup> that metallic europium is divalent, for its type of crystal structure, low density, high compressibility, etc., are out of line with the corresponding constants for most of the rare earths.

The fact that Gd is ferromagnetic and Eu is not, does not constitute an insurmountable difficulty, for the exchange coupling between the magnetic electrons is presumably caused by the indirect Zener-Ruderman-Kittel-Yosida mechanism via conduction electrons, and the constants of the exchange field can conceivably be quite different with two than with three conduction electrons per atom.

There are however still some difficulties in finding an acceptable magnetic theory even in the high temperature region, even with a divalent model. In the first place, the observed effective magneton number is about 5% higher than for the free ion. Possibly the residual contribution is to be attributed to incomplete quenching of the conduction electrons. A more serious difficulty is that it is hard to see how Curie's law can be so nearly obeyed if there is an antiferromagnetic Curie point around  $90^\circ$ , as indicated by the magnetic curves of moment against temperature (see Fig. 1). Antiferromagnetic coupling tends to reduce the moment, giving a Curie denominator of the form  $T + \Delta$  rather than  $T$ .

It is certainly difficult to find a divalent model which will explain the magnetic behavior at low temperatures, in particular the onset of a greatly increased susceptibility below about  $50^\circ\text{K}$ , and a high saturation curvature at low temperatures. A true antiferromagnetic is usually characterized by a susceptibility which has a positive temperature derivative below the Néel point, and is difficult to saturate.

#### THEORY FOR TRIVALENT MODEL

The behavior of europium at low temperatures can, strange to say, be explained fairly well on the basis of a trivalent model. We represent the exchange coupling by the usual artifice of a molecular field, and neglect the crystalline field since metallic europium is body-centered cubic, and the lowest states  $J=0$ ,  $J=1$  are consequently unsplit. The details of such a model are outlined at the beginning of the preceding paper, henceforth

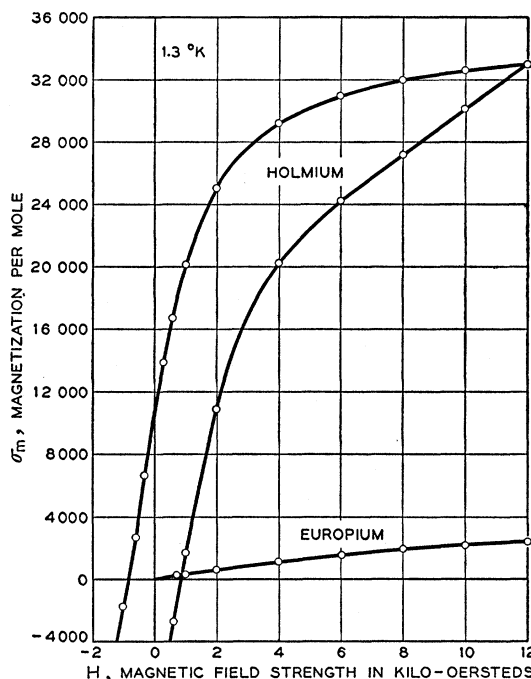


FIG. 3. Lack of hysteresis in Eu, compared with its presence in ferromagnetic Ho.

called l.c., by Wolf and Van Vleck,<sup>7</sup> on europium iron garnet. If  $J_{ij}$  denote the exchange integral connecting atoms  $i$  and  $j$ , the appropriate exchange field can be shown to be

$$H_{\text{ex}} = (\sum_j J_{ij} / N\beta^2) M_{\text{Eu}}, \quad (2)$$

at low temperatures, where practically all of the magnetic moment comes from matrix elements connecting  $J=0$  with  $J=1$ . (At higher  $T$  the ratio of  $H_{\text{ex}}$  to  $M_{\text{Eu}}$  is a slowly varying function of temperatures because of the fact that the ratio of spin to total magnetic moment depends on the relative population of the various states.) In weak fields we can provisionally substitute (2) in Eq. (1) of l.c., and solve the resulting equation for  $M_{\text{Eu}}$  to determine the ratio  $M_{\text{Eu}}/H_0$ . This procedure however, neglects saturation effects, and is obviously incapable of explaining the curvature exhibited in Fig. 2. To allow for saturation, we abandon the use of perturbation theory made under the assumption that the exchange field is small, and instead solve rigorously the secular problem for the pair of states  $J=0$ ,  $1$  joined by an exchange plus applied field, which we suppose directed along the  $z$  axis. The exchange field acts only on the spins. If we neglect all matrix elements except those joining  $J=0$  and  $J=1$ , as is appropriate at low temperature, we find

$$M_{\text{Eu}} = 2\beta N \lambda \left( \frac{2\beta [H_0 + (\frac{1}{8}\Delta M_{\text{Eu}}/N\beta^2)]}{[\frac{1}{4}E_1^2 + 4\beta^2 (H_0 + \frac{1}{8}\Delta M_{\text{Eu}}/N\beta^2)^2]^{\frac{1}{2}}} \right), \quad (3)$$

<sup>7</sup> W. Wolf and J. H. Van Vleck, preceding paper [Phys. Rev. 118, 1490 (1960)].

<sup>6</sup> P. G. de Gennes, Compt. rend. 247, 1836 (1958).

where

$$E_1 = E_{J=1} - E_{J=0}; \quad \Delta = 16 \sum_j J_{ij}. \quad (4)$$

$$\lambda = (n_0 - n_{10})/N. \quad (5)$$

Here  $n_0, n_{10}$  are, respectively, the number of atoms in the states  $J=0$  and  $J=1$ ,  $M_J=0$ . Equation (3) must be solved for  $M_{Eu}$  or, in other words, a consistent value of the magnetization must be used on the left-hand side of the equation and in the exchange field. This minor algebraic difficulty did not occur in the paper on the europium garnet, as the exchange field was considered as arising entirely from the iron rather than Eu, and so could be treated as impressed rather than self-consistent. The relation (3) can conveniently be expressed in the parametric form:

$$2\beta H_0 = \alpha \left( \frac{2\alpha^2 E_1}{1-\alpha^4} + \frac{E_1 - \lambda \Delta}{1+\alpha^2} \right), \quad (6)$$

$$M_{Eu} = 4N\lambda\alpha\beta/(1+\alpha^2). \quad (7)$$

By means of (6) and (7) the magnetic field  $H$  can be computed as a function of  $M_{Eu}$  and the saturation curves drawn.

At very low temperatures the concentration in upper states is negligible and so  $\lambda = 1$  [see Eq. (5)]. The best fit to experiment is obtained if one takes

$$E_1/k = 285^\circ, \quad (E_1 - \Delta)/k = 9.1^\circ. \quad (8)$$

The resulting saturation curve for  $T=0$  is shown by broken lines in Fig. 2 and is in good accord with the experimental results for  $T = 1.3^\circ$  and  $4.2^\circ$ . The value of  $E_1/k$  obtained in this fashion is forty percent lower than the spectroscopic value of  $480^\circ$  for the free ion. The discrepancy is not too alarming in view of the approximations in the theory and especially the sensitivity of the experimental results to the sample. (Earlier, perhaps less accurate measurements on another specimen gave a value of  $E_1$  only 20 percent lower than the spectroscopic one.) It is of interest to compare the size of the exchange integral obtained from (8) with that in metallic gadolinium. From (4) and (8) one finds for Eu,  $\sum_j J_{ij}/k = 18^\circ$ . From the Curie point of gadolinium one deduces a value of  $28^\circ$  for this sum if one uses the standard expression

$$T_c = \frac{2}{3} S(S+1) \sum_j J_{ij}/k \quad (9)$$

of molecular field theory for the Curie point. (The more refined formulas of Brown and Luttinger, etc., are not applicable because the effect of non-nearest neighbors may be substantial.) The agreement is as close as one has any right to expect, for gadolinium and europium have entirely different dimensions and lattice structures, viz. hexagonal close-packed and body-centered cubic. In gadolinium there are six neighbors at 3.57 Å and six more at 3.64, whereas in europium there are eight nearest neighbors at 3.99 Å. Hence one would expect gadolinium to have a larger value of  $\sum_j J_{ij}$ .

We now investigate the consequences of the trivalent theory at temperatures other than  $T=0$ . One can explain the gradual fading out of the saturation curvature as the temperature is raised or, in other words, one can obtain substantially the solid curves in Fig. 2 if  $\lambda$  is treated as an empirical function of the temperature. The problem is to see if theory will yield the proper relation between  $\lambda$  and  $T$ . If one uses the usual Boltzmann expression

$$\lambda = \frac{1 - e^{-E_1/kT}}{1 + 3e^{-E_1/kT}}, \quad (10)$$

for the quantity  $\lambda$  defined in (5), the decrease of  $\lambda$  from its maximum value 1 does not set in rapidly enough. However, an explanation of this fact is found in the blurring of the upper states  $J=1$  by the matrix elements of the exchange potential of the form  $(\Delta J)_i = -(\Delta J)_j = \pm 1$ . One can make approximations analogous to those of the Holstein-Primakoff theory except that now one has excitation waves whereby an atom is excited to the state  $J=1$  rather than turned over. The excitation wave is handed on from atom to atom analogously to the spin reversal in the  $H-P$  theory.

Since there are such grave doubts as to whether metallic europium is really trivalent, we will omit the mathematical details of the theory. The technique shows considerable resemblance to that employed by Bogoliubov<sup>8</sup> in infiltrating the crystalline field into spin wave theory or by Anderson<sup>9</sup> in treating superconductivity. We will simply give the final formulas, which, quite irrespective of whether they are applicable to metallic europium, may be useful for materials (EuN?) where trivalent ions are the only magnetic ingredient and are closely enough spaced so that the exchange field is important. With the excitation wave theory, the formula for the quantity  $\lambda$  to be used in (2) or (6-7) is

$$\lambda_0 - \lambda = \frac{z^{\frac{1}{2}} \Omega^2}{2\pi^2 \Delta^{\frac{1}{2}} E_1^{\frac{1}{2}}} \sum_{n=1}^{\infty} \left[ (1 + \Omega^2 E_1^{-2}) \left( \frac{kT}{\Omega n} \right) K_1 \left( \frac{n\Omega}{kT} \right) + 3\Omega^2 E_1^{-2} \left( \frac{kT}{\Omega n} \right)^2 K_2 \left( \frac{n\Omega}{kT} \right) \right], \quad (11)$$

where  $K(x)$  is a Bessel function of imaginary argument defined and tabulated by Watson,<sup>10</sup>  $\lambda_0$  is the value of  $\lambda$  at  $T=0$ , and

$$\Omega = [E_1(E_1 - \Delta)]^{\frac{1}{2}}, \quad z = \frac{6 \sum_j J_{ij}}{\sum_j (r_{ij}/l)^2 J_{ij}}, \quad (12)$$

with  $l$  the lattice constant. In deriving (11) the quantity  $\lambda$  is interpreted as the expectation value of an operator,

<sup>8</sup> N. N. Bogoliubov and S. V. Tiablikov, *Izvest. Akad. Nauk (S.S.S.R.) Ser. Fiz.* 21, No. 6 (1957).

<sup>9</sup> P. W. Anderson, *Phys. Rev.* 112, 1905 (1958).

<sup>10</sup> G. N. Watson, *A Treatise on the Theory of Bessel Functions* (Cambridge University Press, New York, 1952), 2nd ed., p. 172, Eq. §(4) and Table, p. 737.

which is unity in the state  $J=0$ , is 0 for  $J=1$ ,  $M_J=\pm 1$ , and is  $-1$  for  $J=1$ ,  $M_J=0$ . A difficulty is that at  $T=0$ , the expectation value is not unity because exchange interaction blends in some states of higher  $J$ , and in applying (11) to (3) or (7), we somewhat arbitrarily replace  $\lambda$  by  $1+(\lambda-\lambda_0)$ .

The excitation wave theory gives a specific heat

$$C_v = \frac{3z^{\frac{1}{2}}N\Omega^4}{4\pi^2 E_1^{\frac{1}{2}}\Delta^{\frac{1}{2}}} \sum_{n=1}^{\infty} \left[ \left( \frac{1}{T} + \frac{12k^2T}{n^2\Omega^2} \right) K_2\left(\frac{n\Omega}{kT}\right) + \frac{3k}{\Omega n} K_1\left(\frac{n\Omega}{kT}\right) \right] \quad (13)$$

near  $T=0$  much larger than that obtained from the usual Schottky formula based on a Boltzmann distribution among the multiplet components.

The excitation wave theory has crude, qualitative success in explaining the trend of  $M_{Eu}$  with temperature at low temperatures: the susceptibility drops much more rapidly with increasing temperature than one would expect on the basis of the Boltzmann formula (10). This

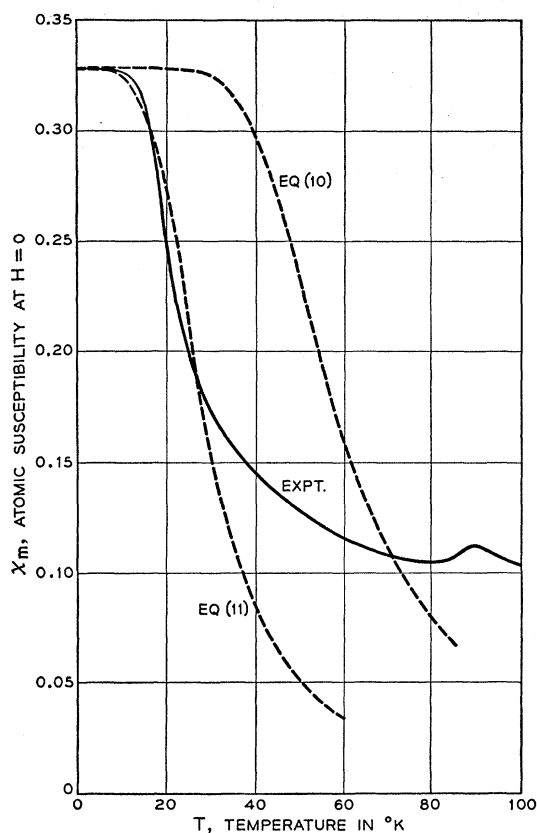


FIG. 4. The susceptibility in weak fields as a function of temperature. The solid curve is experimental. The theoretical curve based on Eq. (10) assumes a Boltzmann distribution among the multiplet states of a Eu ion. The curve labeled Eq. (11) is also for a trivalent ion, but uses the excitation wave approximation which is appropriate for low temperatures.

fact is shown in Fig. 4. The value of  $z$  used in obtaining the curve based on Eq. (11) is 30. The quantity  $z$  defined by (12) would be equal to the number of nearest neighbors (8 for a body-centered lattice) if they were the only ones coupled to the central atom, but with the Z-R-K-Y mechanism this is not the case, and atoms at different distances may make substantial contributions to the sums in (12). The dropoff of the susceptibility predicted by the excitation wave theory is much too rapid except in the low-temperature region, but this is not a cause for undue concern, as the excitation wave theory is only a low-temperature approximation. At higher temperatures one should shift over to a curve based on the Boltzmann distribution. The fact that even the latter yields too low a susceptibility at high temperature has been mentioned earlier, and is evidence that at high temperatures europium is divalent.

As far as magnetic behavior is concerned, it would be convenient to have europium behave divalently at high temperatures, and trivalently at low temperatures. However, from a metallurgical standpoint, this is very unlikely. Barrett<sup>11</sup> has measured the lattice parameters of metallic europium down to 5°K and finds only a minor change (<1%) in the lattice parameters as compared with ambient temperatures. In view of the difficulty of finding a simple magnetic theory with constant valence that works both at high and low temperature, it is perhaps not beyond the range of possibility that there is some kind of valence adjustment that does not affect the lattice parameters or elastic constants but this would be a freak situation. The other possibility is that the magnetic behavior of divalent, or nearly divalent europium is too recondite to be amenable to any simple theory. Conceivably, for instance, there is a mixture of states of different valency with the percentages depending little on temperature, but we have been unable to find a model of this character that will explain the observed magnetic behavior.

#### ENIGMA OF THE MAGNETIC BEHAVIOR OF EU IN PLATINUM COMPOUNDS

We have seen that the magnetic behavior of pure metallic europium is difficult to interpret. That of europium in compounds with platinum metals is even stranger. It is generally accepted by metallurgists that in these metals the europium is trivalent. Bozorth, Matthias, Suhl, Corenzwit, and Davis<sup>12</sup> find that  $\text{EuIr}_2$  is ferromagnetic, with a Curie point 70°K, only about 20% lower than for  $\text{GdIr}_2$ . Such a behavior, however, cannot be obtained from trivalent ions with a value of the exchange integral at all in line with that for other rare earths in an iridium compound. To see why this is the case, we can at this point review why a trivalent model could explain the high saturation curvature of metallic europium at  $T=0$ . The inclusion of exchange

<sup>11</sup> C. S. Barrett, J. Chem. Phys. **25**, 1123 (1956).

<sup>12</sup> R. M. Bozorth, B. T. Matthias, H. Suhl, E. Corenzwit, and D. D. Davis, Phys. Rev. **115**, 1595 (1959).

effects via the molecular field has the effect of replacing the reciprocal  $1/E_1$  of the natural multiplet width by  $1/(E_1 - \Delta)$  in the expression for the susceptibility in weak fields [see Eq. (3) with the approximation  $\frac{1}{2}E_1$  for the radical]. If  $\Delta$  is nearly equal to  $E_1$  the susceptibility at  $T=0$  (where  $\lambda=1$ ) is raised enormously compared to the free ion. Under these conditions, the saturation corrections are far more important than for the free ion. However, as the temperature is raised, and  $\lambda$  becomes less than unity because some atoms are promoted to the state  $J=1$ , the near-balance between the multiplet width and the exchange term is destroyed, and there is a pronounced drop in the susceptibility and saturation curvature, in accord with experiment. Incidentally, because this near-balance is such a delicate effect, the susceptibility of metallic Eu with a trivalent model should be tremendously sensitive to pressure or impurities. If  $\Delta$  exceeds  $E_1$ , ferromagnetism can result, and this is how one would explain the ferromagnetism of  $\text{EuIr}_2$  on the basis of a trivalent model. The Curie point would be zero if  $E_1 = \Delta$ , and somewhat higher if  $\Delta > E_1$ . From Eq. (4) we thus see that the minimum value of  $\sum_j J_{ij}/k$  which can result in ferromagnetism is  $30^\circ$  if one uses the spectroscopic value  $E_1/k = 480^\circ$ . However, if one uses this value of  $\sum_j J_{ij}/k$  in (9) one finds that the Curie temperature of  $\text{GdIr}_2$  should be about  $300^\circ\text{K}$ , over three times the experimental value. One is thus forced to abandon either the trivalent model or the conventional molecular field theory, in which it is assumed that the exchange integral is relatively in-

sensitive to the ion, and which accounts fairly well for the magnetic behavior of the platinum compounds of the other rare earths.

A further difficulty is that the saturation magnetization of  $\text{EuIr}_2$  is as low as found by Bozorth and collaborators. The saturation moment and Curie temperature vary roughly as  $n_B$  and  $n_B^2$ . So it is hard to understand why the saturation magnetization is experimentally only about a tenth as large for  $\text{EuIr}_2$  as for the following compounds  $\text{GdIr}_2$  and  $\text{TbIr}_2$  although the Curie temperature is of roughly the same size. The most natural explanation of this situation is that the ferromagnetism comes from small inhomogeneous regions which contribute most of the moment, but this proposal seems rather unreasonable physically. Possibly there is a mixture of ions of different valencies. In any case it appears that although the conventional molecular field theory works quite well for the right half of the rare earth sequence, it cannot be consistently employed for  $\text{EuIr}_2$  and  $\text{GdIr}_2$  if one makes the usual assumption, explained at the beginning of l.c.,<sup>7</sup> that the exchange field acts only on the spin. This suggests that the actual behavior of the rare earth metals is really much more complicated than envisaged in the usual theory.

#### ACKNOWLEDGMENTS

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