π-Nucleon Phase Shifts in the Energy Range 350 to 600 Mev*

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This paper contains the results of attempts to obtain a set of phase shifts valid in the energy region just above the 3-3 resonance.

We have found it possible to extend the family of phase shifts found by the workers at Dubna at an energy of about 300 Mev. The features of this set of phase shifts is that α_1, α_{11}, are positive and α_{13} is negative and small. We find in addition that δ_{13} is positive and δ_{15} negative. At 600 Mev the results are probably consistent with a resonance in the $d_{\frac{3}{2}}$ state, but do not conclusively indicate such a resonance. This resonant state, however, decays a sizable fraction of the time into a final state with two π mesons. There are indications that at energies of 400 to 500 Mev that most of the single pion production comes from s and $p_{\frac{1}{2}}$ states.

A LARGE peak in the π^--p cross section at about 850 Mev was discovered by counter groups working at the Brookhaven cosmotron.[1]

In 1957 and 1958 workers at Cornell[2] and California Institute of Technology[3] showed that the cross section for the processes

$$\gamma+p\rightarrow\pi^0+p,$$
$$\rightarrow\pi^++n,$$

showed lumps in the energy region corresponding to a π-p scattering energy of about 600 Mev. Wilson[4] demonstrated that the total π^--p cross-section measurements were in fact consistent with there being two peaks in the π^--p cross section in this energy region; one being at 600 Mev and the other at about 900 Mev. Counter measurements confirmed the existence of the two peaks.[5,6]

The angular distribution of the photo-π^0's is dominated by $J=\frac{3}{2}$ amplitudes all of the way from threshold to 900 Mev. With this result it was clear that the 600-Mev peak in the π^--p cross section must be largely attributed to a $p_{\frac{3}{2}}$ or a $d_{\frac{3}{2}}$ state. Peierls[7] pointed out that the photoproduction data was indeed consistent with an enhanced $d_{\frac{3}{2}}$ state. Following a suggestion by Sakurai,[8] Stein[9] has measured the polarization of protons from photoproduction of π^0's. His results indicate a large degree of polarization at 90° in the center-of-mass system. These data thus strongly indicate that the second peak in the π^--p cross section has the opposite parity of the 3-3 resonance and is very likely due to a $d_{\frac{3}{2}}$, $I=\frac{1}{2}$ state. The purpose of this paper is to describe an attempt at a phase shift analysis of the π^--p scattering in this energy region with the existing data. The most extensive data are those of Crittenden *et al.*[10] which were obtained with a propane bubble chamber. The determination of the inelastic and charge exchange cross sections are probably not very reliable. Recently counter data on elastic scattering and inelastic scattering have been added at lower energies.[11] Also there seems to be no data on π^+-p interactions between 500 Mev and 1 Bev. The crucial features of the existing data are (1) the rapid decrease in the charge exchange cross section between 300 and 600 Mev, and (2) the large amount of inelastic scattering at 600 Mev. This indicates a qualitative difference between the resonance in the $J=\frac{3}{2}$, $I=\frac{3}{2}$ state and the resonance (if it is one) at 600 Mev. The 600-Mev resonance is very likely strongly damped by inelastic processes. The other peculiar feature of the data is the rather strange shape of the differential elastic cross section at 370, 430, 460 Mev as shown in Fig. 1. The $d\sigma/d\Omega$ curves are humped in the forward hemisphere and are concave downward.[10,11] This shape is characteristic of an interference between p- and d-wave spin flip terms.

The differences of the two p-wave amplitudes, as shown in Fig. 2, give the p-wave spin flip amplitudes. In order to have the experimentally observed angular distribution, the d waves must be split in a similar fashion, (i.e., δ_{13} positive and δ_{15} negative). Reversing the signs of the p and d phase shifts would not give results consistent with the relatively small charge exchange cross section. This is the prime argument for

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the track of phase shifts investigated. The charge exchange cross section thus resolves a Fermi-Yang type ambiguity for the $I=\frac{1}{2}$ phase shifts.

In the energy region from 350 to 600 Mev we use s, p, and d waves to fit the angular distributions. The data fit are ideally $\pi^{+}+p$, $\pi^{-}+p$, $\pi^{0}+n$ angular distributions and the π^{+} and π^{-} inelastic cross sections. This means that we are fitting 17 data and have at our disposal 20 parameters with which to do this. These parameters are the phase shift and the amplitude of the outgoing wave for each angular momentum and isotopic spin state. This is to be compared to lower energy π-p scattering where one fits 9 data with 6 parameters. The situation at the higher energy is not hopeless however since one expects considerable continuity from lower energy on physical grounds, (causality for example). If the absorptive processes have a cross section not too different from $\pi\lambda^2$ then one can absorb in an important fashion at most two partial waves. The $I=\frac{3}{2}$ absorption in this region is quite small and the $I=\frac{1}{2}$ absorption is not too much larger than $\pi\lambda^2$. Thus we are fitting 17 data with perhaps 12 parameters. Thus up to 500 Mev one can probably do a phase shift analysis. The $I=\frac{3}{2}$ phase shifts except for α_{33} are not changing very rapidly and the peculiar shape of the π^{-} differential elastic distribution can only be attributed to the spin flip interference if we are limited to s, p, and d waves. The lack of much of a peak at 0° in the π^{-} differential elastic distribution makes it impossible to have much absorption in the $d_{\frac{3}{2}}$ state which fact makes it seem most likely that the absorption is in the s and $p_{\frac{1}{2}}$ waves. As soon as a large peak develops in the forward hemisphere, as usually occurs when large absorption takes place, then the results of a phase shift analysis become rather ambiguous. Table I gives the results of our phase shift analysis. At 600 Mev one can note that there is considerable ambiguity in our results. The results are particularly sensitive to the amount of elastic charge exchange scattering. To obtain this information one must distinguish between the processes

$$\pi^{-}+p \rightarrow \pi^{0}+n,$$

and

$$\pi^{-}+p \rightarrow 2\pi^{0}+n.$$

To do this is experimentally difficult.

The errors quoted in the table were obtained by varying the input data within the experimental errors. Unfortunately the $I=\frac{3}{2}$, d wave phase shifts are not very well determined. This is because of the particular shape the π^{+}-p differential cross section.

In the course of this investigation several programs were written to search for phase shifts. The final pro-

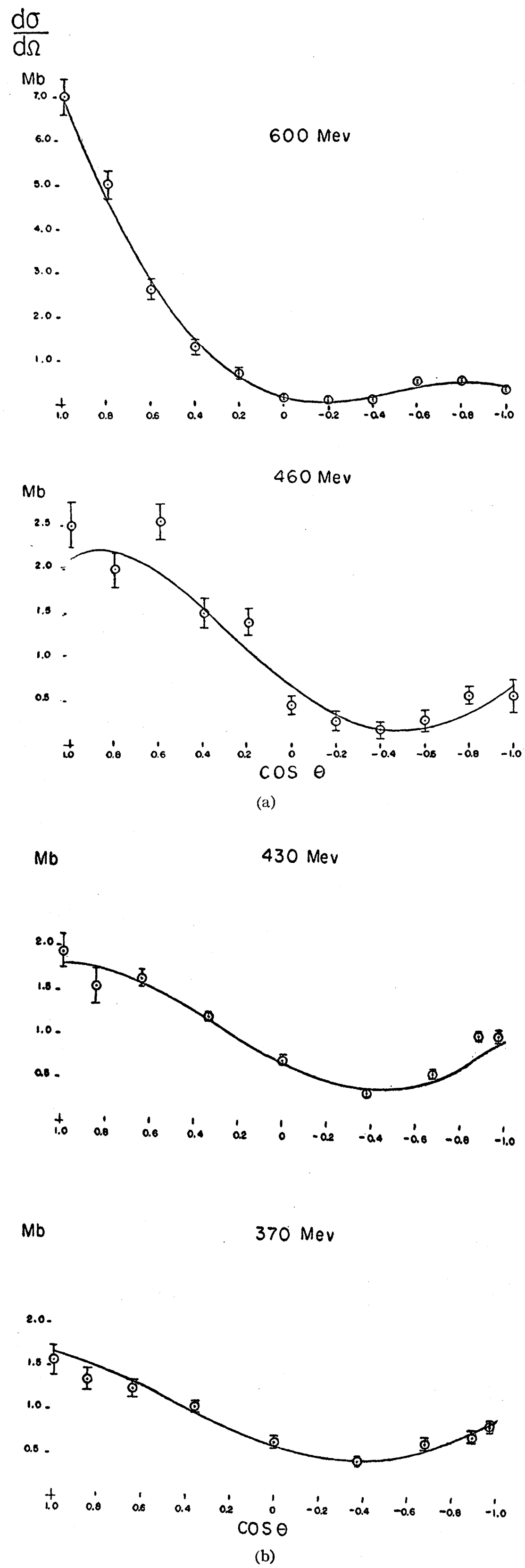

FIG. 1. Experimental data and curves determined in the phase shift analysis. The two lower energy curves were taken from the data of Goodwin *et al.* (see reference 11). Their low-angle point is excluded from consideration because of the possibility of Coulomb interference.

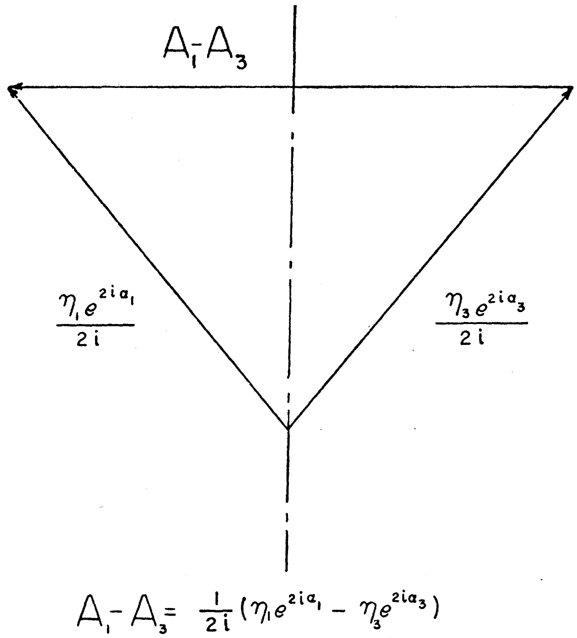

FIG. 2. Vector diagram indicating the preferred orientation of the p-wave amplitudes for the correct interference in the spin flip scattering.

gram fits π^-, π^+, and π^0 differential elastic cross sections and the inelastic cross sections. It is possible to vary any or all of the phase shifts and amplitudes. The computer minimizes the difference between the observed and calculated points by successively varying the various phase shifts and amplitudes.

This particular family of phase shifts were discovered by Zinov and Korenchenko[12] and we have extended it to higher energy. We have made no particular effort to search for other families because this family fits smoothly onto the lower energy Fermi type phase shifts. However, whenever we started the computer a considerable distance from this set of phase shifts at energies below 600 Mev it converged to the set of phase shifts given.

At 370 Mev we used the π^- data of Goodwin[11] *et al.* and the π^+ data of Grigor'ev and Mitin.[13] At 430 we used the π^- data of Goodwin *et al.* and the same π^+ phase shifts as at 370 except for α_{33} which was free to vary. It was found necessary to clamp δ_{33} and δ_{35} at

[12] V. G. Zinov and S. M. Korenchenko, J. Exptl. Theoret. Phys. U.S.S.R. **33**, 1307 (1957) [translation: Soviet Phys. JETP **6**, (33) 1006 (1958)], also B. Pontecorvo's report in the Ninth Annual International Conference on High-Energy Physics, Kiev, 1959 (unpublished).

[13] N. A. Mitin and E. L. Grigor'ev, Soviet Physics—JETP **5**, 378 (1957).

TABLE I. Phase shifts and amplitudes.

	370 Mev	430 Mev	460 Mev	600 Mev
α_1	$37°\pm2°$	$44°\pm2°$	$41.6°\pm5°$	$59.7°\pm7°$
η_1	$0.96{}^{+0.04}_{-0.10}$	0.84 ± 0.10	0.34 ± 0.05	0.62 ± 0.2
α_3	$-32.4°\pm2°$	$-32.4°$[a]	$-28.3°\pm1.0°$	$-32.5°{}^{+3°}_{-0°}$
η_3	1.0[a]	1.0[a]	0.86 ± 0.04	$0.73{}^{+0\ 2}_{-0}$
α_{11}	$9.2°\pm3°$	$12.5°\pm2°$	$28.7°\pm2°$	$10.9°\pm15°$
η_{11}	0.70 ± 0.06	0.62 ± 0.10	$1.00{}^{+0}_{-0.05}$	$0.16{}^{+0.2}_{-0.1}$
α_{31}	$-5.8°\pm3°$	$-5.8°$[a]	$-20°\pm2°$	$-19.5\pm6°$
η_{31}	1.0[a]	1.0[a]	0.98 ± 0.02	$0.93{}^{+0.07}_{-0.30}$
α_{13}	$-2.8°\pm0.3°$	$-6.2\pm2°$	$-0.7°\pm0.5°$	$-12.5°{}^{+4°}_{-6°}$
η_{13}	1.0 ± 0	$1.0{}^{+0}_{-0.01}$	0.99 ± 0.01	0.32 ± 0.10
α_{33}	$147.7°\pm3°$	$150.6°\pm2°$	$160°\pm2°$	$157.5°{}^{+3}_{-7}$
η_{33}	1.0[a]	1.0[a]	$1.00{}^{+0}_{-0.01}$	$1.0{}^{+0}_{-0.3}$
δ_{13}	$4.5°{}^{+2°}_{-0°}$	$7.1°{}^{+2°}_{-1°}$	$18.5°\pm2°$	$18.3{}^{+35°}_{-0°}$
η_{13}'	1.0 ± 0	1.0 ± 0	1.00 ± 0	0.52 ± 0.2
δ_{33}	$2.5°$[a]	$2.5°$[a]	2.8 ± 0.7	$3.5{}^{+0}_{-3°}$
η_{33}'	1.0[a]	1.0[a]	0.99 ± 0.01	1.0 ± 0
δ_{15}	$-1.6\pm1°$	$-1.6\pm1°$	$-0.5°\pm0.1°$	$-5.6\pm2°$
η_{15}	1.00 ± 0	$1.00{}^{+0}_{-0.01}$	1.00 ± 0	1.00 ± 0
δ_{35}	$-3.0°$[a]	$-3.0°$[a]	-3.7 ± 0.1	$-3.3{}^{+2}_{-0}$
η_{35}	1.0[a]	1.0[a]	1.0 ± 0	1.0 ± 0

[a] Indicates a quantity held fixed in a given analysis.

values which are roughly a mean of the values of Foote[14] *et al.* and Grigor'ev and Mitin.[13]

At 460 Mev the π^- and π^0 data are those of Crittenden *et al.* and the π^+ those of Willis.[15] Our $I=\frac{3}{2}$ phase shifts agree very closely with one of the families that Willis had found and also the d-wave phase shifts are similar in character to those of Foote *et al.*[14]

At 600 Mev we used the π^- data of Crittenden *et al.* and the π^+ data of Willis after renormalizing it.

We feel the important results here are as follows:

1. The largest $I=\frac{1}{2}$ amplitude at 460 and 600 Mev is the $d_{\frac{3}{2}}$. The results are probably consistent with a resonance in the $d_{\frac{3}{2}}$ state, but do not conclusively indicate such a resonance. The state decays into a two-meson final state a large fraction of the time.

2. The Dubna phase shifts are extended into a higher energy range. The particular features of these are that α_1 and α_{11} are positive α_{13} is negative.

3. α_1 is positive and seems to increase through this energy range. Both α_1 and α_{11} must have rather considerable imaginary parts. This is not surprising if one makes simple threshold arguments concerning pion production by pions. In the former case one can have an s- and a p-wave pion produced and in the latter a pair of s-wave or a pair of p-wave pions.

[14] J. H. Foote, O. Chamberlain, E. H. Rogers, H. M. Steiner, C. Wiegand, and T. Ypsilantis, Phys. Rev. Letters **4**, 30 (1960).

[15] W. J. Willis, Phys. Rev. **116**, 753 (1959).