Dependence on Atomic Number of the Nuclear Photoeffect at High Energies*

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A measurement was made of the number of neutron-proton coincidences observed when 320-Mev bremsstrahlung bombarded D, Li, Be, C, O, Al, Ti, Cu, Sn, and Pb. If one normalizes the data for the number of neutron-proton pairs in a nucleus (i.e., by dividing by NZ/A) it is found that the observed coincidences decrease as A increases.

It is possible to quantitatively account for this A dependence by correcting for the probability that two nucleons will escape from inside a nucleus without either having a collision. The probability of escape is a function of the nuclear radius, R, and the mean free path, λ, in nuclear matter. For medium weight elements the observed neutron-proton pairs are produced with a cross section given by

$$\sigma_{Z,A}(\text{coincidences})\cong 3.0(NZ/A)\sigma_D P(2R/\lambda),$$

where σ_D is the cross section for the photodisintegration of the deuteron and where $P(2R/\lambda)$ is the probability-of-escape factor. For two nucleons emitted at 180°, the form of $P(x)$ is

$$P(x)=(3/x^3)[2-e^{-x}(x^2+2x+2)]$$

The formula for the cross sections is shown to be what one would expect if the fundamental mechanism in complex nuclei is the same as that suggested by Wilson for the photodisintegration of the deuteron. The constant, 3.0, depends on the cube of a neutron-proton pair interaction distance. A less naive treatment also involves a nucleon pair correlation function.

INTRODUCTION

WHEN nuclei are bombarded by high-energy x rays, neutrons and protons are observed to be emitted in coincidence,[1–4] which is in qualitative agreement with the quasi-deuteron model proposed by Levinger.[5] The model is one in which high-energy x rays interact with a pair of nucleons in a complex nucleus rather than with the entire nucleus.

The measurements described in this paper are part of a series. The other experiments previously reported[2,3,6] by us, and similar work reported by Barton and Smith[4] have shown that the mechanisms responsible for the simultaneous photoejection of a neutron and proton are the same mechanisms which result in the photodisintegration of the deuteron. Also, it has been found that the angular correlation of the neutron and proton are those kinematically required for the deuteron modified by the initial motion of the two nucleons in the nucleus. This paper deals with the dependence of the number of neutron-proton coincidences on the atomic weight, A, (or charge Z) of the nucleus.

Apparatus

The equipment used in studying the Z dependence was identical with that previously reported[2] and is outlined in Fig. 1. The proton counter was set at an angle of 76° relative to the x-ray beam and detected protons with energies 130±10 Mev, corresponding to photons with a mean energy of 262 Mev for the photodisintegration of the deuteron; the MIT synchrotron was operating at 320 Mev. The "large neutron detector" and the associated electronic circuitry are discussed in detail in an article by Christie *et al.*[7] The neutron detector subtended a solid angle of 0.35π steradian in order to include the entire angular spread which arises from the internal momentum of the nucleons.[3,4]

In the case of Li, Cu, and Sn, a series of different target thicknesses were run in order to determine a correction for target thickness. The targets of all the other elements were about 3 g/cm² thick except for Ti which was 5.7 g/cm² thick.

Results

The results of the measurements, corrected for target thickness, are given in Table I. The deuterium data

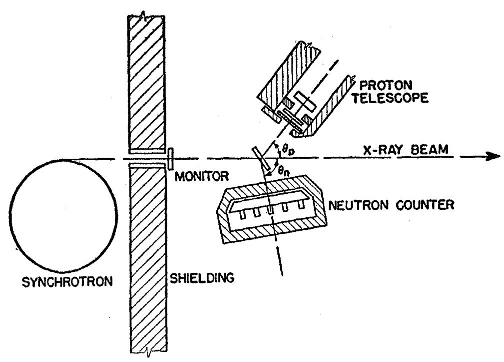

FIG. 1. Experimental arrangement. The angles are 76° in the laboratory. The maximum energy in the beam is 320 Mev. For details on the detectors, see reference 7.

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[1] M. Q. Barton and J. H. Smith, Phys. Rev. **95**, 573 (1954); H. Myers, A. Odian, P. C. Stein, and A. Wattenberg, Phys. Rev. **95**, 576 (1954).
[2] A. Odian, P. C. Stein, A. Wattenberg, B. T. Feld, and R. M. Weinstein, Phys. Rev. **102**, 837 (1956).
[3] A. Wattenberg, A. Odian, P. C. Stein, and H. Wilson, Phys. Rev. **104**, 1710 (1956).
[4] M. Q. Barton and J. H. Smith, Phys. Rev. **110**, 1143 (1958).
[5] J. S. Levinger, Phys. Rev. **84**, 43 (1951).
[6] R. M. Weinstein, A. Odian, P. C. Stein, and A. Wattenberg, Phys. Rev. **99**, 1621 (1955).
[7] E. R. Christie, B. T. Feld, A. C. Odian, P. C. Stein, and A. Wattenberg, Rev. Sci. Instr. **27**, 127 (1956).

was obtained from a D_2O-H_2O subtraction. The neutron-proton coincidence counting rates which are listed in the second column of the table, are expressed in counts per x-ray monitor unit per (g/cm^2) of target.

The next column of the table lists the coincidence results in the form of relative cross sections. They are expressed as a ratio of the cross section for the element relative to the cross section for deuterium. Such a ratio is directly obtained from the experimental data and does not require a knowledge of the absolute photon flux or the efficiencies and solid angles of the counters. The last column of the table gives the relative cross sections normalized for the number of neutron-proton pairs in each nucleus, namely, the values of the previous column are divided by NZ/A. The dependence of NZ/A is expected also on Levinger's[5] model.

The values of the last column of the table are plotted in Fig. 2. The decrease in the cross section (per neutron-proton pair) with increasing A is readily understood

TABLE I. Neutron-proton coincidences.

Element	Coincidences / monitor-g/cm^2	σ/σ_D [a]	$\left(\frac{\sigma/\sigma_D}{NZ/A}\right)$ [b]
$_1D^2$	0.121 ±0.012	1.000	
$_3Li^7$	0.095 ±0.006	2.8±0.3	1.63±0.20
$_4Be^9$	0.063 ±0.006	2.3±0.3	1.06±0.15
$_6C^{12}$	0.045 ±0.003	2.2±0.3	0.74±0.08
$_8O^{16}$	0.047 ±0.003	3.1±0.4	0.78±0.11
$_{13}Al^{27}$	0.039 ±0.004	4.4±0.6	0.65±0.08
$_{22}Ti^{48}$	0.028 ±0.003	5.6±0.8	0.47±0.06
$_{29}Cu^{64}$	0.026 ±0.004	6.8±1.3	0.43±0.08
$_{50}Sn^{119}$	0.0174±0.0021	8.6±1.4	0.30±0.06
$_{82}Pb^{207}$	0.0157±0.0022	13.4±2.1	0.27±0.04

[a] Cross section per atom relative to the cross section for deuterium. To convert these to absolute values, multiply the values in this third column by 63 microbarns which is the value for σ_D at this energy. See J. C. Keck and A. V. Tollestrup, Phys. Rev. **101**, 360 (1956); E. A. Whalin, B. D. Schriever, and A. O. Hanson, Phys. Rev. **101**, 377 (1956).
[b] Cross section per nucleon pair in the nucleus relative to the cross section for deuterium.

with the aid of Serber's[8] semiclassical model of high-energy reactions. Namely, some of the particles scatter before they escape from the nucleus; the probability that both nucleons escape without interacting decreases as the size of the nucleus increases.

Probability of Escape Factor

Keck[9] and Weil and McDaniel[10] in analyzing their data on photoprotons from carbon went to some effort to estimate the effect of the scatterings inside the nucleus. Barton and Smith[4] also estimated the probability of escape of a single nucleon in order to analyze their neutron-proton data from lithium. Although it is difficult to calculate the effect of the scattering on the emission of a single particle, it is possible to obtain readily an analytic expression for the probability of escape of two nucleons (back to back) from a nucleus. An outline of the derivation is given in the Appendix. It is assumed that the mean free path inside nuclear matter is the same for both neutrons and protons; it is also assumed that the density of nuclear matter is uniform. At the energies employed in our measurements it is considered that the two nucleons being emitted back to back is sufficiently close to the real physical situation.

With these assumptions the probability that both nucleons escape without either one undergoing an interaction is

$$P(x)=(3/x^3)[2-e^{-x}(x^2+2x+2)], \qquad (1)$$

where $x=2R/\lambda$, the ratio of the nuclear diameter to the mean free path. For the case where $R=r_0A^{\frac{1}{3}}$, we get $x=(2r_0/\lambda)A^{\frac{1}{3}}$. Various values of $2r_0/\lambda$ were tried in order to study which values provided the best correction for the data.

Figure 3 shows the data of Fig. 2 corrected for the probability of escape. The value of $P(x)$ and the values

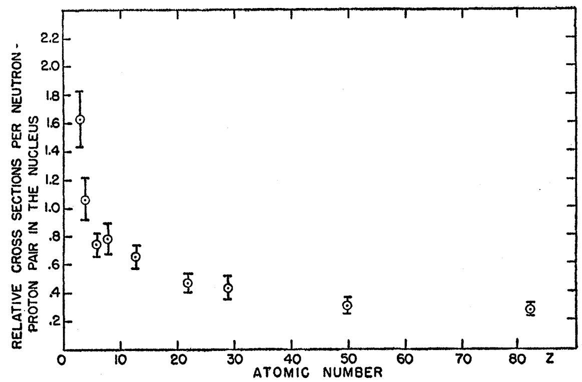

FIG. 2. Relative cross sections per neutron-proton pair in the nucleus versus atomic number. The cross section of the element of interest is divided by the cross section for deuterium and by the factor NZ/A.

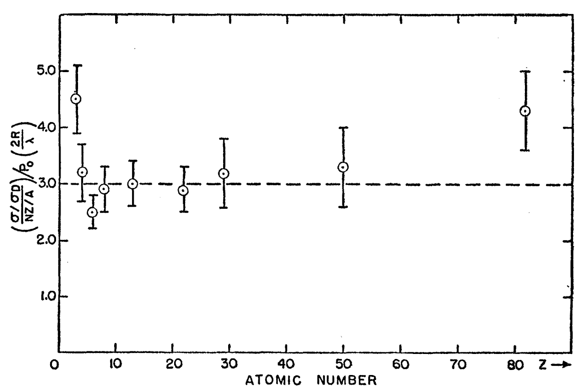

FIG. 3. The relative cross sections per neutron-proton pair corrected for the probability of escape is plotted against atomic number. The probability of escape factor is calculated using $r_0=1.30\times10^{-13}$ cm and $\lambda=3.6\times10^{-13}$ cm. The probability of escape factor is given in expression (1). The data shown are those of Fig. 2 divided by $P(2R/\lambda)$.

[8] R. Serber, Phys. Rev. **72**, 1144 (1947).
[9] J. C. Keck, Cornell University thesis, 1951 (unpublished).
[10] J. W. Weil and B. D. McDaniel, Phys. Rev. **92**, 391 (1953).

TABLE II. Relative cross sections and the probability of escape.

Element	$P(2R/\lambda)$[a]	$\left(\dfrac{\sigma/\sigma_D}{NZ/A}\right)\Big/P\left(\dfrac{2R}{\lambda}\right)$[b]
Li	0.36	4.5 ± 0.6
Be	0.33	3.2 ± 0.5
C	0.30	2.5 ± 0.3
O	0.27	2.9 ± 0.4
Al	0.215	3.0 ± 0.4
Ti	0.161	2.9 ± 0.4
Cu	0.136	3.2 ± 0.6
Sn	0.091	3.3 ± 0.7
Pb	0.061	4.3 ± 0.7

[a] Probability of escape factor calculated with $R=1.30\times10^{-13}A^{1/3}$ cm and with $\lambda=3.6\times10^{-13}$ cm.
[b] Relative cross sections per nucleon pair in the nucleus (last column of Table I) divided by the probability of escape factor.

of $(\sigma/\sigma_D)/[P(x)NZ/A]$ are tabulated in Table II for the case where $2r_0/\lambda=0.58$; if one takes $r_0=1.3\times10^{-13}$ cm, λ is then 3.6×10^{-13} cm, which is slightly longer than used in interpreting other experiments.[11] However, one should note that for our geometry not all neutron events which are scattered will miss the neutron counter as it subtends a solid angle of 0.35π steradian. This increases the "effective" mean free path inside the nucleus. The fact that some scattering will be recorded and that with increasing nuclear size, multiple scattering will become more important are believed to be the reason why in Fig. 3 the points for the heavy elements lie above those for the medium weight elements.

From Fig. 3 we get $(\sigma/\sigma_D)/[P(x)(NZ/A)]=3.0$[12]; therefore

$$\sigma\cong3.0(NZ/A)P(2R/\lambda)\sigma_D. \tag{2}$$

In medium weight elements the cross section for producing correlated neutron-proton pairs is just a constant times the number of neutron-proton pairs multiplied by the cross section for the photodisintegration of the deuteron.

In Fig. 3 it should be noted that the value for Li lies above the straight line. This is not too surprising for a residual nucleus of five nucleons. One can interpret the larger value of Li as due to the breakdown of the probability of escape calculation for such a small nucleus or as lithium being a special nucleus requiring a higher value for the constant. Using escape probabilities estimated in an entirely different way, Barton and Smith[4] obtained from their data on lithium a value of the constant which is in agreement with the value we would obtain from our Li data.

[11] A. Wattenberg, in *Encyclopedia of Physics*, edited by S. Flügge/Marburg (Springer-Verlag, Berlin, 1957), Vol. XL.

[12] J. H. Smith (private communication) has pointed out that this constant is affected by corrections of the order of 10% which tend to compensate each other. For example, the binding energy of the neutron-proton pair in the complex nucleus leads to the effective energy being higher in complex nuclei than in deuterium. The bremsstrahlung has decreased; however, the cross section has increased. If more precise measurements and interpretations are made such corrections should be included.

Discussion

Experiments[2] reported previously showed that the interactions involved in the high-energy photoeffect in deuterium and in complex nuclei were predominantly the same. The fact that the data agrees with the above formula, (2), is further confirmation of this.

It is interesting to note that if one employs Wilson's[13] model for the photodisintegration of the deuteron and applies it to complex nuclei one obtains our empirical formula (2) but with a different value for the constant.

Wilson assumes that in the energy region 200 to 300 Mev the photodisintegration of the deuteron is a process mainly associated with meson production. If the meson production occurs when two nucleons are within a meson Compton wavelength of each other, then instead of the meson escaping, phase space favors two nucleons carrying off the momentum and energy. On this picture one can readily calculate the ratio

$$\frac{\sigma}{\sigma_D}=\frac{\begin{pmatrix}\text{Number of}\\ \text{protons}\end{pmatrix}\times\begin{pmatrix}\text{density of}\\ \text{neutrons}\end{pmatrix}\times(4\pi/3)a^3}{\int_0^a|\psi H(r)|^2r^2dr}. \tag{3}$$

The numerator comes from asking for the probability of finding a neutron within a sphere of one meson compton wavelength $a=(\hbar/\mu c)$ radius (for every proton). This numerator assumes completely uncorrelated nucleons. The denominator is just the integral over the density given by the Hulthen wave function and has the numerical value $\frac{1}{4}$ for $a=1.41\times10^{-13}$ cm. The numerator is readily evaluated for the simple picture of a spherical nucleus with uniform density. The number of protons is Z, the density of neutrons is

$$N/(\tfrac{4}{3}\pi R^3)=N/(\tfrac{4}{3}\pi r_0^3A),$$

substituting these values in (3) gives

$$\sigma/\sigma_D=4(NZ/A)(a^3/r_0^3). \tag{4}$$

Obviously the value of the constant in expression (2) depends on the choice of $(a/r_0)^3$; a value of $r_0=1.30\times10^{-13}$ cm leads to

$$\sigma=4.9(NZ/A)\sigma_D. \tag{5}$$

If one includes in expression (5) a probability of escape factor, then we have obtained a similar form to the expression (2) which fits the experimental data. The disagreement of the numerical constants is of interest. There are really two parameters (r_0/λ) and (a/r_0). The value of the numerical constant is comparatively insensitive to (r_0/λ); specifically, for (r_0/λ) 20% different than the value used, the constant factor changes by about 35%. For shorter mean free paths the formula no longer fits the data. However, the constant varies as the cube

[13] R. R. Wilson, Phys. Rev. **85**, 125 (1952); **104**, 218 (1956).

power of (a/r_0) so that a change of 25% in this ratio changes the numerical constant by almost a factor of two. In other words one could obtain numerical agreement between (2) and (5) by making "a" 18% smaller than the Compton wavelength of a pion. The parameter "a" can be considered an interaction distance parameter. This is close to the value Austern[14] used in fitting his theory of the photodisintegration of the deuteron.

It is reassuring to find that with the aid of naive models one can semiquantitatively understand many of the aspects of the high-energy photoeffect in complex nuclei.

A much more sophisticated treatment of the high-energy photoeffect has recently been given by Gottfried.[15] He points out that the type of data obtained in this experiment bears on the dynamics while our earlier experiments[1,3] dealt more with the kinematical effects. One of the main points in Gottfried's analysis is in the possibility of obtaining information on the nucleon pair correlation function. One finds a dependence on the cube power of a pair correlation parameter. The experimental data therefore sets comparatively narrow limits on such a parameter.[16]

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APPENDIX. PROBABILITY OF ESCAPE FUNCTION

We are considering the situation where there are two detectors (effectively on opposite sides of a target) which determine an axis in space, and we are interested only in those nucleons which are produced (back to back) along this axis. We wish to know what is their probability of escaping from a spherical nucleus without suffering a collision. Cylindrical coordinates are most convenient. It can be seen from Fig. 4 that, for a sphere of radius R, the length of a cylinder at a distance r from the central axis is $2(R^2-r^2)^{\frac{1}{2}}$; the differential volume (between r and $r+dr$) is $4\pi r(R^2-r^2)^{\frac{1}{2}}dr$.

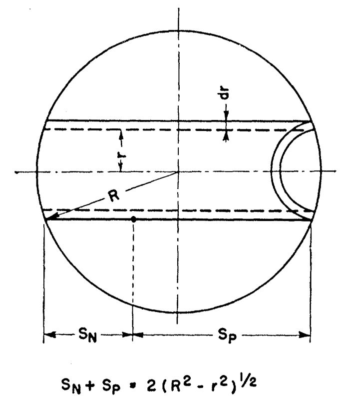

FIG. 4. Convenient coordinate system for calculating the probability of escape factor. The dot on the lower chord through the sphere indicates the point at which the neutron and proton were created by the x ray.

If the path length of the neutron inside the nucleus is S_n and that of the proton is S_p, then $S_n+S_p=2(R^2-r^2)^{\frac{1}{2}}$. The probability that both of these particles escape without either having an interaction is the product $e^{-S_n/\lambda_n}e^{-S_p/\lambda_p}$.

We assume both particles have the same mean free path in nuclear matter namely $\lambda_n=\lambda_p=\lambda$. Then the probability of escape becomes

$$e^{-(S_n+S_p)/\lambda}=\exp[-2(R^2-r^2)^{\frac{1}{2}}/\lambda].$$

The mean probability of escape is the integral of the production times the escape probability divided by the total production or

$$P\left(\frac{2R}{\lambda}\right)=\frac{\int_0^R \exp[-2(R^2-r^2)^{\frac{1}{2}}/\lambda]4\pi r(R^2-r^2)^{\frac{1}{2}}dr}{\int_0^R 4\pi r(R^2-r^2)^{\frac{1}{2}}dr}.$$

The integration is readily performed and yields

$$P\left(\frac{2R}{\lambda}\right)=\frac{3\lambda^3}{8R^3}\left\{2-e^{-2R/\lambda}\left[\left(\frac{2R}{\lambda}\right)^2+2\left(\frac{2R}{\lambda}\right)+2\right]\right\},$$

or

$$P(x)=(3/x^3)\{2-e^{-x}[x^2+2x+2]\}.$$

[14] N. Austern, Phys. Rev. **100**, 1522 (1955).
[15] K. Gottfried, Nuclear Phys. **5**, 557 (1958).
[16] K. Gottfried (private communication) points out that one can see the effect of the correlation function in the naive theory by putting a correlation factor in the numerator of Eq. (3). If one assumes the interaction distance is a meson Compton wavelength, then the correlation factor is less than unity. A correlation factor less than unity would arise from a hard core, in which case the denominator has to be reevaluated. Such considerations are treated in Gottfried's article.[15]