Corrections to the Impulse Approximation for Photon-Deuteron Scattering

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The validity of the impulse approximation for the scattering of 50–120 Mev photons from deuterons is investigated by the use of forward scattering dispersion relations. The only significant deviation from the impulse approximation found is in the spin-independent amplitude. Most of this deviation is shown to be the result of the exchange part of the neutron-proton potential. The exchange force is known to increase the electric dipole photodisintegration cross section; it is shown that the exchange force also increases the electric dipole elastic scattering cross section by about 10–20%.

A RECENT experiment at the Massachusetts Institute of Technology[1] on photon scattering from deuterons has indicated that the total differential cross section at 90° is from 30% to 70% larger than is given by the impulse approximation[2] for energies in the range 50–120 Mev. Since photodisintegration occurs at these energies some "cooperation" between the neutron and proton in absorbing the photon must be taking place and might be expected to lead to corrections to the impulse approximation for Compton scattering. This paper represents an attempt to estimate these effects and as such is an extension of Sec. III. B of an earlier paper by one of the authors.[2]

The amplitude for the elastic forward scattering of a photon from a system of spin 1 may be written in the form

$$\begin{aligned}T_d(k_l)=&A_d(k_l)\mathbf{e}\cdot\mathbf{e}'+B_d(k_l)i\mathbf{J}\cdot\mathbf{e}\times\mathbf{e}'\\&+C_d(k_l)\mathbf{e}\cdot\mathbf{e}'(3J_z^2-2)+D_d(k_l)\\&\times\{\mathbf{J}\cdot\mathbf{e}\mathbf{J}\cdot\mathbf{e}'+\mathbf{J}\cdot\mathbf{e}'\mathbf{J}\cdot\mathbf{e}+\mathbf{e}\cdot\mathbf{e}'(J_z^2-2)\},\end{aligned}\qquad(1)$$

where $\mathbf{e}$ and $\mathbf{e}'$ are the polarization vectors of the incident and outgoing photons, $\mathbf{J}$ is the spin operator of the scatterer, and A, B, C, and D are functions only of photon lab energy $\hbar k_l c$. The differential elastic cross section in the forward direction for an unpolarized beam is given by $d\sigma/d\Omega=\frac{1}{3}(3|A|^2+2|B|^2+6|C|^2+4|D|^2)$.

In the impulse approximation, A, B, C, and D are assumed to be given by the superposition of the amplitudes for scattering from each of the particles making up the system. For the deuteron, the impulse approximation leads to the result $C=D=0$, and $A_d=A_p+A_n$, $B_d=B_p+B_n$, where the subscripts d, p, and n refer to the deuteron, proton, and neutron, respectively, and the scattering amplitude for a particle of spin $\frac{1}{2}$ has been written in the form

$$T(k_l)=A(k_l)\mathbf{e}\cdot\mathbf{e}'+B(k_l)i\boldsymbol{\sigma}\cdot\mathbf{e}\times\mathbf{e}'.\qquad(2)$$

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[1] L. G. Hyman, R. Ely, D. H. Frisch, and M. A. Wahlig, Phys. Rev. Letters **3**, 93 (1959); L. G. Hyman, Massachusetts Institute of Technology, Ph.D. thesis, 1959 (unpublished).

[2] R. H. Capps, Phys. Rev. **106**, 1031 (1957); **108**, 1032 (1957).

I. THE SPIN-INDEPENDENT AMPLITUDE

The spin-independent amplitude may be written in the form $A_d=A_p+A_n+\Delta$, so that Δ is the correction to the impulse approximation. Each of the amplitudes A_d, A_p, and A_n satisfies the dispersion relation

$$\operatorname{Re}A(k_l)-A(0)=\frac{2k_l^2}{\pi}\int_0^\infty\frac{\operatorname{Im}A(k_l')dk_l'}{k_l'(k_l'^2-k_l^2)},\qquad(3)$$

and the unitarity condition

$$\operatorname{Im}A(k_l)=(k_l/4\pi)\sigma_{\text{total}}(k_l).\qquad(4)$$

The total cross section for unpolarized incident photons, $\sigma_{\text{total}}(k_l)$, is predominately the pion photoproduction cross section σ_π and, in the case of the deuteron, the photodisintegration cross section σ_{dis}. We will consider only the lowest order terms in $e^2/\hbar c$ so that the photon scattering cross section itself can be neglected in determining $\operatorname{Im}A$ from Eq. (4). The constant $A(0)$ which appears in Eq. (3) is the Thomson amplitude and depends only on the mass and charge of the particle. Thus, if M and e are the mass and charge of the proton, we have

$$A_d(0)=-e^2/2Mc^2,\quad A_p(0)=-e^2/Mc^2,\quad A_n(0)=0.\qquad(5)$$

Combining Eqs. (3), (4), and (5), we get for the correction to the impulse approximation,

$$\operatorname{Re}\Delta=\frac{e^2}{2Mc^2}+\frac{k_l^2}{2\pi^2}\int_0^\infty\frac{\sigma_{\text{dis}}+\sigma_{\pi d}-\sigma_{\pi p}-\sigma_{\pi n}}{k_l'^2-k_l^2}dk_l',\qquad(6)$$

$$\operatorname{Im}\Delta=(k_l/4\pi)(\sigma_{\text{dis}}+\sigma_{\pi d}-\sigma_{\pi p}-\sigma_{\pi n}).\qquad(7)$$

The quantity $\sigma_{\pi d}-\sigma_{\pi p}-\sigma_{\pi n}$ is, of course, zero below the meson threshold but is not known for higher energies because $\sigma_{\pi n}$ is not known. For π^+ photoproduction the deuteron to proton experimental ratio is given roughly by $\sigma_{\pi^+d}\approx0.9\sigma_{\pi^+p}$.[3] Hence we take the total pion production cross section from deuterons to be given by the relation, $\sigma_{\pi d}\approx0.9(\sigma_{\pi p}+\sigma_{\pi n})$. We further assume $\sigma_{\pi n}\approx\sigma_{\pi p}$. Then, using the experimental cross sections

[3] K. M. Crowe, R. M. Friedman, and D. C. Hagerman, Phys. Rev. **100**, 1799 (A) (1956).

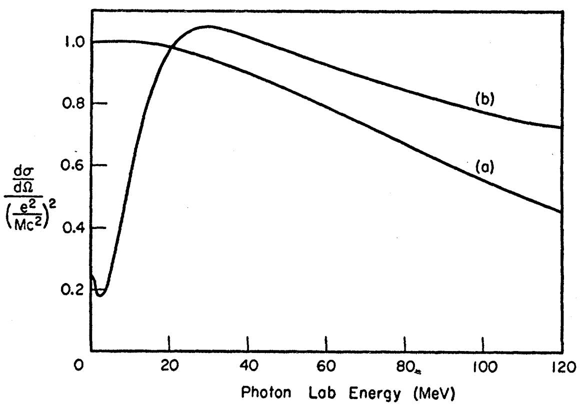

FIG. 1. Total (elastic plus inelastic) differential cross section in the lab system for forward scattering as given by the impulse approximation with (curve b) and without (curve a) the correction Δ to the spin-independent parts.

for σ_{dis} [4] and $\sigma_{\pi p}$,[5] we obtain the results of Table I. (Note that with our assumption about the meson cross sections the photodisintegration contribution has the same sign as the meson contribution only because we are evaluating Re Δ at energies between the peaks in these two cross sections. Above the meson production peak the two effects would tend to cancel under this assumption.) The effect of the correction Δ on the total forward scattering is plotted in Fig. 1; the dipole model of reference 2 is used to evaluate the impulse approximation amplitudes.

Unfortunately, experiments cannot be done for forward scattering, so that some further information about the angular dependence of Δ is necessary in order to extend the 0° predictions to finite angles. In the impulse approximation, the electric dipole is the largest amplitude. Furthermore, there is no interference between electric and magnetic amplitudes at 90° where the experiment of Hyman *et al.*[1] was done, so that large effects at 90° are expected to be the result of the corrections to the electric dipole amplitude. Gauge invariance allows us to write the electric dipole amplitude (in the long wavelength limit) in terms of the wave function of the neutron-proton system.[6] This property allows us to estimate the effects of the neutron-proton potential. As shown in reference 6, if the vector potential $\mathbf{A}$ can be written as the gradient of a scalar, i.e., $\mathbf{A}=\nabla G$, then the Hamiltonian, to second order in e, may be written as

$$H=H_0+(ie/\hbar c)[G,H_0]-(e^2/2\hbar^2c^2)[G,[G,H_0]], \quad (8)$$

TABLE I. Real and imaginary parts of the correction Δ to the spin-independent forward scattering amplitude in units of $e^2/(Mc^2)$. The bottom line gives the amount of the correction that is due to the estimated difference between the meson photoproduction cross sections.

$\hbar k_l c$ (Mev)	0	30	60	90	120
Im Δ	0	0.253	0.182	0.19	0.19
Re Δ	0.50	−0.025	−0.055	−0.10	−0.205
Meson part of Re Δ	0	−0.004	−0.015	−0.030	−0.057

where H_0 is the zero order Hamiltonian. In the long wavelength approximation $\mathbf{A}=\mathbf{e}$, so that $G=\mathbf{e}\cdot\mathbf{r}_p$, where $\mathbf{r}_p$ is the position of the proton. The zero order Hamiltonian is written in terms of the position of the center of mass $\mathbf{R}=\frac{1}{2}(\mathbf{r}_p+\mathbf{r}_n)$ and the distance between the proton and neutron $\mathbf{r}=\mathbf{r}_p-\mathbf{r}_n$, i.e.,

$$H_0=-(\hbar^2\nabla_R^2/4M)-(\hbar^2\nabla_r^2/M)+(1-x)V_1+xV_2\mathcal{P}, \quad (9)$$

where $\mathcal{P}$ is the exchange operator ($\mathcal{P}\mathbf{r}_p=\mathbf{r}_n$; $\mathcal{P}\mathbf{r}_n=\mathbf{r}_p$) and the potential has been divided into its exchange part $xV_2\mathcal{P}$ and its nonexchange part $(1-x)V_1$. The nonexchange part may contain central, spin-orbit, and tensor forces.

We now use the above Hamiltonian [Eqs. (8) and (9)] to calculate the electric dipole scattering amplitude. Since the initial energy E is the sum of the photon energy $\hbar k_l c$ and the deuteron binding energy $(-E_B)$, we get

$$\begin{aligned}T_d'=&(e^2/\hbar^2c^2)\langle\psi_d|[\mathbf{e}\cdot\mathbf{r}_p,[\mathbf{e}'\cdot\mathbf{r}_p,H_0]]|\psi_d\rangle\\&-\frac{e^2}{\hbar^2c^2}\sum_m\bigg(\langle\psi_d|[\mathbf{e}'\cdot\mathbf{r}_p,H_0]|\psi_m\rangle\frac{1}{E-E_m}\\&\times\langle\psi_m|[\mathbf{e}\cdot\mathbf{r}_p,H_0]|\psi_d\rangle+\langle\psi_d|[\mathbf{e}\cdot\mathbf{r}_p,H_0]|\psi_m\rangle\\&\times\frac{1}{E-(E_m+2\hbar k_lc)}\langle\psi_m|[\mathbf{e}'\cdot\mathbf{r}_p,H_0]|\psi_d\rangle\bigg). \quad (10)\end{aligned}$$

If we consider only the spin-independent part of T_d' only the component of $\mathbf{e}'$ along $\mathbf{e}$ will contribute to the sum over m. Then the second term can be rewritten in terms of the electric dipole photodisintegration cross section, which is given by

$$\sigma_{\text{dis}}^{\text{ED}}=\frac{(2\pi)^2}{k_l}\frac{e^2}{\hbar^2c^2}|\langle\psi_d|[\mathbf{e}\cdot\mathbf{r}_p,H_0]|\psi_m\rangle|^2\frac{dm}{dE}. \quad (11)$$

Using Eqs. (9) and (11), Eq. (10) becomes

$$\begin{aligned}T_d'=&-\mathbf{e}\cdot\mathbf{e}'(e^2/Mc^2)+(e^2/\hbar^2c^2)\\&\times\langle\psi_d|xV_2(\mathbf{r})\mathbf{e}\cdot\mathbf{r}\,\mathbf{e}'\cdot\mathbf{r}|\psi_d\rangle\\&+\frac{\mathbf{e}\cdot\mathbf{e}'}{2\pi^2}\int_0^\infty\frac{\sigma_{\text{dis}}^{\text{ED}}k_l'^2dk_l'}{k_k'^2-k_l^2}. \quad (12)\end{aligned}$$

[4] C. A. Barnes, J. H. Carver, G. H. Stafford, and D. H. Wilkinson, Phys. Rev. **86**, 359 (1952); Lew Allen, Jr., Phys. Rev. **98**, 705 (1955); J. C. Keck and A. V. Tollestrup, Phys. Rev. **101**, 360 (1956); E. A. Whalin, B. D. Schriever, and A. O. Hanson, Phys. Rev. **101**, 377 (1956); G. R. Bishop, C. H. Collie, H. Halban, A. Hedgran, K. Siegbahn, S. Du Toit, and R. Wilson, Phys. Rev. **80**, 211 (1950); J. A. Phillips, J. S. Lawson, and P. G. Kruger, Phys. Rev. **80**, 326 (1950).

[5] K. M. Watson, J. C. Keck, A. V. Tollestrup, and R. L. Walker, Phys. Rev. **101**, 1159 (1956).

[6] R. G. Sachs and N. Austern, Phys. Rev. **81**, 705 (1951).

If use is made of the electric dipole sum rule,[7]

$$\frac{1}{2\pi^2}\int_0^\infty \sigma_{\mathrm{dis}}{}^{\mathrm{ED}}dk_l' = \frac{e^2}{2Mc^2} - \frac{e^2}{\hbar^2c^2}\langle\psi_d|xV_2(\mathbf{e}\cdot\mathbf{r})^2|\psi_d\rangle, \quad (13)$$

Eq. (12) may also be written in a form very similar to the forward scattering dispersion relation, Eq. (3), i.e.,

$$T_d' = \mathbf{e}\cdot\mathbf{e}'\left(-\frac{e^2}{2Mc^2} + \frac{k_l^2}{2\pi^2}\int_0^\infty \frac{\sigma_{\mathrm{dis}}{}^{\mathrm{ED}}dk_l'}{k_l'^2 - k_l^2}\right). \quad (14)$$

This equation, however, is not restricted to the forward direction. Furthermore, the photodisintegration cross section up to 150 Mev is almost all electric dipole [8,9] so that Δ is also mainly electric dipole.

In evaluating T_d' for energies in the range 50–120 Mev, it is convenient to use Eq. (12), since the factor $k_l'^2/(k_l'^2 - k_l^2)$ in the integrand is small wherever the cross section is large so that the value of the integral is quite small. The exchange force term may be estimated by using a Hulthén potential for $V_2(r)$ and taking x, the fraction of the potential which has exchange character, to be $\frac{1}{2}$. The result is about $-0.09\ e^2/(Mc^2)$. This term is not included in the impulse approximation and leads to an increase of about 20% over the impulse approximation result for the electric dipole scattering cross section at all angles. For energies below 50 Mev the integral in Eq. (12) will not be small but provides the transition to the zero energy value, which, as seen from Eq. (14), is the Thomson cross section.

Corrections to the long wavelength approximation used above are not determined by gauge invariance but depend on the details of the electromagnetic interactions of the deuteron. These corrections are not expected to be large in this energy region because an extra factor of $(k_lr)^2$ occurs in the matrix elements and, although k_lR (where R is the "radius of the deuteron") is of order unity, this factor of $(k_lr)^2$ usually occurs multiplied by the potential V_2, which has a shorter range than the deuteron. Furthermore, those terms which do not involve the potential [e.g., $\langle\psi_d|(\mathbf{e}\cdot\mathbf{r})^2(\mathbf{k}_l\cdot\mathbf{r})^2|\psi_d\rangle$] are generally terms included in the impulse approximation so that the long wavelength approximation is expected to be fairly good for estimating corrections to the impulse approximation. For the same reason, the inelastic cross section is apparently not much changed by the cooperative effects. In the impulse approximation, the inelastic cross section at 90° is only about 20% of the total cross section for 60-Mev photons and about 40% at 120 Mev, so that only a large change in the inelastic scattering could be seen in the total cross section. In the experiment of Hyman *et al.*[1] only the outgoing photon was observed, so that the incident photon energy is higher for inelastic events than for elastic events with the same outgoing photon energy. This complicates the determination of cross sections from the data.

II. THE SPIN-DEPENDENT TERMS

The major contributions to the spin-dependent scattering amplitude are magnetic and thus not specified by gauge invariance. For the forward direction we can write dispersion relations for the functions $B(k_l)$, $C(k_l)$, and $D(k_l)$ of Eq. (1); e.g.,

$$\mathrm{Re}\,B(k_l) - k_l\frac{dB}{dk_l}(0) = \frac{2k_l^3}{\pi}\int_0^\infty \frac{\mathrm{Im}\,B(k_l')dk_l'}{k_l'^2(k_l'^2 - k_l^2)}. \quad (15)$$

The dispersion relations for C and D are similar to that for A, Eq. (3). If only dipoles interact, the unitarity conditions are

$$\begin{aligned}
\mathrm{Im}\,B &= (k_l/4\pi)(\tfrac{3}{4}\sigma_2 - \tfrac{3}{4}\sigma_1 - \tfrac{3}{2}\sigma_0),\\
\mathrm{Im}\,C &= (k_l/4\pi)[(1/20)\sigma_2 - \tfrac{1}{4}\sigma_1 + \tfrac{1}{2}\sigma_0],\\
\mathrm{Im}\,D &= (k_l/4\pi)[(3/20)\sigma_2 - \tfrac{3}{4}\sigma_1 + \tfrac{3}{2}\sigma_0],
\end{aligned} \quad (16)$$

where σ_J is the total cross section for states of total angular momentum J from unpolarized incident photons. Spin-independent cross sections will not contribute to Eqs. (16).

Since the integral in Eq. (15) emphasizes the low-energy values of Im B, the major contribution is from the magnetic dipole photodisintegration to the singlet S state. However, this is large only at the very lowest energies, and for scattering at energies above 30 Mev the integral combines with the zero energy term to give approximately the impulse approximation result. This can be seen from a perturbation calculation with the same interaction as is used to calculate the low-energy magnetic dipole photodisintegration,[10] i.e.,

$$\begin{aligned}
H' &= \mu_p\boldsymbol{\sigma}_p\cdot\nabla\times\mathbf{A} + \mu_n\boldsymbol{\sigma}_n\cdot\nabla\times\mathbf{A}\\
&= (\mu_p+\mu_n)\tfrac{1}{2}(\boldsymbol{\sigma}_p+\boldsymbol{\sigma}_n)\cdot\nabla\times\mathbf{A}\\
&\qquad + (\mu_p-\mu_n)\tfrac{1}{2}(\boldsymbol{\sigma}_p-\boldsymbol{\sigma}_n)\cdot\nabla\times\mathbf{A}, \quad (17)
\end{aligned}$$

where μ and $\boldsymbol{\sigma}$ are the magnetic moment and spin operator for the proton (subscript p) and neutron (subscript n). If the D-wave part of the deuteron wave function is neglected, the angular momentum $\mathbf{J}$ of the deuteron is equal to $\frac{1}{2}(\boldsymbol{\sigma}_p+\boldsymbol{\sigma}_n)$. Then, in the long wavelength approximation, the first term in Eq. (17) leads to scattering only through deuteron intermediate states. On the other hand, the second term leads to transitions through the singlet S states, ψ_S, and can be related [in the same way as in Eqs. (10), (11) and (12)] to the magnetic dipole photodisintegration cross section

[7] J. S. Levinger and H. A. Bethe, Phys. Rev. **78**, 115 (1950).
[8] J. J. DeSwart and R. E. Marshak, Phys. Rev. **111**, 272 (1958).
[9] W. Zernik, M. L. Rustgi, and G. Breit, Phys. Rev. **114**, 1358 (1959).

[10] J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley & Sons, New York, 1952), p. 608.

TABLE II. The electric dipole photodisintegration cross sections in microbarns for total angular momentum 2, 1, and 0 as calculated from the matrix elements given by DeSwart and Marshak.[a] The bottom line is related to Im B by Eq. (16).

$\hbar k_l c$ (Mev)	22.4	53.5	80
$\sigma_0(\mu b)$	0	1	16
$\sigma_1(\mu b)$	51	82	216
$\sigma_2(\mu b)$	37	64	238
$\frac{3}{4}(\sigma_2-\sigma_1-2\sigma_0)$	-10.5	-15	-7.5

[a] See reference 8.

$\sigma_{\rm dis}{}^{\rm MD}$ which is given by

$$\sigma_{\rm dis}{}^{\rm MD}=\frac{1}{6}\sum_{\substack{\text{spins,}\\ \text{pol.}}}\frac{(2\pi)^2}{k_l}(\mu_p-\mu_n)^2 \times|\langle\psi_S|\tfrac{1}{2}(\boldsymbol{\sigma}_p-\boldsymbol{\sigma}_n)\cdot\mathbf{k}_l\times\mathbf{e}|\psi_d\rangle|^2\frac{dm}{dE}. \quad (18)$$

The quantity dm/dE denotes the density of final states. Then, if $\mathbf{n}$ is a unit vector in the $\mathbf{k}_l\times\mathbf{e}$ direction, the spin-dependent scattering amplitude is

$$T_d''=\left(-(\mu_p+\mu_n)^2\frac{k_l}{\hbar c}+\frac{3k_l^3}{4\pi^2}\int_0^\infty\frac{\sigma_{\rm dis}{}^{\rm MD}dk_l'}{k_l'(k_l'^2-k_l^2)}\right)\times i\mathbf{J}\cdot\mathbf{n}'\times\mathbf{n}. \quad (19)$$

Since $\sigma_{\rm dis}{}^{\rm MD}$ is large only near the resonance in the singlet state, we can put $k_l'^2-k_l^2$ equal to $-k_l^2$ whenever the photon energy is much above the resonance energy. The resulting integral can be evaluated by using the following sum rule, derived from Eq. (18),

$$\int_0^\infty(\sigma_{\rm dis}{}^{\rm MD}/k_l')dk_l'=\tfrac{4}{3}\pi^2(\mu_p-\mu_n)^2/\hbar c. \quad (20)$$

We then get for the scattering amplitude

$$\begin{aligned}T_d''&=[-(\mu_p+\mu_n)^2(k_l/\hbar c)\\&\qquad-(\mu_p-\mu_n)^2(k_l/\hbar c)]i\mathbf{J}\cdot\mathbf{n}'\times\mathbf{n} \quad (21)\\&=2(\mu_p{}^2+\mu_n{}^2)(k_l/\hbar c)i\mathbf{J}\cdot\mathbf{n}\times\mathbf{n}',\end{aligned}$$

which is the impulse approximation result for the magnetic dipole contribution to B.

Thus the largest part of the photodisintegration cross section contribution to Eq. (15) gives the impulse approximation result. However, in the 10–150 Mev region, although the cross section is much smaller than at lower energies, the spin dependence is more complicated[8,9] and the simple theory of Eqs. (17)–(21) above is incomplete. To use Eqs. (15) and (16) we need the values of σ_2, σ_1, and σ_0, which are not simply related to any experimental quantities. DeSwart and Marshak[8] have been able to explain the isotropic and $\sin^2\theta$ parts of the disintegration cross section at 3 energies in this region using spin-orbit forces and the electric dipole approximation. We can calculate the electric dipole cross sections $\sigma_2{}^{\rm ED}$, $\sigma_1{}^{\rm ED}$, and $\sigma_0{}^{\rm ED}$ using the matrix elements given in Table I of their paper.[8] The particular combination $\frac{3}{4}\sigma_2{}^{\rm ED}-\frac{3}{4}\sigma_1{}^{\rm ED}-\frac{3}{2}\sigma_0{}^{\rm ED}$, which appears in Eq. (16), does not vary greatly with energy as shown in Table II. The total magnetic dipole disintegration cross section as calculated by Zernik *et al.*[9] with forces similar to those used by DeSwart and Marshak is about 6–10 microbarns in this region. No matter how this cross section is distributed among σ_0, σ_1, and σ_2 we can say that $(k_l/4\pi)(-10\mu b)$ will be a fair estimate for the total photodisintegration contribution to Im B for energies in the range 10–150 Mev. Using Eq. (15), and assuming the low-energy part of the integral to be given by Eq. (21), the forward amplitude should then be increased over the impulse approximation by about

$$\frac{k_l}{2\pi^2}(10\mu b)\int_{10\text{ Mev}}^{150\text{ Mev}}\frac{k_l^2dk_l'}{(k_l^2-k_l'^2)k_l'}. \quad (22)$$

For a photon energy of 60 Mev, this increase amounts to about 0.019 $e^2/(Mc^2)$ compared with the impulse approximation spin-dependent amplitude of 0.309 $e^2/(Mc^2)$. This crude estimation thus indicates that the corrections to the impulse approximation for the forward spin-dependent amplitude B are small. Similar results hold for C and D. Hence it is probable that the corrections to the spin-dependent amplitudes at other angles are not very large.

III. SUMMARY

From these considerations we conclude that part of the discrepancy between the experimental results of Hyman *et al.*[1] and the predictions of the impulse approximation is due to the exchange forces between the neutron and proton. The exchange force increases the electric dipole spin-independent elastic scattering cross section by approximately 10–20% over the impulse approximation for energies between 30 and 120 Mev. From an approximate evaluation of forward scattering dispersion relations, we find no evidence for any major modifications of the spin-dependent scattering. Thus our results indicate that the difference between the impulse approximation and the actual cross section should have approximately the angular dependence $\frac{1}{2}(1+\cos^2\theta)F^2(\frac{1}{2}q)$, where q is the momentum transfered to the deuteron and $F^2(\frac{1}{2}q)$ is the "sticking factor" of the deuteron (see reference 2) given by $F(k)=\int e^{i\mathbf{k}\cdot\mathbf{r}}\times|\psi_d|^2d^3r$. In the energy region of 100–150 Mev, the spin-independent amplitude is smaller because of the destructive interference between the Thomson amplitude and meson effects. Hence, we expect the effect on the total cross section of the spin-independent correction term estimated here also to be smaller. The results of Hyman *et al.*[1] show qualitative agreement with this energy dependence.

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