# Small-Angle Proton-Proton Scattering at 435 Mev*

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This paper presents the results of measurements of the scattering cross sections of protons by protons in the angular range 5°–20° (center of mass). An ionization chamber was used to measure the direct beam and the scattered protons were detected by means of photographic plates. The results are substantially in agreement with other work in this range of energy and angle although there is some indication of a slight minimum in the curve at the edge of the Coulomb region.

## I. EXPERIMENTAL ARRANGEMENT

THE 435-Mev unpolarized proton beam from the Carnegie Institute of Technology synchrocyclotron was used for the experiment. The experimental setup is shown in Fig. 1. Two sections of collimator were used. The first, 20 in. long, was placed in the shield wall around the cyclotron room and defined the beam by means of three sets of slits to a size of 1 in.$\times\frac{1}{2}$ in. The second section, 40 in. long, was constructed with three sets of defining slits and an adjustable angular spread. The slit locations in both collimators and the spread of the outer section were adjusted to optimize the beam collimation. For the optimum collimator geometry, see Fig. 1, the full beam spread at half maximum was $\frac{1}{4}°$ (laboratory) at the location of the photographic plates.

The target box was made of styrofoam and consisted of an outer and an inner chamber. The inner box, which had a 0.003-in. Al lining and contained liquid hydrogen, was 20 in. long and was cooled by the outer styrofoam container holding vapor from a pan of liquid nitrogen.

The beam was monitored by an ion chamber containing argon placed between the end of the collimator and the target box. The monitor was calibrated by measuring the charge built-up across a standard condensor. The windows of the ion chamber were made of brass 0.002 in. thick.

In order to reduce scattering, the region between the hydrogen target and the photographic plate holder was filled by helium at atmospheric pressure contained in a plastic balloon. Just beyond the helium balloon and in front of the photographic plates on one side of the beam (see Fig. 1) a copper absorber $2\frac{1}{2}$ in. thick was placed. The absorber was thick enough to remove from the beam $\pi$ mesons, deuterons, and slow protons from the $p+p\rightarrow\pi+d$ and $p+p\rightarrow\pi+p+n$ reactions. It did, of course, also scatter and partially absorb the elastically scattered protons under investigation. No absorber was used on the opposite side of the beam.

The photographic plate detectors were Ilford *G*5 emulsions 200 microns thick. They were mounted in a plate holder which inclined them at 40° to the beam. In addition to the plates at small angles, two small plate holders were mounted at 20° and 42° in the laboratory (44° and 90° in the center of mass). These plates permitted a check to be made of the cross sections at large angles. No correction was necessary at these angles for deuterons and the contribution due to $\pi$ mesons and protons was not over 1%. No copper absorber was used.

The determination of the cross section required an evaluation of the effective absorption factor for the copper placed in front of the plates at small angles. This factor was measured in the direct beam using the technique shown in Fig. 2. The beam impinged on a plate partially obscured by a $2\frac{1}{2}$-in. copper absorber in position $A$. After a short exposure the absorber was moved to position $B$ and another exposure made on a new plate. The two plates were scanned in the $A$ and $B$ regions. Using the plateau values $A_1$, $B_1$ on the first exposure and $A_2$, $B_2$ on the second, the copper absorption factor was taken to be $(A_1B_2/A_2B_1)^{\frac{1}{2}}$. The use of two exposures eliminated possible error arising from a small spatial variation of the beam.

Scanning the photographic plates was complicated by multiple scattering of the protons in the copper absorber. This scattering made necessary the inclusion of a range of incident angles for protons in the emulsions. This range was chosen to be $\pm5.7°$ with respect to the mean projected direction of the tracks. An added complication in determining the acceptable angular limits on a particular plate was the distortion of the emulsion during processing and drying. Because this

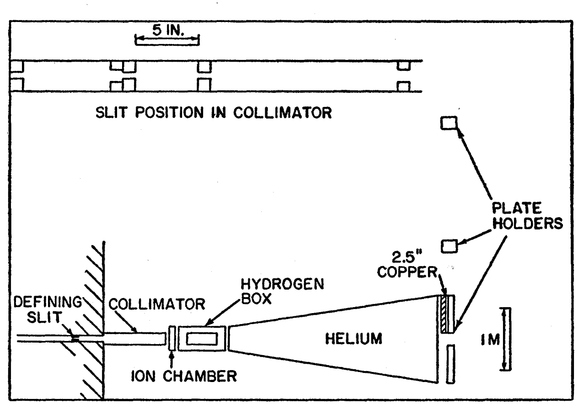


Fig. 1. Experimental arrangement.

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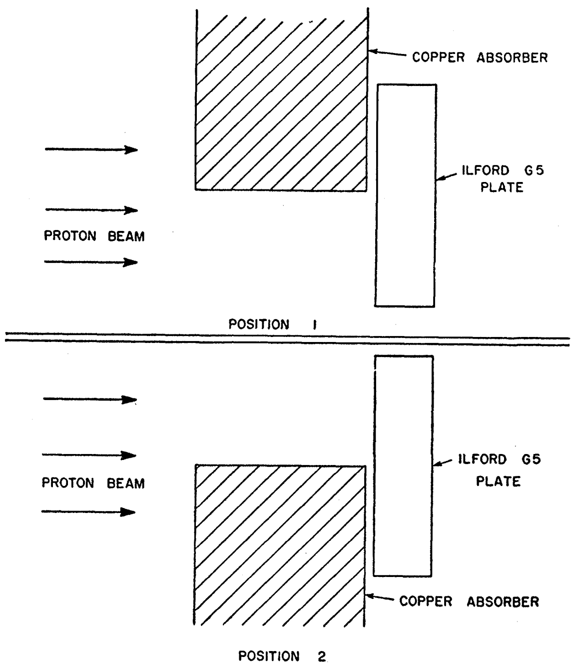


FIG. 2. Setup for determination of copper absorption.

distortion was not negligible, amounting to several degrees, and because it varied from plate to plate, a rough calibration of the mean direction was necessary for each plate position.

Many background particles in the plates were rejected on the basis of grain density. The grain density discrimination was done visually without grain count for most of the points, but before this procedure was adopted a careful grain count-angle study was carried out on one point. This work showed that the grain density distribution was sufficiently peaked to permit a qualitative distinction between the elastically scattered protons and other particles.

## II. RESULTS

The scattering cross section was measured using one effect plus background measurement preceded by a background run with the target box filled with warm helium and followed by another where the box contained cold hydrogen gas but no liquid hydrogen. The details of the background measurements are described in Appendix I. The set of three runs was done several times, but the data presented are from the last set. Figure 3 and Table I show the cross section obtained from this last set of plates. The values were corrected for absorption of the beam in the liquid hydrogen target and for other minor effects. These corrections are discussed in Appendixes II and III. The errors indicated represent the standard deviations for the points as determined by the counting statistics. The horizontal lines below the points indicate the angular resolution. The solid curve plotted is the sum of the Møller[1] formula, corrected for the resolution, and a smooth curve drawn through the points beyond the Coulomb region and extrapolated to smaller angles as a straight line. Also shown in Fig. 3 are the results of another experiment[2] in the same region of energy and angle. The agreement between them is fairly good although the present experiment indicates a slight minimum at about 9° in the center-of-mass system. The three points defining this minimum were taken from the same plate. Considering the difficulty in scanning and the large background at small angles, the minimum is not regarded as definitely established. A slight minimum appears in the cross section of Fischer and Goldhaber[3] at 330 Mev, but not in the curve of Holt, Kluyver, and Moore[4] at 380 Mev.

The present results, like those of Fischer and Goldhaber[3] and Holt, Kluyver, and Moore[4] indicate that there is some destructive interference between the nuclear scattering and Coulomb scattering amplitudes, but it is relatively weak at this energy.

## ACKNOWLEDGMENTS

The authors would like to thank Dr. Gerson Goldhaber for the plate holders used in this experiment and

TABLE I. Center-of-mass cross sections.

| Center-of-mass scattering angle (deg) | Center-of-mass scattering cross section (millibarns/steradian) | Ratio of net effect to background |
|---|---|---|
| 5.1 | 16.6 ±1.9 | 0.20 |
| 6.0 | 7.0 ±1.1 | 0.16 |
| 6.9 | 5.9 ±0.6 | 0.28 |
| 7.8 | 4.4 ±0.3 | 0.35 |
| 8.7 | 3.75±0.25 | 0.48 |
| 9.6 | 3.4 ±0.2 | 0.61 |
| 10.6 | 4.55±0.2 | 0.99 |
| 11.4 | 5.3 ±0.3 | 1.34 |
| 12.2 | 4.6 ±0.2 | 1.63 |
| 13.3 | 4.6 ±0.25 | 1.79 |
| 14.0 | 4.35±0.3 | 1.54 |
| 14.75 | 4.6 ±0.3 | 1.88 |
| 15.9 | 4.3 ±0.25 | 2.39 |
| 16.7 | 4.30±0.25 | 2.70 |
| 17.4 | 4.15±0.25 | 2.58 |
| 18.6 | 4.2 ±0.2 | 2.50 |
| 19.3 | 4.1 ±0.2 | 3.51 |
| 20.1 | 4.10±0.2 | 3.86 |
| 44° | 4.11±0.2 | |
| 90° | 3.53±0.17 | |

[1] C. Møller, Ann. Physik **14**, 531 (1932).

[2] R. B. Sutton, T. H. Fields, J. G. Fox, J. A. Kane, W. E. Mott, and R. A. Stallwood, Phys. Rev. **97**, 783 (1955); **109**, 1713–1715 (1958).

[3] David Fischer and Gerson Goldhaber, Phys. Rev. **95**, 1350 (1954).

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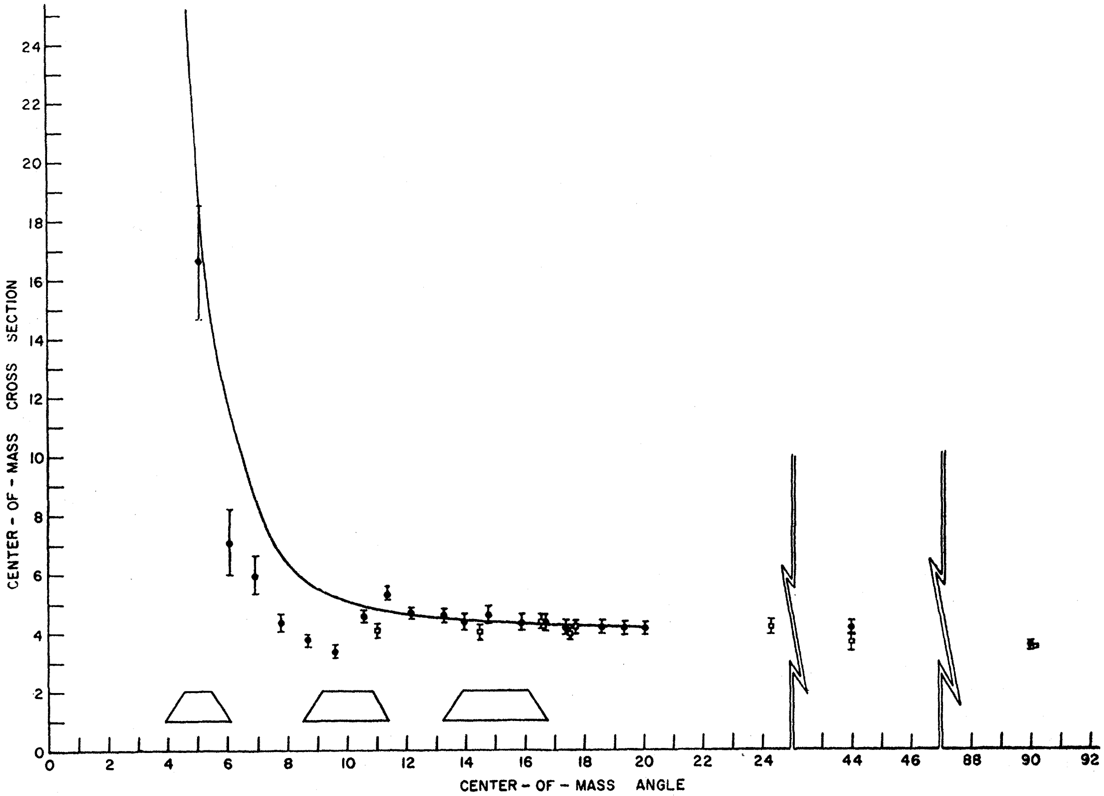


FIG. 3. Differential scattering cross section. Circles ⊙ show the results of the present experiment. Squares ⊡ indicate the measurements of Sutton *et al.* at the same energy. The curve is the sum of the Møller cross section and a constant nuclear cross section of 4.2 barns (gradually tapering off to 4.1 barns above 16°).

Dr. Arnold Clark for help during the initial stages of the experiment.

### APPENDIX I. BACKGROUND SUBTRACTION

The main body of the report indicates that two background measurements were made, the first for a target box containing helium at room temperature and the second with cold hydrogen gas. Experimentally these two backgrounds were found to differ by less than 10%.

The contribution of the gas in the empty target box to the scattered background can be estimated using the usual scattering cross section,

$$\sigma(\theta)=\frac{z^2e^4[1-\beta^2\sin^2(\theta/2)](1-\beta^2)}{4m^2v^4\sin^4(\theta/2)}.$$

At 6° c.m. neither the warm helium nor the cold hydrogen, assumed to be in equilibrium with liquid hydrogen at the boiling point 20.4°K, produced more than 2% of the observed background.

### APPENDIX II. NUCLEAR INTERACTION AND MULTIPLE SCATTERING IN THE LIQUID HYDROGEN

Assuming a total nuclear scattering cross section,[5] elastic and inelastic, of 27 mb for hydrogen, a calculation was made showing the proton beam to be attenuated by 6% in passing through the liquid hydrogen in the target. For small angle scattering up to 22° c.m. (or 11° lab) the scattered beam for almost all scattering locations in the target traverses the entire length of the hydrogen. Therefore, for all points in this region the background plus effect count was increased by 1.06.

For the points at 44° and 90° c.m. the average path length in the hydrogen was less than 20 inches. However, the total cross section was larger for the scattered beam because of its lower energy. Calculations were carried out on the mean corrections in the background plus effect value and they showed the correction to lie between 1.05 and 1.07.

Scattering in air and helium was evaluated for the large and smaller angle scattering. The reductions in intensity were found to be (0.7±0.1)% for air and 0.1% for helium. Multiple scattering in the liquid hydrogen influenced the differential cross section in this experiment in the Coulomb region in the usual way, namely to produce a broadening of the forward peak. The root mean square scattering angle was calculated to be 0.19°. There was another source of broadening in the experiment, namely the sizable volume of hydrogen from which the beam scattered,

[5] Wilmot N. Hess, Revs. Modern Phys. **30**, 368 (1958).

a volume of dimensions about 1 in.$\times\frac{1}{2}$ in.$\times$20 in. The mean angular spread from this source combined with the original beam spread is shown by the resolution lines in Fig. 3.

### APPENDIX III

Several minor effects in the experiment were estimated and found to be small. Nuclear absorption in the helium reduced the beam by only 0.1%, but for the large scattering angles (44° and 90° c.m.) where the protons traversed air rather than He the value was 0.7% to 0.9%. The root mean square scattering angle for the He in the bag was 0.02°, for air in the case of the larger angles 0.1°, for the 0.006 in. of aluminum in the container 0.043° and for styrofoam 0.024° for a 1-cm thick layer.

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# Mu-Mesonic Molecules. I. Three-Body Problem*

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An approximate method is developed for treating a generalized hydrogen-molecule ion in which two heavy particles have positive unit charges and one light particle has a negative unit charge. The expansion parameter of this approximation is the ratio of the light to the heavy mass. In first order, the method requires finding a solution to a pair of ordinary, second-order differential equations, which are coupled unless the masses of the heavy particles are equal. Explicit expressions for the coefficients in these equations are derived. The asymptotic forms of these coefficients for large nuclear separations give to first order the reduced-mas corrections to the binding energy of the light particle on either of the two heavy particles. The usual scattering theory is extended to obtain formulas for the various possible cross sections associated with this system. An iterative, variational technique for obtaining eigenvalues and eigenfunctions for bound states of the system is presented.

## I. INTRODUCTION

THE experimental observation of $\mu^-$-meson-induced fusion in a hydrogen bubble chamber[1] has led to an increased interest in the three-body system consisting of a light negatively charged particle in the presence of two heavier positively charged nuclei. This system, the generalized hydrogen molecular ion, has been treated in the past by the approximation of Born and Oppenheimer.[2] In this approximation the expansion parameter is the fourth root of the ratio of the mass of the light particle to that of the heavier particles. For electronic molecules this quantity is small ($\sim$1/7) and the approximation is sufficiently accurate to be useful in many calculations. For $\mu$-mesonic molecules, however, the corresponding value is nearly one ($\sim\frac{2}{3}$), and the approximation is open to question.

In this paper we develop a method based on a variational approximation to the wave function of this three-body system. Although this method has the same starting point as the Born-Oppenheimer approximation —namely, the solution for the motion of the light particle with the heavy ones held fixed—it leads to an expansion parameter that is the ratio of the masses themselves. In the present approximate treatment first-order terms in this parameter have been included, while second-order ones are ignored. When the masses of the two nuclei are not equal, it is essential that the first-order terms be included, because they lead to the distinctive features of the unequal-mass case. Thus, for example, the difference in binding energy of the light particle on one or the other of the two nuclei is contained in these terms; clearly, if the positions of the nuclei are fixed, their mass differences can play no role. In this unequal-mass case, it will be shown that the wave function of the system is obtained from the solution of a pair of coupled, ordinary, second-order differential equations in which the coupling terms come from the first-order corrections. On the other hand, if the masses of the two nuclei are equal, the pair of equations is uncoupled and the first-order terms serve only to improve the accuracy of the calculation. The development of the equations for the wave functions is given in Sec. II.

In Sec. III, the scattering states for these systems are treated. By use of the asymptotic behavior of the system of equations, explicit expressions for the elastic and exchange cross sections are derived. For unequal nuclear masses one obtains different expressions depending on whether the total energy is less than or

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