Current-Carrier Transport and Photoconductivity in Semiconductors with Trapping

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Fundamental differential equations are derived under the unrestricted approximation of electrical neutrality that admits trapping. Applied magnetic field is taken into account. The general transport equations derived hold without explicit reference to detailed trapping and recombination statistics. Modified ambipolar diffusivity, drift velocity, and lifetime function, which depend on two phenomenological differential "trapping ratios," apply in the steady state. The same diffusion length is shown to hold for both carriers, and a general "diffusion-length lifetime" is defined. Mass-action statistics are considered for cases of (one or) two energy levels. Certain "effective"—rather than physically proper—electron and hole capture and release frequencies or times that apply to concentration increments are defined, and a restriction from detailed balance to which they are subject is derived. Found widely useful is "capture concentration," the concentration of centers at equilibrium that are occupied times the fraction unoccupied. Criteria are given for minority-carrier trapping, recombination, and majority-carrier trapping, and for "shallow" and "deep" traps. Applications of the formulation include: the diffusion-length lifetime corresponding to the Shockley-Read electron and hole lifetimes, and that for recombination centers in the presence of (nonrecombinative) traps; linear and nonlinear steady-state and transient photoconductivity; the photomagnetoelectric effect; and drift of an injected pulse. The small- and large-signal nonlinearities that may occur with saturation of deep traps provide a single-level model for superlinearity. Photomagnetoelectric current is found to be decreased by minority-carrier trapping, through an increase in diffusion length. A simple general criterion is given for the local direction of drift of a concentration disturbance. With trapping, there may be "reverse drift," whose direction is normally that for the opposite conductivity type. With solutions of one type obtained for drift of an injected pulse, multiple trapping ultimately results in Gaussian mobile-carrier distributions which spread as if through diffusion and which drift at a fraction of the ambipolar velocity. With solutions of another type, related to reverse drift is the occurrence of local regions of mobile-carrier depletion which may in practice extend over appreciable distances.

1. INTRODUCTION

THE space charge associated with carrier injection in homogeneous semiconductors is frequently quite negligible, and, in phenomenological transport theory, implications of this local electrical neutrality have been worked out in some detail. However, the simple neutrality condition widely employed—that of constant excess of one mobile-carrier concentration over the other—is a restricted one that applies as an approximation in some cases. Upon injection, changes generally occur in concentrations of fixed charges associated with various impurities or crystal imperfections, including those on which equilibrium conductivity and those on which equilibrium lifetime, as a rule, largely depend. In a general sense, these concentration changes constitute *trapping*. Extending results previously reported,[1] this paper gives general ambipolar theory based on the unrestricted neutrality condition that admits trapping, with some theoretical applications to problems in transport and photoconductivity.[2]

In Sec. 2, fundamental differential equations are derived that take into account diffusion, drift, recombination and trapping and include an applied magnetic field. This section also contains: a specialization to the steady state, which exhibits how trapping (of arbitrary statistics) modifies recombination and the transport processes; definitions of certain "effective" frequencies and times that properly characterize trapping and recombination as they apply to concentration increments above thermal equilibrium; certain fundamental relations from detailed balance; and criteria for classifying centers with respect to their trapping and recombination properties.

In Sec. 3, the general ambipolar formulation is applied to investigate trapping in various connections. From theory for the steady state, diffusion lengths and lifetimes are evaluated, and the influence of trapping on the photomagnetoelectric (PME) effect is determined. Small- and large-signal steady-state nonlinearities are analyzed. A treatment of transient photoconductivity is given; the present formulation provides results of comparative formal simplicity. This treatment involves a formalism that recurs in the theory of time-dependent transport. An analysis is given of such transport, namely, the drift with trapping of an injected pulse.

2. GENERAL FORMULATION

The formulation is accomplished in two stages: In Sec. 2.1, general differential equations for the transport are derived along the lines of previous treatments.[3,4] These equations involve no specific reference to the detailed trapping and recombination statistics. In Sec. 2.2, the formulation is completed with equations for the time rates of change of concentrations of trapped carriers.

2.1 The Transport Equations

The unrestricted neutrality condition is that the total concentration of positive charges, the sum of the

[1] W. van Roosbroeck, Bull. Am. Phys. Soc. 2, 152 (1957).
[2] For a more detailed and extended treatment with further applications, see: W. van Roosbroeck, Bell System Tech. J. 39, 515 (1960).
[3] W. van Roosbroeck, Phys. Rev. 91, 282 (1953).
[4] W. van Roosbroeck, Phys. Rev. 101, 1713 (1956).

concentration p of mobile holes and the concentration $\hat{p}$ of all fixed positive charges, is equal to the corresponding concentration of negative charges:

$$m \equiv p+\hat{p} = n+\hat{n}. \tag{1}$$

For the total concentration m, two forms of continuity equation may be written which are extensions of the familiar (nonambipolar) continuity equations for holes and for electrons that apply for no trapping:

$$\begin{aligned} \partial m/\partial t &= \partial p/\partial t+\partial \hat{p}/\partial t = -e^{-1}\,\mathrm{div}\mathbf{I}_p+g-\mathfrak{R}_m \\ &= \partial n/\partial t+\partial \hat{n}/\partial t = e^{-1}\,\mathrm{div}\mathbf{I}_n+g-\mathfrak{R}_m. \end{aligned} \tag{2}$$

Here, $\mathbf{I}_p$ and $\mathbf{I}_n$ are the hole and electron current densities, and the volume generation rate function g is that for interband excitations. The volume rate $\mathfrak{R}_m$ is associated with trapping and recombination. It depends directly only on the various concentrations and not explicitly on coordinates and time; $\mathfrak{R}_m$ plus $\partial\hat{p}/\partial t$ and $\partial\hat{n}/\partial t$, respectively, gives volume rates $\mathfrak{R}_p$ and $\mathfrak{R}_n$ for p and n. The use of the same volume rate $\mathfrak{R}_m$ in each of Eqs. (2) is consistent with the neutrality condition and with the condition[5]

$$\mathrm{div}\mathbf{I}=0, \quad \mathbf{I}=\mathbf{I}_p+\mathbf{I}_n, \tag{3}$$

which applies in regions containing no sources or sinks of (total) current.

Hole and electron current densities that include the effect, for small Hall angles θ_p and θ_n, of steady applied magnetic field are given by[6]

$$\begin{aligned} \mathbf{I}_p &= \mathbf{I}_p{}^*+\theta_p\mathbf{I}_p{}^*\times\mathbf{k}, \\ \mathbf{I}_n &= \mathbf{I}_n{}^*+\theta_n\mathbf{I}_n{}^*\times\mathbf{k}, \end{aligned} \tag{4}$$

where $\mathbf{k}$ is a unit vector in the direction of the magnetic field and $\mathbf{I}_p{}^*$ and $\mathbf{I}_n{}^*$ are defined by[7]

$$\begin{aligned} \mathbf{I}_p{}^* &\equiv \sigma_p\mathbf{E}-eD_p\,\mathrm{grad}p, \\ \mathbf{I}_n{}^* &\equiv \sigma_n\mathbf{E}+eD_n\,\mathrm{grad}n. \end{aligned} \tag{5}$$

Equations (4) and (5) hold under the assumption of Boltzmann statistics. They result in

$$\begin{aligned} -\mathrm{div}\mathbf{I}_p &= -\mathrm{div}(\sigma_p\mathbf{E})+eD_p\,\mathrm{div}\,\mathrm{grad}p \\ &\quad -\theta_p[\mathrm{grad}\sigma_p,\mathbf{E},\mathbf{k}] = \mathrm{div}\mathbf{I}_n = \mathrm{div}(\sigma_n\mathbf{E}) \\ &\quad +eD_n\,\mathrm{div}\,\mathrm{grad}n+\theta_n[\mathrm{grad}\sigma_n,\mathbf{E},\mathbf{k}], \end{aligned} \tag{6}$$

in which the heavy brackets denote scalar triple products.

The ambipolar continuity equation for m is obtained by a procedure similar to that previously employed in the no-trapping case[3,4] except that, for the required generality, p and n are treated formally as unrelated variables: The respective forms of Eqs. (6) for $\mathrm{div}\mathbf{I}_p$ and $\mathrm{div}\mathbf{I}_n$ are introduced into Eqs. (2). The two forms for $\partial m/\partial t$ that result are then multiplied, respectively, by σ_n and σ_p and added, to eliminate $\mathrm{div}\mathbf{E}$; and $\mathbf{E}$ is replaced by the expression obtained by solving for $\mathbf{E}$ to the first order in Hall angles in

$$\begin{aligned} \mathbf{I} &= \sigma\mathbf{E}+e\,\mathrm{grad}(D_nn-D_p\hat{p})+(\theta_p\sigma_p+\theta_n\sigma_n)\mathbf{E}\times\mathbf{k} \\ &\quad -e\,\mathrm{grad}(\theta_pD_p\hat{p}-\theta_nD_nn)\times\mathbf{k}. \end{aligned} \tag{7}$$

Equation (7), the sum of the equations for $\mathbf{I}_p$ and $\mathbf{I}_n$, exhibits the respective drift, Dember, Hall, and PME contributions to total current density. To simplify the result for $\partial m/\partial t$, use is made of $\mathrm{curl}\mathbf{E}=0$; a time-dependent contribution to magnetic field from time dependence of $\mathbf{I}$ generally has quite negligible effect. Use is also made of the proportionality of the hole and electron mobilities μ_p and μ_n to the corresponding diffusion constants D_p and D_n in accordance with Einstein's relation.

The continuity equation for m that results[8] may be written in the form[9]

$$\begin{aligned} \partial m/\partial t-g+\mathfrak{R}_m &= -e^{-1}\,\mathrm{div}\mathbf{I}_{Dn}-e^{-1}\,\mathrm{div}\mathbf{I}_{Dp} \\ &\quad -\mathbf{v}_n\cdot\mathrm{grad}n-\mathbf{v}_p\cdot\mathrm{grad}p, \end{aligned} \tag{8}$$

which exhibits current densities $\mathbf{I}_{Dn}$ and $\mathbf{I}_{Dp}$ that involve the diffusion of electrons and holes, respectively, and velocity functions[10] $\mathbf{v}_n$ and $\mathbf{v}_p$ that give their drift:

$$\begin{aligned} \mathbf{I}_{Dn} &\equiv -e\sigma^{-1}\{\sigma_pD_n\,\mathrm{grad}n \\ &\quad +[\theta_p-\tfrac{1}{2}\theta(\sigma_p/\sigma)]\sigma_pD_n\,\mathrm{grad}n\times\mathbf{k}\}, \\ \mathbf{I}_{Dp} &\equiv -e\sigma^{-1}\{\sigma_nD_p\,\mathrm{grad}p \\ &\quad +[\theta_n+\tfrac{1}{2}\theta(\sigma_n/\sigma)]\sigma_nD_p\,\mathrm{grad}p\times\mathbf{k}\}, \\ \mathbf{v}_n &\equiv -e\mu_n\mu_p\sigma^{-2}p[\mathbf{I}-\theta(\sigma_p/\sigma)\mathbf{I}\times\mathbf{k}], \\ \mathbf{v}_p &\equiv e\mu_n\mu_p\sigma^{-2}n[\mathbf{I}+\theta(\sigma_n/\sigma)\mathbf{I}\times\mathbf{k}]. \end{aligned} \tag{9}$$

Here $\theta\equiv\theta_p-\theta_n$ is the sum of the magnitudes of the Hall angles. Tensor ambipolar diffusivities for electrons and holes under the magnetic field may be written from $\mathbf{I}_{Dn}$ and $\mathbf{I}_{Dp}$. The vector-product contributions to these current densities involve PME currents as well as the combined action of the magnetic and Dember fields. It is easily seen that the divergences of these vector products vanish for the linear small-signal case; and,

[5] W. van Roosbroeck, Bell System Tech. J. **29**, 560 (1950).

[6] See reference 4. Small Hall angles are assumed partly because appreciable magnetoresistance is otherwise involved. As indicated in this reference, arbitrary Hall angles (and injection levels) could suitably be taken into account by theory involving the phenomenological magnetoresistance without added carriers. Note that small or moderate magnetic field will generally not affect the occupation probabilities for the traps.

[7] The notation employed is consistent with that of references 3 and 4.

[8] This equation specialized to the case of $\Delta\hat{p}=0$ and no applied magnetic field can be shown to be consistent with a continuity equation for Δp derived under the assumption of a common lifetime function for electrons and holes; E. S. Rittner, *Photoconductivity Conference*, edited by R. G. Breckenridge, B. R. Russell, and E. E. Hahn (John Wiley & Sons, Inc., New York, 1956), Chap. IIIA.

[9] Compare with Eq. (21) of reference 2.

[10] In Eqs. (133) of reference 2, velocities $\mathbf{v}_n$ and $\mathbf{v}_p$ (written with carets) are define which are for the linear small-signal case and no applied magnetic field.

containing $[\text{grad}n, \text{grad}p, \mathbf{k}]$ as a factor, they vanish also whenever the concentration gradients are collinear, as in the no-trapping case, or in the steady state, or in certain cases of simple flow geometry.

The neutrality and continuity equations in conjunction with equations for the time rates of change of concentrations of fixed charges at each of the various trapping levels are equations equal in number to the number of unknown concentrations. These equations accordingly suffice as fundamental differential equations provided $\mathbf{I}$ is a known function of the space coordinates and time. If $\mathbf{I}$ must be determined from boundary conditions, then use is made of the fundamental differential equation, Eq. (3), which expresses the solenoidal property of $\mathbf{I}$. This may be written in terms of $\mathbf{E}$ or electrostatic potential V as additional dependent variable by use of Eq. (7); and $\mathbf{I}$ may then similarly be eliminated from the continuity equation.

The current densities in ambipolar form obtained by eliminating $\mathbf{E}$ may be written as

$$\begin{aligned}\mathbf{I}_p &= (\sigma_p/\sigma)\mathbf{I}+\mathbf{I}^{\equiv}=\mathbf{I}_{p0}+\Delta\mathbf{I},\\ \mathbf{I}_n &= (\sigma_n/\sigma)\mathbf{I}-\mathbf{I}^{\equiv}=\mathbf{I}_{n0}-\Delta\mathbf{I},\end{aligned} \tag{10}$$

with

$$\begin{aligned}\mathbf{I}^{\equiv} \equiv &-e\sigma^{-1}(\sigma_p D_n \,\text{grad}n+\sigma_n D_p \,\text{grad}p)\\ &+\theta(\sigma_n\sigma_p/\sigma^2)\mathbf{I}\times\mathbf{k}+e\sigma^{-2}(\theta_p\sigma_n+\theta_n\sigma_p)\\ &\cdot(\sigma_p D_n \,\text{grad}n+\sigma_n D_p \,\text{grad}p)\times\mathbf{k},\end{aligned} \tag{11}$$

and

$$\begin{aligned}\Delta\mathbf{I} \equiv\ & e^2\mu_n\mu_p\sigma_0^{-1}\sigma^{-1}(n_0\Delta p-p_0\Delta_n)\mathbf{I}\\ &-e\sigma^{-1}(\sigma_p D_n \,\text{grad}n+\sigma_n D_p \,\text{grad}p)\\ &+\theta e^2\mu_n\mu_p\sigma^{-2}[\sigma_0^{-1}(\sigma_{n0}-\sigma_{p0})(n_0\Delta p-p_0\Delta n)\\ &-n_i^2(\Delta\sigma/\sigma_0)^2+\Delta n\Delta p]\mathbf{I}\times\mathbf{k}+e\sigma^{-2}(\theta_p\sigma_n+\theta_n\sigma_p)\\ &\cdot(\sigma_p D_n \,\text{grad}n+\sigma_n D_p \,\text{grad}p)\times\mathbf{k}.\end{aligned} \tag{12}$$

Subscripts zero denote equilibrium values, and Δn, Δp, and $\Delta\sigma$ are increments above equilibrium; $n_i^2=n_0p_0$ is independent of conductivity. The current densities for equilibrium carrier concentrations are given by

$$\begin{aligned}\mathbf{I}_{p0} &= (\sigma_{p0}/\sigma_0)\mathbf{I}+\theta e^2\mu_n\mu_p n_i^2\sigma_0^{-2}\mathbf{I}\times\mathbf{k},\\ \mathbf{I}_{n0} &= (\sigma_{n0}/\sigma_0)\mathbf{I}-\theta e^2\mu_n\mu_p n_i^2\sigma_0^{-2}\mathbf{I}\times\mathbf{k}.\end{aligned} \tag{13}$$

The current density $\mathbf{I}^{\equiv}$ represents the amount by which the electron and hole flow densities exceed the corresponding drift flow densities under $\mathbf{I}/\sigma$, the ohmic contribution to the electrostatic field. The current density $\Delta\mathbf{I}$ is the current density of added mobile carriers: For given $\mathbf{I}$, it represents the amount by which the electron and hole flow densities exceed their values for no added carriers. It is easily seen that the right-hand member of the continuity equation, Eq. (8), may be written as $-e^{-1}\,\text{div}\Delta\mathbf{I}-\theta e\mu_n\mu_p n_i^2\sigma_0^{-2}\,\text{curl}\mathbf{I}\cdot\mathbf{k}$, the second term of which is a contribution from PME circulating current,[4] associated with diffusion under the magnetic field.

2.11 Formulation for the Steady State

A number of results for the steady state can be established from the general differential equations without specifying in detail the trapping and recombination statistics. Differential trapping ratios

$$r_n \equiv d\hat{n}/dm, \quad r_p \equiv d\hat{p}/dm, \tag{14}$$

are introduced. These apply since, in the steady state, $\hat{n}$ and $\hat{p}$ each depend directly only on total concentration m of negative or positive charges. In the immediate context, r_n and r_p will be considered simply as factors which depend in general on m and which, multiplying $\text{grad}m$, give $\text{grad}\hat{n}$ and $\text{grad}\hat{p}$, respectively. They apply, of course, for any number of trapping levels.

With Eqs. (14), it follows from Eqs. (8) and (9) that the continuity equation for the steady state may be written as

$$\text{div}(D'\,\text{grad}\Delta m)-\mathbf{v}'\cdot\text{grad}\Delta m+\Delta g-\Delta m/\tau_m=0, \tag{15}$$

in which D' and $\mathbf{v}'$ are modified ambipolar diffusivity and drift velocity given by

$$\begin{aligned}D' &\equiv \sigma^{-1}[(1-r_p)D_p\sigma_n+(1-r_n)D_n\sigma_p]\\ &= kT\mu_n\mu_p\sigma^{-1}[(1-r_p)n+(1-r_n)p],\\ \mathbf{v}' &\equiv e\mu_n\mu_p\sigma^{-2}\{[(1-r_p)n-(1-r_n)p]\mathbf{I}\\ &\quad+\theta(e/\sigma)[(1-r_p)\mu_n n^2+(1-r_n)\mu_p p^2]\mathbf{I}\times\mathbf{k}\},\end{aligned} \tag{16}$$

and in which the net generation rate $g-\mathcal{R}_m$ has been written as the increment $\Delta g-\Delta m/\tau_m$ in this rate over thermal equilibrium, with Δg and Δm the corresponding increments in g and m and τ_m a lifetime function for Δm. The modified diffusivity and velocity do not apply to time-dependent cases; $\mathbf{v}'$ would, for example, give the effect of applied field on apparent diffusion length, but is not, for example, drift velocity for an injected pulse. The current densities given by Eqs. (10) may be written for the steady state in accordance with

$$\begin{aligned}\mathbf{I}^{\equiv} = &-eD'\,\text{grad}\Delta m+\theta(\sigma_n\sigma_p/\sigma^2)\mathbf{I}\times\mathbf{k}\\ &+e\sigma^{-1}(\theta_p\sigma_n+\theta_n\sigma_p)D'\,\text{grad}\Delta m\times\mathbf{k}.\end{aligned} \tag{17}$$

The equilibrium lifetimes for electrons and holes differ in general, but are nevertheless always associated with the same diffusion length. This result follows readily from Eq. (15), whose linear small-signal form is

$$D_0'\,\text{div}\,\text{grad}\Delta m-\mathbf{v}_0'\cdot\text{grad}\Delta m+\Delta g-\Delta m/\tau_m=0, \tag{18}$$

the zero subscripts denoting thermal-equilibrium values. The lifetime function τ_m is here constant; and since Δn and Δp now equal $(1-r_n)\Delta m$ and $(1-r_p)\Delta m$, with r_n and r_p the thermal-equilibrium trapping ratios, Eq. (18) implies

$$\begin{aligned}(1-r_n)^{-1}D_0'\,\text{div}\,\text{grad}\Delta n-(1-r_n)^{-1}\mathbf{v}_0'\cdot\text{grad}\Delta n\\ +\Delta g-\Delta n/(1-r_n)\tau_m=0,\end{aligned} \tag{19}$$

for electrons and a similar equation for holes. Thus, for Δn the lifetime is τ_m multiplied by $(1-r_n)$, while—as may be established in greater generality from Eqs. (14) and (15)—the diffusivity and velocity are those for Δm multiplied by the reciprocal of this factor, and similarly for Δp. It follows, in particular, that the product of equilibrium diffusivity and lifetime, which is the square of L_0, the diffusion length, is the same for Δn, Δp, and Δm, independently of the particular trapping and recombination statistics.[1] A "diffusion-length lifetime" τ_0, based on the unmodified ambipolar diffusivity[3] $D_0=kT\mu_n\mu_p(n_0+p_0)/\sigma_0$ may accordingly be defined by[11]

$$\begin{aligned}\tau_0\equiv L_0^2/D_0&=(D_0'/D_0)\tau_m\\&=[1-(r_pn_0+r_np_0)/(n_0+p_0)]\tau_m\\&=(n_0\tau_p+p_0\tau_n)/(n_0+p_0),\end{aligned}\quad(20)$$

in which τ_p and τ_n are the equilibrium lifetimes for Δp and Δn. The diffusion length and lifetime τ_0 that correspond to the (equilibrium) Shockley-Read electron and hole lifetimes as well as the τ_0 for recombination in the presence of nonrecombinative traps are evaluated in Sec. 3.

2.2 Mass-Action Theory

Relationships of the mass-action type provide a simple[12] and general[13] basis for trapping and recombination. Levels from two types of centers will be considered and, partly by way of notational convention, these will be taken as acceptor and donor levels. This case, involving both negative and positive fixed charges, is the simplest for which both steady-state trapping ratios occur. With suitable interpretation of the notation, the mass-action equations for this case apply to (one or) two kinds of single-level centers in general. An extension for centers of a single type but with two energy levels will also be given. In multilevel cases, two successive levels generally suffice for analysis of the trapping at a given time. Levels appreciably lower and higher than these may contribute to recombination, but will not contribute to trapping, since the lower ones remain full (or else saturated) and the higher ones empty.

2.21 Single-Level Centers of Two Types

Mass-action equations for the two types of single-level centers present together are:

$$\begin{aligned}g-\mathfrak{R}_m&=g-C_inp-C_{p1}[p\hat{n}-p_1(\mathfrak{N}_1-\hat{n})]\\&\quad-C_{n2}[n\hat{p}-n_2(\mathfrak{N}_2-\hat{p})],\\\partial\hat{n}/\partial t&=\mathfrak{R}_n-\mathfrak{R}_m=C_{n1}[n(\mathfrak{N}_1-\hat{n})-n_1\hat{n}]\\&\quad-C_{p1}[p\hat{n}-p_1(\mathfrak{N}_1-\hat{n})],\\\partial\hat{p}/\partial t&=\mathfrak{R}_p-\mathfrak{R}_m=-C_{n2}[n\hat{p}-n_2(\mathfrak{N}_2-\hat{p})]\\&\quad+C_{p2}[p(\mathfrak{N}_2-\hat{p})-p_2\hat{p}].\end{aligned}\quad(21)$$

The first equation gives $\mathfrak{R}_m$, and it (as well as the other two) is obtained by considering the photoconductive case of uniform concentration and no transport, $g-\mathfrak{R}_m$ being the contribution to $\partial m/\partial t$ which does not involve transport. Four processes are taken into account for each type of center. In the second equation, for example, the term $C_{p1}p\hat{n}$ is the volume rate of neutralization of fixed negative charges by holes; C_{p1} is a phenomenological capture coefficient which depends in general on temperature and not on concentration. The second term in the same brackets gives the rate for the inverse process, $C_{p1}p_1$ being the emission coefficient for hole emission from a neutral acceptor center. Here $\mathfrak{N}_1$ is the total concentration of the acceptor centers, and the concentration p_1, constant at given temperature, is defined by the condition that the quantity in brackets vanish at thermal equilibrium, in accordance with detailed balance. The brackets to the left relate to the interactions of the same centers with electrons, the term $C_{n1}n(\mathfrak{N}_1-\hat{n})$ being the volume rate of capture of electrons by the neutral acceptor centers, and $C_{n1}n_1$ the coefficient for electron emission from the charged ones. The concentrations n_1 and p_1 are those of the Hall-Shockley-Read theory,[14,15] and are here introduced without explicit reference to Boltzmann statistics. The third equation expresses the dependence of $\partial\hat{p}/\partial t$ on the analogous processes for the donor centers. In the first equation, which includes the rate C_inp of direct electron-hole recombination, are involved only interactions which change the total concentration m.

Though written symmetrically for fixed charges of both signs, Eqs. (21) may formally be transformed so as to apply to two types of donor or acceptor centers. This possibility is related to the circumstance that the fixed charges are not properly considered as trapped carriers in that the trapping processes are manifest through changes in fixed-charge concentrations rather than in these concentrations themselves. For example, centers of the acceptor type function as electron or hole traps according to whether the concentration of the charged centers increases or decreases with carrier injection.

For theoretical applications, it is desirable to replace Eqs. (21) by equations in concentration increments above thermal equilibrium and to define, from the latter equations, suitable capture and release frequencies and times for mobile electrons and holes.

[11] R. N. Zitter, Phys. Rev. **112**, 852 (1958), discusses phenomenological lifetime for any model derived from the PME effect (in the thick slab). This is the same as τ_0, and Zitter relates it to a diffusion length.

[12] A. Hoffmann, *Halbleiterprobleme*, edited by W. Schottky (Friedrich Vieweg und Sohn, Braunschweig, 1955), Vol. II, Chap. 5. See also; E. Spenke, *Elektonische Halbleiter* (Springer-Verlag, Berlin, 1955), pp. 304–307.

[13] F. W. G. Rose, Proc. Phys. Soc. (London) **B71**, 699 (1958).

[14] R. N. Hall, Phys. Rev. **83**, 228 (1951); **87**, 387 (1952).

[15] W. Shockley and W. T. Read, Phys. Rev. **87**, 835 (1952).

Subtracting from Eqs. (21) the corresponding thermal-equilibrium equations, in which the time derivatives and the quantities in the various square brackets are zero, gives equations for $\Delta g-\Delta\mathfrak{R}_m$, $\partial\Delta\hat{n}/\partial t$, and $\partial\Delta\hat{p}/\partial t$. From these, if volume generation and direct recombination are neglected, it follows that the respective contributions to $\partial\Delta n/\partial t$ and $\partial\Delta p/\partial t$ other than the terms involving transport processes as such are

$$\begin{aligned}-\Delta\mathfrak{R}_n&=-\Delta\mathfrak{R}_m-\partial\Delta\hat{n}/\partial t\\&=-C_{n1}[(\mathfrak{N}_1-\hat{n}_0)\Delta n-n_0\Delta\hat{n}-\Delta n\Delta\hat{n}]+C_{n1}n_1\Delta\hat{n}\\&\quad-C_{n2}[\hat{p}_0\Delta n+n_0\Delta\hat{p}+\Delta n\Delta\hat{p}]+C_{n2}n_2\Delta(\mathfrak{N}_2-\hat{p}),\\-\Delta\mathfrak{R}_p&=-\Delta\mathfrak{R}_m-\partial\Delta\hat{p}/\partial t\\&=-C_{p1}[\hat{n}_0\Delta p+p_0\Delta\hat{n}+\Delta p\Delta\hat{n}]+C_{p1}p_1\Delta(\mathfrak{N}_1-\hat{n})\\&\quad-C_{p2}[(\mathfrak{N}_2-\hat{p}_0)\Delta p-p_0\Delta\hat{p}-\Delta p\Delta\hat{p}]+C_{p2}p_2\Delta\hat{p}.\end{aligned}\tag{22}$$

In Eqs. (22), the magnitudes of the contributions involving brackets are capture rates, while the remaining terms on the right are release rates. The capture and release frequencies, introduced in accordance with

$$\begin{aligned}-\Delta\mathfrak{R}_n&=-\nu_{tn1}\Delta n+\nu_{gn1}\Delta\hat{n}\\&\qquad-\nu_{tn2}\Delta n+\nu_{gn2}\Delta(\mathfrak{N}_2-\hat{p})\\-\Delta\mathfrak{R}_p&=-\nu_{tp1}\Delta p+\nu_{gp1}\Delta(\mathfrak{N}_1-\hat{n})\\&\qquad-\nu_{tp2}\Delta p+\nu_{gp2}\Delta\hat{p},\end{aligned}\tag{23}$$

may be suitably identified by comparison with Eqs. (22). In both sets of equations, the top and bottom rows of each right-hand member give contributions associated, respectively, with the acceptor and donor centers.

The capture and release frequencies must evidently entail concentration dependence. As will appear, Eqs. (23) do not impose unique definitions with respect to this dependence, while the uniquely determined constant frequencies that apply near thermal equilibrium are certain "effective" rather than physically proper quantities. These circumstances result because the capture rates, as they appear in Eqs. (22), cannot be written with Δn or Δp as a factor and thus expressed in terms of capture frequencies. To obtain the physically proper capture frequencies would necessitate solution of the particular problem; they would depend in general on coordinates and time. The contributions to the capture rates that contain $\Delta\hat{n}$ and $\Delta\hat{p}$ as factors are associated, however, with trap saturation: These contributions, for carriers of given charge, represent the decreases and increases in capture rate with the filling of centers that assume, respectively, the same and the opposite charges. They may, in a phenomenological sense, be deleted from the capture rates and assigned to the release rates, by which the difference between these rates for centers of each type remains unchanged. Note that the quadratic terms also contain Δn or Δp as a factor; assigning them entirely to the release rates is a matter of convenience. The "effective" capture and release frequencies and times are accordingly as follows:

Electron capture by neutral acceptors:

$$\nu_{tn1}\equiv\tau_{tn1}^{-1}\equiv C_{n1}(\mathfrak{N}_1-\hat{n}_0),$$

Electron release from charged acceptors:

$$\nu_{gn1}\equiv\tau_{gn1}^{-1}\equiv C_{n1}(n+n_1),$$

Hole capture by charged acceptors:

$$\nu_{tp1}\equiv\tau_{tp1}^{-1}\equiv C_{p1}\hat{n}_0,$$

Hole release from neutral acceptors:

$$\nu_{gp1}\equiv\tau_{gp1}^{-1}\equiv C_{p1}(p+p_1),$$

Electron capture by charged donors: (24)

$$\nu_{tn2}\equiv\tau_{tn2}^{-1}\equiv C_{n2}\hat{p}_0,$$

Electron release from neutral donors:

$$\nu_{gn2}\equiv\tau_{gn2}^{-1}\equiv C_{n2}(n+n_2),$$

Hole capture by neutral donors:

$$\nu_{tp2}\equiv\tau_{tp2}^{-1}\equiv C_{p2}(\mathfrak{N}_2-\hat{p}_0),$$

Hole release from charged donors:

$$\nu_{gp2}\equiv\tau_{gp2}^{-1}\equiv C_{p2}(p+p_2).$$

Note, for example, that ν_{tn1} is the "effective" average frequency per electron of electron capture by a neutral acceptor center and hence the reciprocal of the corresponding electron capture or trapping time, τ_{tn1}; and ν_{gn1} is the "effective" average frequency per charged center of electron release from a charged acceptor center and hence the reciprocal of the corresponding electron release time, τ_{gn1}. The saturation terms that originate from the true capture rates appear as the contributions from n and p in the "effective" release frequencies, while the "effective" capture frequencies do not depend on the injection level.

These "effective" quantities are generally the ones on which theoretical expressions depend. For example, for the decay of photoconductivity associated with trapped minority carriers, with recombination of comparatively short lifetime τ_3 entirely in other centers, for which decay times are[16] a release time τ_g for nearly full traps and τ_g plus the multiple-trapping time $(\tau_g/\tau_t)\tau_3$ for nearly empty traps, τ_t and τ_g are correctly identified as equilibrium values of "effective" capture and release times. Thus, τ_g is in general not a constant for the traps but depends also on conductivity, which should be taken into account in calculating the trapping level from the product of τ_g and the capture cross section.

[16] J. A. Hornbeck and J. R. Haynes, Phys. Rev. **97**, 311 (1955); J. R. Haynes and J. A. Hornbeck, *Photoconductivity Conference*, Chap. IIIF (reference 8).

2.211 Thermal-Equilibrium Relationships

The definitions

$$\begin{aligned} n_1 &\equiv n_0(\mathfrak{N}_1-\hat{n}_0)/\hat{n}_0, \quad p_1 \equiv p_0\hat{n}_0/(\mathfrak{N}_1-\hat{n}_0), \\ n_2 &\equiv n_0\hat{p}_0/(\mathfrak{N}_2-\hat{p}_0), \quad p_2 \equiv p_0(\mathfrak{N}_2-\hat{p}_0)/\hat{p}_0, \end{aligned} \tag{25}$$

are required by detailed balance. It is evident from these equations that

$$n_1p_1=n_2p_2=n_0p_0=n_i^2, \tag{26}$$

hold, where n_i is the thermal-equilibrium electron or hole concentration in intrinsic material. Note that Eqs. (26) state, in effect, that the product—$(C_{n1}n_1)(C_{p1}p_1)$ or $(C_{n2}n_2)(C_{p2}p_2)$—of the electron and hole emission coefficients equals n_i^2 times the product of the corresponding capture coefficients.[17] It is readily found from Eqs. (25) that fractions of charged acceptor and donor centers are given, respectively, by

$$\hat{n}_0/\mathfrak{N}_1=(1+\alpha_{10}^{-1})^{-1}, \quad \hat{p}_0/\mathfrak{N}_2=(1+\alpha_{20}^{-1})^{-1}, \tag{27}$$

with

$$\alpha_{10}\equiv n_0/n_1=p_1/p_0, \quad \alpha_{20}\equiv p_0/p_2=n_2/n_0. \tag{28}$$

Through familiar considerations involving equilibrium Boltzmann statistics, the concentrations n_1 or p_1 (and n_2 or p_2) have been shown to equal electron concentration in the conduction band or hole concentration in the valence band for the Fermi level coincident with the energy level of the centers.[15] The relationship

$$\begin{aligned} n_1=n_i^2/p_1&=n_i\exp[(\mathcal{E}_1-\mathcal{E})/kT] \\ &=n_i\exp[e(\Phi-\Psi_1)/kT] \end{aligned} \tag{29}$$

for acceptor centers is here employed, and a similar one for donor centers. Here $\Psi_1\equiv-e^{-1}\mathcal{E}_1$ and $\Phi\equiv-e^{-1}\mathcal{E}$ are the equivalent electrostatic potentials of the energy level $\mathcal{E}_1$ of the centers and the Fermi energy $\mathcal{E}$ for intrinsic material. This relationship is more phenomenological than those involving the energies of the conduction- and valence-band edges and which give n_1 and p_1 in units of the effective densities of states in the bands. Note that the temperature dependence of the energy gap is involved through n_i, while the difference between the effective densities of states or the effective masses with nonspherical energy surfaces in momentum space is reflected simply in a difference between Φ and the midgap potential. If statistical weights associated with spin degeneracy are taken into account, then the definitions of Eqs. (25) are of course retained, but Eqs. (29) are modified. The right-hand members (for n_1) are multiplied by 2; the exponentials for p_1 are multiplied by $\frac{1}{2}$. In the similar result for donor centers, the exponentials for n_2 and p_2 are multiplied by $\frac{1}{2}$ and 2, respectively. For given n_1 and n_2, these modifications[18] produce comparatively minor changes in $\mathcal{E}_1$ and $\mathcal{E}_2$ or Ψ_1 and Ψ_2.

The four effective trapping and release times or frequencies for each type of center satisfy a fundamental restriction, namely

$$\frac{\tau_{gnj}\tau_{tpj}}{\tau_{tnj}\tau_{gpj}}=\frac{\nu_{tnj}\nu_{gpj}}{\nu_{gnj}\nu_{tpj}}=\frac{p_0}{n_0}\frac{1+\Delta p/(p_0+p_j)}{1+\Delta n/(n_0+n_j)}, \quad j=1,2. \tag{30}$$

Thus, only three are independent. As will appear, this restriction is widely useful for calculations and physical interpretations. It is essentially a consequence of detailed balance: For thermal equilibrium, it follows readily from the definitions of Eqs. (25), while the factor on the right that depends on Δn and Δp results simply from the concentration dependence of the effective release frequencies.

2.212 Trapping and Recombination Ranges; Shallow and Deep Traps

Three linear small-signal ranges, respectively, characterized primarily by minority-carrier trapping, recombination, and majority-carrier trapping may be defined for each type of center by use of Eq. (30). The "minority-carrier trapping range" is defined by the condition that the equilibrium minority-carrier to majority-carrier release frequency ratio exceeds unity. In p-type material, this ratio, ν_{gnj}/ν_{gpj}, is $C_{nj}n_j/C_{pj}p_0=C_{nj}n_0/C_{pj}p_j$, from Eqs. (24), (26), (27), and (28); and from Eq. (30), ν_{tnj}/ν_{tpj} is larger by the factor p_0/n_0. The "majority-carrier trapping range" is defined by the condition that the majority- to minority-carrier capture frequency ratio exceeds unity, for which the equilibrium majority- to minority-carrier release frequency ratio is larger by the factor p_0/n_0 for p-type material, or n_0/p_0 for n type. The "recombination range" is defined as that not included in either trapping range. Thus, the recombination range is given by $n_0/n_j=p_j/p_0\leq C_{nj}/C_{pj}\leq p_j/n_0=p_0/n_j$ for p-type material, the electron-trapping range by $C_{nj}/C_{pj}>p_j/n_0=p_0/n_j$, and the hole-trapping range by $C_{nj}/C_{pj}<p_j/p_0=n_0/n_j$. A "minority-carrier capture range," which includes the trapping and recombination ranges, may be defined by $\nu_{tnj}/\nu_{tpj}>1$. Similar results, obtainable by interchanging n and p, hold for n-type material.

The three ranges may be specified in terms of the equality densities. These are the equilibrium carrier concentrations for the Fermi level coincident with the equality level.[19] They are defined in the present context

[17] A. Hoffmann, reference 12; Chih-Tang Sah and W. Shockley, Phys. Rev. **109**, 1103 (1958); W. Shockley, Proc. Inst. Radio Engrs. **46**, 973 (1958).

[18] These are derived from: F. W. G. Rose, Proc. Phys. Soc. (London) **B70**, 801 (1957). See also; P. T. Landsberg, Proc. Phys. Soc. (London) **A65**, 604 (1952); C. H. Champness, Proc. Phys. Soc. (London) **B69**, 1335 (1956).

[19] This is the Fermi level for which the (equilibrium) rates of electron and hole capture and release are all equal: Chih-Tang Sah and W. Shockley, W. Shockley, reference 17. The equality level is similar in purport to the demarcation level of Rose, which is the trapping level for which the rates are equal: A. Rose, Phys. Rev. **97**, 322 (1955); *Progress in Semiconductors*, edited by A. F. Gibson (John Wiley & Sons, Inc., New York, 1957), Vol. II, pp. 111–136.

by

$$n_j^* \equiv C_{pj}p_j/C_{nj} = p_0\nu_{tpj}/\nu_{tnj} = n_0\nu_{gpj}/\nu_{gnj},$$
$$p_j^* \equiv C_{nj}n_j/C_{pj} = n_0\nu_{tnj}/\nu_{tpj} = p_0\nu_{gnj}/\nu_{gpj}, \quad (31)$$

in which the release frequencies are equilibrium values. Thus, the recombination range is given by $n_0 \leq p_j^* \leq p_0$ or $p_0 \geq n_j^* \geq n_0$ for p-type material, the electron-trapping range by $n_j^* < n_0$ or $p_j^* > p_0$, and the hole-trapping range by $n_j^* > p_0$ or $p_j^* < n_0$, and similarly for n-type material. The ranges may evidently also be specified in terms of the equality level, the Fermi level $\mathcal{E}$ for intrinsic material, the actual Fermi level $\mathcal{E}_F$, and the "reflected Fermi level" $\mathcal{E}_F' \equiv 2\mathcal{E} - \mathcal{E}_F$, the reflection of $\mathcal{E}_F$ about $\mathcal{E}$: For the recombination range, the equality level is between $\mathcal{E}_F$ and $\mathcal{E}_F'$; for the minority-carrier trapping range, it is between $\mathcal{E}_F$ and the edge of the majority-carrier band; and for the majority-carrier trapping range, it is between $\mathcal{E}_F'$ and the edge of the minority-carrier band. Note that if the capture coefficients are equal, then $n_j^* = p_j$ (or $p_j^* = n_j$) holds and the respective trapping ranges are given by conditions on the trapping level $\mathcal{E}_j$ obtained by interchanging those on the equality level.

The volume rates of electron and of hole transitions at equilibrium are, respectively, $C_{n1}n_0(\mathfrak{N}_1 - \hat{n}_0) = C_{n1}n_1\hat{n}_0 = n_0\nu_{tn1}$ and $C_{p1}p_0\hat{n}_0 = C_{p1}p_1(\mathfrak{N}_1 - \hat{n}_0) = p_0\nu_{tp1}$ for acceptor-type centers. From Eq. (30), these rates are proportional to ν_{gn1} and ν_{gp1}. Hence each definition given for a trapping range insures that the transition rate at equilibrium for the particular carriers is the larger, and also that the transition rate ν_{tn1} or ν_{tp1} per mobile carrier is the larger too. The definitions for minority- and majority-carrier trapping reflect the circumstance that a transition rate will be the larger if either the cross section or the concentration of the particular carriers is sufficiently large. The recombination range is that for which a larger transition rate per mobile minority carrier is associated with a total transition rate for majority carriers which is the larger.

For shallow minority-carrier traps, since relatively few are occupied by minority carriers at equilibrium so that they can capture majority carriers, the condition for the minority-carrier trapping range may be met even though the capture coefficients are comparable in magnitude. For deep traps, since relatively few can capture minority carriers, the minority-carrier trapping generally requires a minority-carrier capture coefficient considerably the larger. Suitable conditions for "shallow" traps and "deep" traps are, in view of the condition on C_{nj}/C_{pj} for the electron-trapping range, respectively, $p_j \ll n_0$ (or $n_j \gg p_0$) and $n_j \ll p_0$ (or $p_j \gg n_0$) in p-type material. That is, "shallow" and "deep" traps for minority carriers are appreciably removed from the reflected Fermi level, $\mathcal{E}_F'$, towards the edges of the minority- or majority-carrier bands, respectively. Similarly, for majority-carrier trapping, "shallow" and "deep" traps are appreciably removed from the Fermi level, $\mathcal{E}_F$, towards the edges of the majority- or minority-carrier bands, respectively. The proper criteria are essentially that $\mathcal{E}_F'$ separates the "shallow" and "deep" traps for minority carriers and $\mathcal{E}_F$ separates them for majority carriers. Thus, for minority carriers, levels in extrinsic material considerably shallower than the midgap may still be "deep" levels.

2.22 Centers with Two Energy Levels

The formalism for centers of two types is readily modified to yield equations for one type of center with two energy levels. With the assumption that the centers can each assume single negative or positive charge or be neutral, $\hat{n}$ and $\hat{p}$ denote concentrations of centers in the respective charged states. It is thus clear that the fundamental mass-action equations for this case are formally the same as Eqs. (21) with the modification that both $\mathfrak{N}_1 - \hat{n}$ and $\mathfrak{N}_2 - \hat{p}$ are replaced by $\mathfrak{N} - \hat{n} - \hat{p}$, where $\mathfrak{N}$ is the total concentration of the centers. The equations in concentration increments that result if direct recombination is neglected are accordingly

$$\begin{aligned}
\Delta g - \Delta\mathfrak{R}_m &= \Delta g - (\nu_{tp1} + \nu_{tn2})\Delta m \\
&\quad + (\nu_{tn2} - \nu_{gp1} - C_{n2}n_2)\Delta\hat{n} \\
&\quad + (\nu_{tp1} - \nu_{gn2} - C_{p1}p_1)\Delta\hat{p}, \\
\partial\Delta\hat{n}/\partial t &= (\nu_{tn1} - \nu_{tp1})\Delta m \\
&\quad - (\nu_{tn1} + \nu_{gn1} + \nu_{gp1})\Delta\hat{n} \\
&\quad + (\nu_{tp1} - C_{n1}n - C_{p1}p_1)\Delta\hat{p}, \\
\partial\Delta\hat{p}/\partial t &= (\nu_{tp2} - \nu_{tn2})\Delta m \\
&\quad + (\nu_{tn2} - C_{p2}p - C_{n2}n_2)\Delta\hat{n} \\
&\quad - (\nu_{tp2} + \nu_{gp2} + \nu_{gn2})\Delta\hat{p}.
\end{aligned} \quad (32)$$

Effective capture and release frequencies are here employed whose definitions are provided by Eqs. (24) if $\mathfrak{N}_1 - \hat{n}_0$ and $\mathfrak{N}_2 - \hat{p}_0$ are both replaced by $\mathfrak{N} - \hat{n}_0 - \hat{p}_0$. Aside from these modified definitions, Eqs. (32) are formally identical with corresponding equations for single-level centers of two types except for the additional "constraint" terms in which the capture coefficients appear explicitly.

For thermal equilibrium, definitions of n_1, p_1, n_2, and p_2 apply which are Eqs. (25) with both $\mathfrak{N}_1 - \hat{n}_0$ and $\mathfrak{N}_2 - \hat{p}_0$ replaced by $\mathfrak{N} - \hat{n}_0 - \hat{p}_0$. It follows that the restriction

$$n_0^2/n_1n_2 = p_1p_2/p_0^2 = \hat{n}_0/\hat{p}_0, \quad (33)$$

holds for this two-level case. As is easily verified, Eqs. (26) still apply, while the fractions of charged centers are

$$\begin{aligned}
\hat{n}_0/\mathfrak{N} &= (1 + n_1/n_0 + n_1n_2/n_0^2)^{-1} \\
&= (1 + p_0/p_1 + p_0^2/p_1p_2)^{-1} = \alpha_{10}/(1 + \alpha_{10} + \alpha_{20}), \\
\hat{p}_0/\mathfrak{N} &= (1 + p_2/p_0 + p_1p_2/p_0^2)^{-1} \\
&= (1 + n_0/n_2 + n_0^2/n_1n_2)^{-1} = \alpha_{20}/(1 + \alpha_{10} + \alpha_{20}),
\end{aligned} \quad (34)$$

with α_{10} and α_{20} given, as before, by Eqs. (28). Rela-

tionships formally identical with Eqs. (29) give n_1 and n_2 in terms of the two energy levels.

For this two-level case, the four effective trapping and release times or frequencies associated with each energy level satisfy the fundamental restriction that is formally identical with Eq. (30). It is also easily verified that the various conditions given for the recombination and trapping ranges and for shallow and deep traps apply without formal modification.

By suitable notational generalization of the fundamental mass-action equations, the results of this section can be shown to apply to two-level centers in general, whose states (differing successively by one electronic charge) may include ones that are multiply charged, either positively or negatively. Through use of the phenomenological capture coefficients, statistical weights associated with multiply charged states do not enter explicitly.

3. SOME THEORETICAL APPLICATIONS

3.1 Linear Steady-State Photoconductivity; Diffusion-Length Lifetimes

The mass-action equations written for the steady state and linearized by neglect of the quadratic terms give concentration increments that are proportional; solving for $\Delta\hat{n}/\Delta m$ and $\Delta\hat{p}/\Delta m$ provides the thermal-equilibrium trapping ratios, and lifetimes τ_n, τ_p, τ_m, and diffusion-length lifetime τ_0 are readily evaluated. These procedures will be illustrated in detail for the single-level case.

For formal simplicity of the results in this connection (and in various others as well), a concentration which will be called the "capture concentration" is introduced. For, say, acceptor centers, the capture concentration $\mathfrak{N}_1^*$ is defined as follows:

$$\mathfrak{N}_1^* \equiv \mathfrak{N}_1/(1+n_1/n_0)(1+p_1/p_0) = n_0\nu_{tn1}/\nu_{gn1} = p_0\nu_{tp1}/\nu_{gp1} = \mathfrak{N}_1(\hat{n}_0/\mathfrak{N}_1)(1-\hat{n}_0/\mathfrak{N}_1). \quad (35)$$

The various forms for $\mathfrak{N}_1^*$ are obtained by use of the definitions of Eqs. (24), the equilibrium relationships, Eqs. (25), (27), and (28), and the fundamental restriction, Eq. (30). As the last form shows, $\mathfrak{N}_1^*$ is the concentration $\mathfrak{N}_1$ of centers multiplied by the respective equilibrium fractions of centers occupied and unoccupied. Values of it which are small or large result, respectively, in negligible capture frequencies or in large capture frequencies with negligible release frequencies. If the centers are nearly all ionized or unionized, then $\mathfrak{N}_1^*$ is small; the last form shows that its largest value is $\frac{1}{4}\mathfrak{N}_1$, which it assumes for $\hat{n}_0/\mathfrak{N}_1=\frac{1}{2}$, that is, for the Fermi level coincident with the energy level of the centers. Note also that the volume rates of electron and hole transitions at equilibrium,[20] $n_0\nu_{tn1}$ and $p_0\nu_{tp1}$, are equal to $\mathfrak{N}_1^*$ times the corresponding effective release frequencies. With an obvious notational change, entirely similar results hold for donor centers.

The trapping ratio for the case of acceptor centers and the equilibrium lifetimes, which are the Shockley-Read lifetimes,[15] are given by:

$$\begin{aligned}
r_n &= \frac{\nu_{tn1}-\nu_{tp1}}{\nu_{tn1}+\nu_{gn1}+\nu_{gp1}} \\
&= \frac{\tau_{p0}(\mathfrak{N}_1-\hat{n}_0)-\tau_{n0}\hat{n}_0}{\tau_{p0}(\mathfrak{N}_1-\hat{n}_0+n_0+n_1)+\tau_{n0}(p_0+p_1)} \\
&= \frac{\mathfrak{N}_1^*(\tau_{tp1}-\tau_{tn1})}{(\mathfrak{N}_1^*+n_0)\tau_{tp1}+p_0\tau_{tn1}} = 1-\tau_n/\tau_p, \\
\tau_n &= (1-r_n)\tau_m = (\nu_{tp1}+\nu_{gp1}+\nu_{gn1})/\Delta_1 \\
&= \frac{\tau_{n0}(\hat{n}_0+p_0+p_1)+\tau_{p0}(n_0+n_1)}{\mathfrak{N}_1^*+n_0+p_0} \\
&= \frac{(\mathfrak{N}_1^*+p_0)\tau_{tn1}+n_0\tau_{tp1}}{\mathfrak{N}_1^*+n_0+p_0}, \\
\tau_p &= \tau_m = (\nu_{tn1}+\nu_{gn1}+\nu_{gp1})/\Delta_1 \\
&= \frac{\tau_{p0}(\mathfrak{N}_1-\hat{n}_0+n_0+n_1)+\tau_{n0}(p_0+p_1)}{\mathfrak{N}_1^*+n_0+p_0} \\
&= \frac{(\mathfrak{N}_1^*+n_0)\tau_{tp1}+p_0\tau_{tn1}}{\mathfrak{N}_1^*+n_0+p_0}.
\end{aligned} \quad (36)$$

Here, τ_{n0} and τ_{p0} given by

$$\begin{aligned}
\tau_{n0} &\equiv (C_{n1}\mathfrak{N}_1)^{-1} = (1-\hat{n}_0/\mathfrak{N}_1)\tau_{tn1} \\
&= (1+p_1/p_0)^{-1}\tau_{tn1}, \quad (37) \\
\tau_{p0} &\equiv (C_{p1}\mathfrak{N}_1)^{-1} = (\hat{n}_0/\mathfrak{N}_1)\tau_{tp1} = (1+n_1/n_0)^{-1}\tau_{tp1},
\end{aligned}$$

are the respective limiting lifetimes[21] in strongly extrinsic p- and n-type materials (in which they are also τ_{tn1} and τ_{tp1}); and Δ_1 given by

$$\Delta_1 \equiv \nu_{tn1}\nu_{gp1}+\nu_{tn1}\nu_{tp1}+\nu_{tp1}\nu_{gn1} = C_{n1}C_{p1}\mathfrak{N}_1(\mathfrak{N}_1^*+n_0+p_0) \quad (38)$$

is always positive if neither C_{n1} nor C_{p1} is zero.

The diffusion length L_0 and lifetime τ_0 corresponding to the Shockley-Read lifetimes may be evaluated from D_0' and τ_m or from τ_n and τ_p, thus from Eqs. (16) or (20) and Eqs. (36). These equations give

$$\begin{aligned}
L_0^2 &= D_0'\tau_m = D_0\tau_0 = D_0[1-r_n p_0/(n_0+p_0)]\tau_p \\
&= kT\mu_n\mu_p\sigma_0^{-1}(n_0\tau_{tp1}+p_0\tau_{tn1}) \\
&= \sigma_0^{-1}(\sigma_{p0}D_n\tau_{tn1}+\sigma_{n0}D_p\tau_{tp1}), \quad (39)
\end{aligned}$$

where σ_{n0} and σ_{p0} are $e\mu_n n_0$ and $e\mu_p p_0$. Other forms may be written by expressing τ_{tn1} and τ_{tp1} in terms of τ_{n0} and τ_{p0} by use of Eqs. (37). The diffusion-length life-

[20] See Sec. 2.212.

[21] Conditions for these lifetimes are $p_0 \gg \mathfrak{N}_1^*+p_1+p_1^*$ and $n_0 \gg \mathfrak{N}_1^*+n_1+n_1^*$.

time for this case,

$$\tau_0=(\nu_{gn1}+\nu_{gp1})/(\nu_{tn1}\nu_{gp1}+\nu_{tp1}\nu_{gn1})$$
$$=[\tau_{p0}(n_0+n_1)+\tau_{n0}(p_0+p_1)]/(n_0+p_0)$$
$$=(n_0\tau_{tp1}+p_0\tau_{tn1})/(n_0+p_0), \quad (40)$$

is formally similar to the familiar common lifetime[14,15] for both electrons and holes for the limiting case of $\mathfrak{N}_1$ small, as inspection of Eqs. (36) serves to verify.[22] This common lifetime otherwise applies as such only under a condition restricting the capture concentration which is frequently severe: The condition, obtained from Eqs. (36) by use of $|\tau_p-\tau_0|/\tau_0\ll 1$ and $|\tau_n-\tau_0|/\tau_0\ll 1$, requires for the minority-carrier trapping range that this concentration be small compared with the equilibrium minority-carrier concentration.

A diffusion-length lifetime which is also of interest is that for traps in conjunction with recombination centers. For recombination centers in extrinsic material, a lifetime τ_3 for minority carriers may be specified. Assume negligible recombination in the traps themselves, so that, for p-type material, $\tau_m=\tau_n/(1-r_n)$ is $\tau_3/(1-r_n)$. Then, for nonrecombinative electron traps of the acceptor type, Eqs. (36) give $r_n=\nu_{tn1}/(\nu_{tn1}+\nu_{gn1})$, $r_p=0$, and, from Eqs. (16), $D_0'/D_0=[1-p_0\mathfrak{N}_1^*/(n_0+p_0)(\mathfrak{N}_1^*+n_0)]$ results. Or, if the electron traps are of the donor type, then $r_n=0$, $r_p=-\nu_{tn2}/\nu_{gn2}$, $D_0'/D_0=[1+\mathfrak{N}_2^*/(n_0+p_0)]$, and $\tau_m=\tau_3$ are obtained. Essentially the same diffusion-length lifetime, namely

$$\tau_0=(D_0'/D_0)\tau_m=[1+\mathfrak{N}_j^*/(n_0+p_0)]\tau_3, \quad j=1, 2, \quad (41)$$

results for both types of traps.[23] Thus, minority-carrier trapping increases diffusion-length lifetime.[24] As the analysis shows, this increase results directly from an increase in the ambipolar minority-carrier diffusivity. The effect is appreciable for capture concentration at least comparable with the equilibrium concentration of majority carriers. Similar analysis for nonrecombinative majority-carrier traps gives a τ_0 which is that of Eq. (41) modified by division by $1+\mathfrak{N}_j^*/n_0$ for n-type material or $1+\mathfrak{N}_j^*/p_0$ for p-type. Thus, majority-carrier trapping decreases diffusion-length lifetime, but only by a factor no smaller than $(1+p_0/n_0)^{-1}$ or $(1+n_0/p_0)^{-1}$, respectively.

[22] As W. L. Brown has pointed out, this formal similarity must hold because diffusion length does not depend on release times but on the capture times, the times the carriers are free. It can be shown that it holds for any number M of types of centers, for which τ_0 is given by

$$\Big[\sum_{j=1}^{M}(n_0\tau_{tpj}+p_0\tau_{tnj})^{-1}\Big]^{-1}/(n_0+p_0).$$

[23] Equation (20) provides an equivalent derivation. It can be shown that if different types of traps are present, the $\mathfrak{N}_j^*$ in Eq. (41) is replaced by the sum of the respective capture concentrations.

[24] A. K. Jonscher, Proc. Phys. Soc. (London) **B70**, 230 (1957) gives an increase of diffusion length with trap concentration which is bounded and always essentially negligible, a result at variance with that given here. See reference 2, footnote on p. 525.

General conditions for the validity of the linear analysis of this section may be formulated as conditions for the neglect of the quadratic terms. For this purpose, the steady-state equations for uniform concentrations and volume generation rate Δg are employed. The conditions are then obtained in a self-consistent manner as restrictions on (positive) Δg or on concentration increments by substituting for the concentration increments their values from the linearized equations in terms of Δg and the equilibrium capture and release frequencies. It is thus found, for example, that for nonrecombinative electron traps of the acceptor type, Δp and $\Delta\hat{n}$ must be small compared with the concentration $\mathfrak{N}_1-\hat{n}_0$ of unoccupied traps, and Δn small compared with n_0+n_1. These may be severe conditions for p-type material. They could in practice require an injection level much lower than small-signal ones meeting the familiar condition[3] based on conductivity change.

3.2 The Photomagnetoelectric Effect

The steady-state PME effect with trapping in an infinite slab to the faces of which the applied magnetic field is parallel will be considered for a linear small-signal case. Equations (7), (10), and (17) for current densities may be suitably specialized, as in the treatment previously given for the no-trapping case,[4] and the short-circuit PME current along the slab obtained as an integral across the slab of the PME current density. The latter is evaluated from the solution of the suitably specialized continuity equation for boundary conditions corresponding to recombination of carriers at the respective surfaces with generation at the illuminated one. The concentration variable is now Δm, the modified ambipolar diffusivity D_0' is employed, and a surface recombination velocity s_m for Δm implies velocities s_n for Δn and s_p for Δp. These are clearly such that $s_m\Delta m=s_n\Delta n=s_p\Delta p$ holds, from which the relationships

$$s_m=(1-r_n)s_n=(1-r_p)s_p,$$
$$s_m\tau_m=s_n\tau_n=s_p\tau_p\equiv L_s \quad (42)$$

follow.

The increase ΔG in conductance of the slab is the integral of $\Delta\sigma$ across the slab, or

$$\Delta G=e\int_{-y_0}^{y_0}(\mu_n\Delta n+\mu_p\Delta p)dy$$
$$=e(\mu_n+\mu_p)\int_{-y_0}^{y_0}(\tau_c/\tau_m)\Delta m dy. \quad (43)$$

The second form follows from $\Delta n/\tau_n=\Delta p/\tau_p=\Delta m/\tau_m=\mathfrak{R}_m$, with

$$\tau_c\equiv\Delta\sigma/e(\mu_n+\mu_p)\Delta g=(\mu_n\tau_n+\mu_p\tau_p)/(\mu_n+\mu_p), \quad (44)$$

a lifetime function that determines the conductivity

increase $\Delta\sigma$ for the uniform volume generation rate $\Delta g=\mathfrak{R}_m$.

The PME method of the high-recombination-velocity dark surface is best employed, since it generally provides better accuracy for the conductance change than does the thick-slab method which it otherwise subsumes as a limiting case.[25] Optimum slab thickness is about one or two diffusion lengths. For large dark-surface recombination velocity, the small-signal results for no trapping[26] give, for the present case,

$$I^{(sc)}=-\theta e\mathfrak{L}L_0(S_1+\coth 2Y_0)^{-1}$$
$$=-\theta(\mu_n+\mu_p)^{-1}(L_0/\tau_c)(\coth Y_0)\Delta G, \quad (45)$$

in which $L_0\equiv(D_0\tau_0)^{\frac{1}{2}}$ is the diffusion length, Y_0 is y_0/L_0, and S_1 is $s_{m1}L_0/D_0'=L_{s1}/L_0$, subscript 1 referring to the illuminated surface. Note that ΔG now involves τ_c as a factor.

For nonrecombinative traps in extrinsic material with recombination of lifetime τ_3 in other centers,[27] τ_0 is given by Eq. (41). For ΔG, Eqs. (43) hold with $\tau_c/\tau_m=[(1-r_n)\mu_n+(1-r_p)\mu_p]/(\mu_n+\mu_p)$ for the linear small-signal case. The solution for Δm is readily obtained by comparison with that for the corresponding no-trapping case,[28] and

$$\Delta G=K_G e(\mu_n+\mu_p)\tau_3\mathfrak{L}(\cosh 2Y_0-1)/$$
$$(S_1\sinh 2Y_0+\cosh 2Y_0) \quad (46)$$

results for the present linear small-signal case, where, for p-type material, $K_G\equiv\tau_c/\tau_3$ is given by

$$K_G=1+(b+1)^{-1}\mathfrak{N}_j^*/n_0 \quad \text{(electron trapping)}$$
$$K_G=[1+(1+b^{-1})^{-1}\mathfrak{N}_j^*/p_0]/[1+\mathfrak{N}_j^*/p_0] \quad (47)$$
$$\text{(hole trapping)}.$$

For hole and electron trapping, respectively, in n-type material, n_0 and p_0 in these equations are interchanged and $b\equiv\mu_n/\mu_p$ replaced by its reciprocal. Note that the expression which K_G multiplies also depends on trapping, since L_0 does.

Dimensionless PME current-conductance ratio

$$\mathcal{I}/(\Delta G/G_0)\equiv -2y_0(\mu_n+\mu_p)I^{(sc)}/\theta D_0\Delta G$$
$$=(K_\tau/K_G)2Y_0\coth Y_0 \quad (48)$$

follows from Eqs. (45) and (46), with

$$K_\tau\equiv\tau_0/\tau_3=1+\mathfrak{N}_j^*/(n_0+p_0), \quad (49)$$

for nonrecombinative minority-carrier traps; for majority-carrier traps, this K_τ is modified as in connection with Eq. (41). Apparent lifetime τ_r on the assumption of no trapping, obtained by equating $\mathcal{I}/(\Delta G/G_0)$ to $[2y_0/(D_0\tau_r)^{\frac{1}{2}}]\coth[y_0/(D_0\tau_r)^{\frac{1}{2}}]$, is accordingly given by $\tau_r\tanh^2[y_0/(D_0\tau_r)^{\frac{1}{2}}]=(K_G^2/K_\tau)\tau_3\tanh^2 Y_0$, and equals $(K_G^2/K_\tau)\tau_3$ for the thick slab. As trap concentration increases, diffusion length increases and a slab of any given thickness becomes a "thin" slab, for which $Y_0\coth Y_0\sim 1$; and $\mathcal{I}/\Delta G/G_0)$ approaches a constant value which is independent of the thickness. For example, if the half-thickness y_0 is of order $(D_0\tau_3)^{\frac{1}{2}}$, then $K_\tau\gg 1$ or $\mathfrak{N}_j^*\gg n_0+p_0$ also gives small Y_0. From the expressions for K_τ and Eqs. (47) for K_G it is found that $\mathcal{I}/(\Delta G/G_0)$ approaches $2(b+1)n_0/(n_0+p_0)$ for electron trapping and $2(b+1)p_0/b(n_0+p_0)$ for hole trapping, regardless of conductivity type. On the other hand, if the slab is so thick that $y_0^2\gg D_0\tau_3\mathfrak{N}_j^*/(n_0+p_0)$ holds, then the condition $\mathfrak{N}_j^*\gg n_0+p_0$ for large trap concentration gives[29] τ_3/τ_τ equal to K_τ/K_G^2 or $(b+1)^2n_0^2/(n_0+p_0)\mathfrak{N}_j^*$ for electron trapping and $(b+1)^2p_0/b^2(n_0+p_0)$ for hole trapping in p-type material, with similar results for n-type obtained as in connection with Eqs. (47).

The recombination lifetime τ_3 can be determined from suitable measurements with traps saturated. With τ_3 known, measurement of $\mathcal{I}/(\Delta G/G_0)$ serves to determine τ_0, since, from Eqs. (47) and (49), K_τ/K_G may be written as $(b+1)n_0[n_0+p_0-(p_0-bn_0)\tau_3/\tau_0]^{-1}$ for electron trapping in p-type material, or as an analogous expression for hole trapping in n-type. If the equilibrium concentration of empty traps and the release time (which, if τ_3 is comparatively short, is the photoconductive decay time for the traps nearly full) are also determined, then trap concentration, energy level, and capture cross section can easily be calculated.

3.3 Nonlinear Steady-State Photoconductivity

Lifetime functions τ_n and τ_p for Δn and Δp resulting from arbitrary steady-state injection levels may be evaluated from $\mathfrak{R}_m$. For centers of the acceptor type and no direct recombination, Eqs. (21) give

$$\Delta n/\tau_n=\Delta p/\tau_p=\mathfrak{R}_m=C_{n1}[n(\mathfrak{N}_1-\hat{n})-n_1\hat{n}]$$
$$=C_{p1}[p\hat{n}-p_1(\mathfrak{N}_1-\hat{n})]$$
$$=\frac{np-n_i^2}{\tau_{p0}(n+n_1)+\tau_{n0}(p+p_1)}. \quad (50)$$

The familiar last form,[15] which results from elimination of $\hat{n}$ and use of Eqs. (37), provides expressions for τ_n and τ_p in terms of Δn and Δp. Certain other forms[2] result if both n and p are eliminated instead by use also of the neutrality condition. Then τ_n and τ_p as well as

[25] Reference 4, Sec. 3.42.

[26] Reference 4, Eq. (50).

[27] This case has been treated by A. Amith, Bull. Am. Phys. Soc. 4, 28 (1959); Phys. Rev. **116**, 793 (1959). See also reference 11.

[28] In Eq. (44) of reference 4, Δp is replaced by Δm; the D_0 that appears explicitly originates from the boundary conditions and is replaced by D_0' and S_1 is $s_{m1}L_0/D_0'=L_{s1}/L_0$.

[29] Amith (reference 27 and private communication) has pointed out that trapping usually influences the PME current-conductance ratio mainly through the effect on conductance. For minority-carrier trapping in the thick slab, τ_3/τ_τ is proportional to $\mathfrak{N}_j^{-2}$ in the intermediate range in which $\mathfrak{N}_j^*$ is large compared with minority-carrier concentration n_0 or p_0 but small compared with p_0 or n_0 so that the change in diffusion length may be neglected. For majority-carrier trapping in general, τ_3 and τ_r are substantially equal in this range.

Δn and Δp are written in terms of $\Delta\hat{n}$ as a single independent parameter which can be related to the steady generation rate $\Delta g=\mathfrak{R}_m$.

The lifetime function[15] for $|\Delta\hat{n}|\ll\Delta n\sim\Delta p=\Delta m$,

$$\tau_n\sim\tau_p\sim\frac{\tau_{p0}(n_0+n_1+\Delta p)+\tau_{n0}(p_0+p_1+\Delta p)}{n_0+p_0+\Delta p}, \quad (51)$$

follows readily from Eqs. (50). That this lifetime function apply in general requires $|\Delta\hat{n}|$ relatively small for all Δp, a condition which subsumes, as may be expected, equilibrium lifetimes substantially equal to τ_0. This common lifetime then applies for low injection levels for which the condition $\Delta p\ll(\nu_{gn1}+\nu_{gp1})/(C_{n1}+C_{p1})$ holds, in which equilibrium release frequencies are employed. In the minority-carrier trapping range, both conditions may be severe: Equilibrium lifetimes are τ_0 for $\mathfrak{N}_1^*$ small compared with minority-carrier concentration n_0 or p_0; and τ_0 then applies for Δp small compared with n_0+n_1 or p_0+p_1, which follows also from the condition of Sec. 3.1 for neglect of $C_{n1}\Delta n\Delta\hat{n}$ or $C_{p1}\Delta p\Delta\hat{n}$ suitably specialized. If the condition on Δp is not met, then (with the condition on $\mathfrak{N}_1^*$) Eq. (51) gives a lifetime which increases rapidly with injection level at low injection levels.[30] But such observed behavior with extrinsic material, as these considerations indicate, cannot usually be properly analyzed by use of Eq. (51). The τ_n and τ_p in the small-signal range generally either result primarily from recombination or majority-carrier trapping and are both τ_0 and substantially constant, or else have differing equilibrium values given by Eqs. (36). Thus, unless trap concentration is quite small, Eq. (51) has significant application in the former case only to the transition from τ_0 to the lifetime $\tau_{n0}+\tau_{p0}$ for the large-signal range.

It can be shown[2] that in the latter case of differing lifetimes small-signal trap saturation generally obtains with which apparent diffusion-length lifetime increases to a value given by τ_{gn1} for n-type material or τ_{gp1} for p-type. Further increase then occurs in the approach to a large-signal lifetime which is substantially τ_{n0} for n-type material or τ_{p0} for p type, that is, the (small-signal) lifetime in the limit of strongly extrinsic material of the opposite conductivity type. Such increases of lifetime can account for certain cases of superlinearity, or the more-rapid-than-linear increase of photoconductivity with injection level, on the basis of a single trapping level.[31]

The steady-state fractions of ionized centers can be represented by simple formal generalizations of the equilibrium relationships of Eqs. (27) and (34) for single- and two-level centers: In these equations, $\hat{n}_0$ and $\hat{p}_0$ are replaced by $\hat{n}$ and $\hat{p}$ and α_{10} and α_{20} by

$$\begin{aligned}\alpha_1&\equiv\frac{C_{n1}n+C_{p1}p_1}{C_{p1}p+C_{n1}n_1}=\alpha_{10}\frac{1+\Delta n/(n_0+n_1^*)}{1+\Delta p/(p_0+p_1^*)},\\ \alpha_2&\equiv\frac{C_{p2}p+C_{n2}n_2}{C_{n2}n+C_{p2}p_2}=\alpha_{20}\frac{1+\Delta p/(p_0+p_2^*)}{1+\Delta n/(n_0+n_2^*)},\end{aligned} \quad (52)$$

as can readily be shown[32] by solving for the ionized fractions from Eqs. (21) and also from the corresponding two-level equations of Sec. 2.22.

3.4 Time-Dependent Photoconductivity

The decay of photoconductivity is governed in the general case by nonlinear differential equations which are rather intractable analytically.[33] Solutions[2] of the nonlinear equations are obtainable in closed form, however, for cases of nonrecombinative trapping and for sufficiently small concentration of centers or large concentrations of mobile excess carriers such that the steady-state lifetimes are substantially equal. The latter solution has the restricted general application of the lifetime function of Eq. (51), since it is the integrated form corresponding to this function.

With the neglect of direct recombination, the limiting large-signal equations are linear and give an exponential decay with lifetime equal to the steady-state large-signal lifetime. During this decay, the concentrations of carriers in traps remain constant. The lifetime for centers of a single type is $\tau_{n0}+\tau_{p0}$, and the concentration in the centers is $\Delta\hat{n}=(\nu_{tn1}-\nu_{tp1})/(C_{n1}+C_{p1})$ or $\Delta\hat{p}=(\nu_{tp2}-\nu_{tn2})/(C_{n2}+C_{p2})$. If single-level centers of the acceptor and donor types are present together, then these values of $\Delta\hat{n}$ and $\Delta\hat{p}$ clearly still apply; and the lifetime is the harmonic mean of the lifetimes $\tau_{n0}+\tau_{p0}$ for each type of center. For two-level centers, on the other hand, Eqs. (32) give

$$\Delta\hat{n}=C_{n1}C_{n2}\mathfrak{N}/(C_{n1}C_{n2}+C_{n2}C_{p1}+C_{p1}C_{p2})-\hat{n}_0$$

and

$$\Delta\hat{p}=C_{p1}C_{p2}\mathfrak{N}/(C_{n1}C_{n2}+C_{n2}C_{p1}+C_{p1}C_{p2})-\hat{p}_0,$$

with large-signal lifetime equal to

$$(1+C_{n1}/C_{p1}+C_{p2}/C_{n2})/(C_{n1}+C_{p2})\mathfrak{N}.$$

[30] As a result of saturation of centers available for minority-carrier capture, this lifetime increases essentially linearly in the small-signal range from the equilibrium value $\tau_{p0}(n_0+n_1)/p_0$ or $\tau_{n0}(p_0+p_1)/n_0$ and asymptotically to the large-signal value τ_{p0} or τ_{n0}.

[31] A multilevel model for superlinearity has been given by A. Rose, R C A Rev. **12**, 362 (1951); Phys. Rev. **97**, 322 (1955); Proc. Inst. Radio Engrs. **43**, 1850 (1955); *Photoconductivity Conference*, Chap. 1A (reference 8). See also: R. H. Bube, J. Phys. Chem. Solids **1**, 234 (1957).

[32] The equation given in the abstract of the paper of Sah and Shockley (reference 17) rewritten in the present notation yields $\hat{n}/(\mathfrak{N}-\hat{n}-\hat{p})=\alpha_1$ and $(\mathfrak{N}-\hat{n}-\hat{p})/\hat{p}=\alpha_2^{-1}$, from which the ionized fractions for the two-level case here given follow as solutions of simultaneous linear equations.

[33] Certain analytical approximations have been considered by W.-H. Isay, Ann. Physik **13**, 327 (1953). A treatment which includes numerically computed solutions has been given by K. C. Nomura and J. S. Blakemore, Phys. Rev. **112**, 1607 (1958).

In the linear small-signal case, the photoconductive decay is given by a sum of exponential modes with (real and positive) decay constants whose number exceeds by one the number of types of centers present. This decay will here be considered for centers of a single type.[12,34–40] The solution for centers of the acceptor type is readily found to be given by

$$\Delta m=\Delta p=\sum_{j=1}^{2} A_j e^{-\nu_j t}, \quad \Delta\hat{n}=\sum_{j=1}^{2} r_{nj}A_j e^{-\nu_j t}, \tag{53}$$

in which A_j are constants determined by the initial conditions; the r_{nj} are trapping ratios for the respective decay modes determined by

$$\begin{aligned} -\nu_{tp1}+\nu_j-\nu_{gp1}r_{nj}&=0,\\ \nu_{tn1}-\nu_{tp1}+(-\nu_{tn1}-\nu_{gn1}-\nu_{gp1}+\nu_j)r_{nj}&=0, \end{aligned} \tag{54}$$

with the decay constants ν_j the roots of the equation obtained by equating to zero the determinant of Eqs. (54). The A_j are found in terms of the trapping ratios and the initial concentrations Δm_1, $\Delta\hat{n}_1$, and $\Delta\hat{p}_1$ by setting t equal to zero in Eqs. (53) and solving. The trapping ratios are thus given by

$$\begin{aligned} r_{nj}&=(\nu_{tn1}-\nu_{tp1})/(\nu_{tn1}+\nu_{gn1}+\nu_{gp1}-\nu_j)\\ &=(\nu_j-\nu_{tp1})/\nu_{gp1}, \end{aligned} \tag{55}$$

and the decay constants by

$$\nu_j=\tfrac{1}{2}[(-1)^{j-1}\nu_r+\nu_s], \quad j=1, 2, \tag{56}$$

with

$$\begin{aligned} \nu_r&\equiv(\nu_s^2-4\Delta_1)^{\frac{1}{2}}=[(\nu_{tn1}+\nu_{gn1}-\nu_{tp1}-\nu_{gp1})^2\\ &\qquad+4\nu_{gn1}\nu_{gp1}]^{\frac{1}{2}},\\ \nu_s&\equiv\nu_{tn1}+\nu_{gn1}+\nu_{tp1}+\nu_{gp1}, \end{aligned} \tag{57}$$

and Δ_1 defined by Eq. (38). Equilibrium values of the effective release frequencies are, of course, employed. The corresponding time constants $\tau_1\equiv\nu_1^{-1}$ and $\tau_2\equiv\nu_2^{-1}$ are also equal, respectively, to ν_2/Δ_1 and ν_1/Δ_1. Nonoscillatory decay is easily verified for this case: The second form for ν_r shows that the ν_j are real; and since $\nu_r<\nu_s$, the ν_j are positive.

A subcase that provides some physical interpretations is that of $\mathfrak{N}_1$ sufficiently small so that capture frequencies are small compared with release frequencies. As Eqs. (38) and (57) show, the condition $\nu_s^2\gg4\Delta_1$ then holds, and expansion of the radical gives

$$\begin{aligned} &\tau_1\sim\nu_s^{-1}=\tau_{gn1}\tau_{gp1}/(\tau_{gn1}+\tau_{gp1})\\ &\ll\tau_2\sim\nu_s/\Delta_1\sim\tau_0. \end{aligned} \tag{58}$$

Thus, for this subcase, τ_2 is the steady-state lifetime[36–38] τ_0 of Eq. (40). It is large compared with τ_1, the time constant for the adjustment of $\Delta\hat{n}$ to a fixed fraction of Δp substantially equal to the equilibrium trapping ratio, r_n. This interpretation of τ_1 follows readily from $r_{n2}\sim(\nu_{tn1}-\nu_{tp1})/(\nu_{gn1}+\nu_{gp1})\sim r_n$. If the initial trapping ratio is r_n, then the trapping mode does not occur in Δn, $\Delta\hat{n}$, or Δp; it does not occur either for "critical recombination" with which $\Delta\hat{n}$ remains identically zero as result of equal capture frequencies ν_{tn1} and ν_{tp1} or, for this subcase, equal capture rates for Δn and Δp. For small $\mathfrak{N}_1$, the capture rates are in all cases substantially in the ratio ν_{tn1}/ν_{tp1}. In linear cases, they also decay in the lifetime mode after this mode predominates. The release rates behave similarly, their ratio being equal to ν_{gn1}/ν_{gp1}, or to $(n_0/p_0)(\nu_{tn1}/\nu_{tp1})$ in accordance with Eq. (30).

The condition for neglect of the capture frequencies may be severe: For the minority-carrier trapping range, the condition $\nu_{tn1}+\nu_{tp1}\ll\nu_{gn1}+\nu_{gp1}$ for the approximate form of ν_s (which subsumes $\mathfrak{N}_1^*\ll n_0+p_0$ for neglect of $\nu_{tn1}\nu_{tp1}$ in Δ_1 and, generally, $\nu_s^2\gg4\Delta_1$ as well) is the same as that for steady-state lifetimes equal to τ_0; it requires $\mathfrak{N}_1^*$ small compared with minority-carrier concentration n_0 or p_0.

The release frequencies may be neglected under the condition $\mathfrak{N}_1^*\gg n_0+p_0$. The solution is then simply[41] $\Delta n/\Delta n_1=\exp(-t/\tau_{tn1})$ and $\Delta p/\Delta p_1=\exp(-t/\tau_{tp1})$. For $\mathfrak{N}_1^*$ large, Eqs. (36) show that τ_{tn1} and τ_{tp1} are, respectively, the steady-state lifetimes[36,38] τ_n and τ_p. The condition $\nu_s^2\gg4\Delta_1$ is accordingly $\frac{1}{4}(\tau_n/\tau_p+\tau_p/\tau_n)+\frac{1}{2}\gg1$, namely that one of τ_n or τ_p be small compared with the other. If τ_n or τ_p is the smaller, then substantially all of Δn or Δp, respectively, is transformed comparatively rapidly into positive or negative $\Delta\hat{n}$, after which a slower recombinative decay of $\Delta\hat{n}$ and the concentration of the other mobile carriers takes place as these carriers are captured.

The condition $\nu_s^2\gg4\Delta_1$ implies $\tau_1\ll\tau_2$, with τ_1 essentially a characteristic time for trapping and τ_2 essentially a lifetime. This interpretation does not apply if ν_s^2 and $4\Delta_1$ are comparable so that τ_1 and τ_2 do not differ by much. For small $\mathfrak{N}_1$ and the recombination or majority-carrier trapping range, for example, $\tau_1\sim\tau_2$ may hold. The case of $\nu_s^2\sim4\Delta_1$ for $\mathfrak{N}_1$ large, for which τ_1, τ_2, τ_{tn1}, τ_{tp1}, τ_n, and τ_p are all substantially equal, is a case of recombination with but slight trapping.

The general trapping time and lifetime, obtained

[34] H. Y. Fan, Phys. Rev. **92**, 1424 (1953); **93**, 1434 (1954).
[35] E. S. Rittner, reference 8. This reference includes some nonlinear cases.
[36] E. I. Adirovich and G. M. Guro, Doklady Akad. Nauk S.S.S.R. **108**, 417 (1956) [translation: Soviet Phys. Doklady **1**, 306 (1956)].
[37] D. J. Sandiford, Phys. Rev. **105**, 524 (1957).
[38] D. H. Clarke, J. Electronics and Control **3**, 375 (1957).
[39] W. Shockley, reference 17.
[40] G. K. Wertheim, Phys. Rev. **109**, 1086 (1958).
[41] This result easily follows directly from the differential equations. Or, note that the quantity in brackets in Eqs. (57) is $(\nu_{tn1}-\nu_{tp1})^2$.

from Eq. (56) and related equations, are[37]

$$\begin{aligned}\tau_1=\nu_s^{-1}&=\tau_{gn1}\tau_{gp1}/[(\mathfrak{N}_1^*/p_0+1)\tau_{gn1}\\&\qquad+(\mathfrak{N}_1^*/n_0+1)\tau_{gp1}]\\&=\tau_{tn1}\tau_{tp1}/[(1+p_0/\mathfrak{N}_1^*)\tau_{tn1}\\&\qquad+(1+n_0/\mathfrak{N}_1^*)\tau_{tp1}]\\\ll\tau_2=\nu_s/\Delta_1&=(\mathfrak{N}_1^*+n_0+p_0)^{-1}[n_0\tau_{gn1}+p_0\tau_{gp1}\\&\qquad+n_i^2(\tau_{gn1}+\tau_{gp1})/\mathfrak{N}_1^*]\\&=(\mathfrak{N}_1^*+n_0+p_0)^{-1}[(\mathfrak{N}_1^*+p_0)\tau_{tn1}\\&\qquad+(\mathfrak{N}_1^*+n_0)\tau_{tp1}].\end{aligned}\tag{59}$$

Comparison with Eqs. (36) and (40) shows that this lifetime τ_2 is larger than the steady-state lifetimes τ_n, τ_p, and τ_0; all are equal in the limit of $\mathfrak{N}_1$ small. For $\mathfrak{N}_1$ large in intrinsic material, τ_2 equals $2\tau_0$. Furthermore, these lifetimes all decrease monotonically to zero as $\mathfrak{N}_1$ increases indefinitely.

The decrease of τ_2 with increasing $\mathfrak{N}_1$ may, however, proceed essentially in two ranges, with approximate constancy of τ_2 in an intermediate range.[42] From the first form for τ_2 of Eqs. (59), this intermediate range occurs provided there are capture concentrations $\mathfrak{N}_1^*$ that are small compared with n_0+p_0 and also large compared with $(\nu_{gn1}+\nu_{gp1})/(\nu_{gn1}/n_0+\nu_{gp1}/p_0)$, that is, if the strong inequality

$$\tau_{gn1}+\tau_{gp1}<<<(n_0/p_0)\tau_{gn1}+(p_0/n_0)\tau_{gp1}\tag{60}$$

holds. It can hold for sufficiently strongly extrinsic material if the majority-carrier release time is not too small. For small $\mathfrak{N}_1$, τ_2 varies inversely with $\mathfrak{N}_1$, as Eqs. (40) for τ_0 show. For large $\mathfrak{N}_1$ such that $\mathfrak{N}_1^*\gg n_0+p_0$, τ_2 varies similarly, equalling the value $(n_0\tau_{gn1}+p_0\tau_{gp1})/(n_0+p_0)$ of approximate constancy divided by $\mathfrak{N}_1^*/(n_0+p_0)$. With the second or third form for $\mathfrak{N}_1^*$ of Eqs. (35), this τ_2 reduces to $\tau_{tn1}+\tau_{tp1}$. Since τ_1 for large $\mathfrak{N}_1^*$ is the harmonic mean of τ_{tn1} and τ_{tp1}, τ_1 is the smaller of these capture times and τ_2 the larger, as previously discussed for this case. It can be shown that, for the minority-carrier trapping range, the inequalities that $\mathfrak{N}_1^*$ must satisfy for approximate constancy of τ_2 generally imply the condition $\nu_s^2\gg 4\Delta_1$ on which the calculation is based. A similar situation has been shown to obtain with the inequality for the case of negligible capture frequencies. But since this case involves a condition for neglect of the capture frequencies which is usually severe for the minority-carrier trapping range, it is the present case which would usually apply in practice in this range.

The decay times associated with a small-amplitude pulse of added carriers above a steady generation level Δg are readily evaluated. The equations for $d\delta n/dt$ and $d\delta p/dt$ linear in the concentration increments δn and δp resulting from the pulse may be obtained from Eqs. (23) and (24). Written with capture and release frequencies that are concentration-dependent, they are formally the same as the linear small-signal ones for $d\Delta n/dt$ and $d\Delta p/dt$. For the release frequencies, the definitions of Eqs. (24) apply; for the capture frequencies, $\hat{n}_0$ and $\hat{p}_0$ in these definitions are replaced by $\hat{n}$ and $\hat{p}$. The condition $\nu_s^2\gg 4\Delta_1$ of this section generalized in this way is the condition for a lifetime $\bar{\tau}_2$ for δn and δp which is equal to the generalized ratio ν_s/Δ_1 and which is large compared with the corresponding time constant for trapping. The lifetime $\bar{\tau}_2$ depends on the steady-state values of Δn, $\Delta\hat{n}$, and Δp; it reduces to τ_2 of Eqs. (59) for the linear small-signal case and to $\tau_{n0}+\tau_{p0}$ for Δg large.

3.5 Transport of Injected Carriers

Steady-state transport as well as drift of an injected pulse will be considered in this section on the basis of general differential equations of Sec. 2 specialized to the linear small-signal case for centers of a single type and no applied magnetic field. After this readily effected specialization, $\Delta\hat{n}$ for centers of the acceptor type may be eliminated between the continuity equation in $\Delta m=\Delta p$ and the mass-action equation for $\partial\Delta\hat{n}/\partial t$. With the introduction of $\mathfrak{N}_1^*$ and Δ_1 from Eqs. (35) and (38), the following third-order equation results:

$$\begin{aligned}&\partial^2\Delta p/\partial t^2-D_0\operatorname{div}\operatorname{grad}(\partial\Delta p/\partial t)+\mathbf{v}_0\cdot\operatorname{grad}(\partial\Delta p/\partial t)\\&\quad+\nu_s\partial\Delta p/\partial t-\nu_D D_0\operatorname{div}\operatorname{grad}\Delta p+\nu_v\mathbf{v}_0\cdot\operatorname{grad}\Delta p\\&\qquad+\Delta_1\Delta p=\partial\Delta g/\partial t+(\nu_s-\nu_{tp1})\Delta g.\end{aligned}\tag{61}$$

Here, $D_0\equiv kT\mu_n\mu_p(n_0+p_0)/\sigma_0$ and $\mathbf{v}_0\equiv e\mu_n\mu_p(n_0-p_0)\mathbf{I}/\sigma_0^2$ are the equilibrium ambipolar diffusivity and velocity; ν_s is defined in Eqs. (57) and ν_D and ν_v by

$$\begin{aligned}\nu_D&\equiv[1+\mathfrak{N}_1^*/(n_0+p_0)](\nu_{gn1}+\nu_{gp1}),\\\nu_v&\equiv\nu_{gn1}+\nu_{gp1}+(\nu_{gn1}-\nu_{gp1})\mathfrak{N}_1^*/(n_0-p_0).\end{aligned}\tag{62}$$

The frequency ν_v will be referred to as the "straggle constant." It is readily shown that the linear differential equations which Δn and $\Delta\hat{n}$ satisfy are entirely similar to Eq. (61) except for suitable modifications of the right-hand member; all the concentrations satisfy the same equation if there is no volume generation. For Δn it suffices to replace ν_{tp1} where it occurs explicitly by ν_{tn1}, while for $\Delta\hat{n}$ only the generation term $(\nu_{tn1}-\nu_{tp1})\Delta g$ occurs, $\partial\Delta g/\partial t$ being absent. It can also be shown that recombination of lifetime τ_3 in other centers in extrinsic material can be taken into account by adding τ_3^{-1} to the coefficient ν_s of $\partial\Delta n/\partial t$ and $\partial\Delta p/\partial t$ and $(\nu_s-\nu_{tp1})/\tau_3$ or $(\nu_s-\nu_{tn1})/\tau_3$ for n- or p-type material to the coefficient Δ_1 of Δn and Δp.

3.51 Steady-State Transport; Reverse Drift

A simple case which yields qualitative information of interest is that of injection into a filament in the steady state with applied field. For this case,

$$\nu_D D_0 d^2\Delta p/dx^2-\nu_v v_0 d\Delta p/dx-\Delta_1\Delta p=0\tag{63}$$

[42] The approach to constancy with increasing concentration of centers is discussed in reference 40.

is to be solved for, say, Δp zero for distance x along the filament negatively or positively infinite and continuous at the origin at which there is carrier injection with zero injected total-current density. Equation (63) is easily shown to be equivalent to Eqs. (15) and (16) specialized for no volume generation and acceptor centers only; ν_D and ν_v are $\nu_s-\nu_{tp1}$ times D_0'/D_0 and v_0'/v_0, respectively, and τ_p from Eqs. (36) is $(\nu_s-\nu_{tp1})/\Delta_1$.

The solutions in the semi-infinite regions separated by the origin are $\exp(r_1x)$ and $\exp(r_2x)$ where r_1 and r_2 are given by

$$\begin{pmatrix} r_1 \\ r_2 \end{pmatrix} = \tfrac{1}{2}\left[\nu_v v_0/\nu_D D_0 \pm \left[(\nu_v v_0/\nu_D D_0)^2+4\Delta_1/\nu_D D_0\right]^{\frac{1}{2}}\right] \sim \begin{pmatrix} \nu_v v_0/\nu_D D_0 \\ -\Delta_1/\nu_v v_0 \end{pmatrix}, \quad (64)$$

as obtained from Eq. (63). The case of recombination without appreciable trapping[5] presents no unfamiliar features; the approximation given will accordingly be considered, which is that for Δ_1 small, as may result from one of the capture coefficients small. The magnitude of r_1 is thus large compared with that of r_2. With the condition $v_0>0$, which may be assumed without loss of generality, $\exp(r_1x)$ gives the familiar sharply varying field-opposing solution to the left of the origin and $\exp(r_2x)$ gives the corresponding gradually varying field-aiding solution to the right, provided ν_v is positive; thus r_1 and r_2 are, respectively, positive and negative. But negative ν_v can occur, for which an anomalous behavior obtains, the field-opposing and field-aiding solutions then being, respectively, the gradually and sharply varying exponentials $\exp(r_2x)$ and $\exp(r_1x)$. For this case, in the limit of no diffusion, added carrier concentration appears only in the direction opposite to that of the ambipolar drift velocity, that is, opposite to the direction of drift normally determined by conductivity type.

This "reverse drift" associated with trapping may be understood in terms of properties of the current density $\Delta\mathbf{I}$ of added carriers. From Eq. (12), added carriers drift in the direction of the total current density, or the contribution to $\Delta\mathbf{I}$ from drift has the sign of $\mathbf{I}$, if $n_0\Delta p-p_0\Delta n$ or $\Delta p/\Delta n-p_0/n_0$ is positive, that is, if injection results in proportionately more holes than electrons than is the case at thermal equilibrium. This behavior is, of course, that which normally occurs in n-type material; with no trapping, $\Delta p/\Delta n$ equals unity and added carriers drift with or opposite to $\mathbf{I}$ according to whether the semiconductor is n-type or p-type, with no drift in intrinsic material.[3,5] Thus, the normal behavior requires the conditions that $\Delta p/\Delta n-p_0/n_0$ be positive in n-type material and negative in p-type. It is easily shown by writing these conditions by means of Eqs. (36) for the steady-state value $(1-r_n)^{-1}$ of $\Delta p/\Delta n$ that both are tantamount in the steady state to the single condition, $\nu_v>0$. This condition clearly always holds for the majority-carrier trapping range, while reverse drift results for sufficient minority-carrier trapping in not too strongly extrinsic material. From Eqs. (62), $\nu_v>0$ gives

$$p_0-n_0>(\nu_{gn1}-\nu_{gp1})\mathfrak{N}_1^*/(\nu_{gn1}+\nu_{gp1}) \quad (65)$$

for p-type material, and a similar inequality for n-type obtainable by changing the sign of each side. Equating the two sides gives the condition for no drift which, for no trapping, holds for intrinsic material. For electron trapping without recombination, the right-hand side reduces to $\mathfrak{N}_1^*$; for this case, since $\mathfrak{N}_1^*$ equals $n_0\nu_{tn1}/\nu_{gn1}$ from Eqs. (35), reverse drift obtains if n_0/p_0 in p-type material exceeds $\tau_{tn1}/(\tau_{tn1}+\tau_{gn1})$, the fraction of the time electrons are free. A similar result holds for hole trapping in n-type material.

3.52 Drift of an Injected Pulse

The differential equation for drift with negligible diffusion and no volume generation in one Cartesian dimension with trapping by centers of a single type is

$$\partial^2\Delta p/\partial t^2+v_0\partial^2\Delta p/\partial x\partial t+\nu_s\partial\Delta p/\partial t+\nu_v v_0\partial\Delta p/\partial x+\Delta_1\Delta p=0, \quad (66)$$

from Eq. (61). To solve Eq. (66) for a pulse of carriers injected into a doubly infinite filament, for which a suitable technique is that of the bilateral or two-sided Laplace transform with respect to the distance variable, the equation is put into a particular dimensionless form: Independent variables

$$X\equiv x/L, \quad U\equiv t/\tau, \quad (67)$$

are introduced, and with distance and time units given by

$$L\equiv v_0\tau, \quad \tau\equiv(|\nu^2|)^{-\frac{1}{2}},$$
$$\nu^2\equiv 4[\nu_v(\nu_s-\nu_v)-\Delta_1] \equiv 4n_i^2(\nu_{tn1}-\nu_{tp1})^2(p_0-n_0)^{-2}\cdot[(\nu_{tn1}+\nu_{tp1})(p_0-n_0)/(\nu_{tn1}-\nu_{tp1})\mathfrak{N}_1^*-1], \quad (68)$$

subject to the restrictions $\nu^2\neq 0$ and $n_0\neq p_0$, the reduced equation

$$\partial^2\Delta p/\partial U^2+\partial^2\Delta p/\partial X\partial U+\zeta\partial\Delta p/\partial U+\tfrac{1}{2}(\zeta+\kappa)\partial\Delta p/\partial X+\tfrac{1}{4}(\zeta^2-\kappa^2\mp 1)\Delta p=0, \quad (69)$$

results, where κ and ζ are the parameters

$$\kappa\equiv(2\nu_v-\nu_s)\tau=[\nu_{gn1}+\nu_{gp1}+(n_0+p_0)(\nu_{tn1}-\nu_{tp1})/(n_0-p_0)]\tau, \quad (70)$$
$$\zeta\equiv\nu_s\tau=(\nu_{rn1}+\nu_{gn1}+\nu_{tp1}+\nu_{gp1})\tau>0.$$

Coefficient unity for the second term of Eq. (69) results from the definition of L. The double sign in the last term of the equation results from the necessity of defining a real (and positive) τ, the upper and lower signs applying, respectively, for positive and negative ν^2.

Solutions for the initial delta pulse at the origin of $\mathcal{P}$ carrier pairs injected per unit area of cross section are, as may be verified without difficulty,

$$\Delta P \equiv \Delta p/(\mathcal{P}/L) = \{\exp[\kappa X - \tfrac{1}{2}(\zeta+\kappa)U]\}\Big\{\delta(U-X) + \tfrac{1}{2}\Big[(\xi-\kappa)\frac{I_0}{J_0}([X(U-X)]^{\frac{1}{2}}) \pm \frac{I_1}{J_1}([X(U-X)]^{\frac{1}{2}}) \cdot \frac{X}{[X(U-X)]^{\frac{1}{2}}}\Big]\mathbf{1}[X(U-X)]\Big\}, \quad (71)$$

$$\Delta \hat{N} \equiv \Delta \hat{n}/(\mathcal{P}/L) = \tfrac{1}{2}(\xi-\eta)\{\exp[\kappa X - \tfrac{1}{2}(\zeta+\kappa)U]\} \cdot \frac{I_0}{J_0}([X(U-X)]^{\frac{1}{2}})\mathbf{1}[X(U-X)],$$

where ξ and η are the parameters

$$\xi \equiv (\nu_s - 2\nu_{tp1})\tau, \quad \eta \equiv (\nu_s - 2\nu_{tn1})\tau. \quad (72)$$

For $\Delta N \equiv \Delta n(\mathcal{P}/L)$, ξ in ΔP is replaced by η. The modified Bessel functions I_0 and I_1 apply for the upper sign in Eq. (69), that is, for ν real, while the Bessel functions J_0 and J_1 apply for ν imaginary. The term in ΔP and ΔN with the delta function $\delta(U-X) = v_0\tau \cdot \delta(v_0 t - x)$ represents a contribution that drifts at velocity v_0. The continuous distributions are confined to the interval $0 \leq x \leq v_0 t$, $\mathbf{1}[X(U-X)] = \mathbf{1}[x(v_0 t - x)]$ being the step function that is, respectively, zero and unity for negative and positive values of its argument.[43,44]

An illustrative case of minority-carrier trapping in strongly extrinsic material, for which ν is real and the interpretation comparatively straightforward, will be presented first. For strongly extrinsic material, since the parameter ξ or η for minority carriers is substantially equal to κ, the minority-carrier concentration does not include the term with the Bessel function I_0. If, also, the trapping is nonrecombinative, then $\zeta = (\nu_g + \nu_t)\tau$ and $\kappa = (\nu_g - \nu_t)\tau$ hold with $\nu^2 = 4\nu_t\nu_g$, where ν_t and ν_g are ν_{tn1} or ν_{tp1} and ν_{gn1} or ν_{gp1}, respectively, and refer to the minority carrier. Figure 1 shows distributions of mobile minority carriers for $\zeta = 5/4$ and $\kappa = -3/4$, as for release time equal to 4 times the trapping time. The continuous distribution is shown for different times after injection at the origin of the neutral delta pulse. This distribution is led by a delta pulse which drifts at the ambipolar velocity v_0. This remnant of the initial pulse is composed of untrapped carriers; it is attenuated comparatively rapidly by the exponential factor $[\exp(-U)$ for the particular values of the parameters$]$ as the area under the distribution of actual concentration increases asymptotically, as can be shown,[2] to $\frac{1}{5}\mathcal{P}$, the initial pulse strength multiplied by the fraction of the time the carriers are free. The abrupt front of the distribution for the shorter times results from most carriers having been trapped only slightly—at least once, but not much more. A relative maximum within the drift range appears comparatively soon, and the abrupt front then progressively disappears as a result of multiple trapping. Furthermore, there is a reduction of apparent mobility: The maximum drifts, as will be shown, at a velocity which decreases asymptotically to $\frac{1}{5}v_0$, the fraction of v_0 equal to the fraction of the time the carriers are free. It appears from the figure that this limiting velocity is approached comparatively slowly. By suitable approximation involving large U, it is readily shown that[45] the distribution also becomes increasingly Gaussian in shape, particularly about its maximum, spreading as if the carriers were subject only to drift and diffusion with diffusivity (for nonrecombinative trapping) given by $v_0 L/4\zeta^3 = v_0^2\nu_t\nu_g/(\nu_t+\nu_g)^3$.

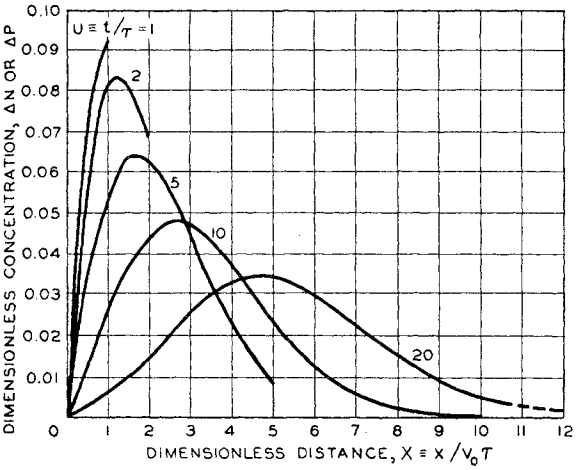

FIG. 1. Continuous concentration distributions at different times of mobile minority carriers from an injected neutral delta pulse for drift with trapping. A strongly extrinsic semiconductor and $\zeta = 5/4$, $\kappa = -3/4$ are assumed, as for nonrecombinative trapping with release time, τ_g, equal to 4 times the trapping time, τ_t.

Some of these results depend on certain general properties of the parameters. From the first forms for κ and ζ of Eqs. (70) and the definitions of τ and ν^2 of Eqs. (68),

$$\zeta = (\kappa^2 \pm 1 + 4\Delta_1\tau^2)^{\frac{1}{2}} \geq (\kappa^2 \pm 1)^{\frac{1}{2}} \quad (73)$$

follows. The inequality sign is associated with recombination, Δ_1 being zero for nonrecombinative trapping. The parameter ζ is real and never negative. For ν imaginary, so that the lower sign applies, a similar

[43] Fan (reference 34 and private communication) has given a solution of this drift problem which applies for negligible majority-carrier capture frequency. Clarke (reference 38) has, in effect, pointed out this restriction, to which solutions in references 34 and 35 for the decay of photoconductivity are also subject.

[44] A. K. Jonscher, Proc. Phys. Soc. (London) **B70**, 223 (1957) has given solutions for drift of minority carriers with recombination and nonrecombinative trapping at variance with solutions given here. See reference 2, footnote on pp. 526, 527.

[45] Use is made of the approximations

$$I_0(z) \sim I_1(z) \sim (2\pi z)^{-\frac{1}{2}}\exp(z) \quad \text{for} \quad |z| \text{ large.}$$

calculation gives

$$\kappa^2=1+(\nu_s^2-4\Delta_1)\tau^2\geq 1, \quad \nu^2<0; \tag{74}$$

the condition $\nu_s^2-4\Delta_1\geq 0$ implies real decay constants and holds from Eqs. (57). For ν real, κ is not restricted. For example, for nonrecombinative trapping in strongly extrinsic material κ is $\frac{1}{2}[(\nu_g/\nu_t)^{\frac{1}{2}}-(\nu_t/\nu_g)^{\frac{1}{2}}]$ and can be zero or have any positive or negative value. Thus, $\zeta\geq 1$ holds for ν real and $\zeta\geq 0$ holds for ν imaginary. Also, for nonrecombinative trapping, the parameters do not depend on the capture coefficient and $\zeta=(\kappa^2\pm 1)^{\frac{1}{2}}$ equals ξ or η according to whether electrons or holes are trapped.

The maxima for sufficiently large U of the continuous distributions of mobile and trapped carriers for cases of real ν occur substantially together. They are found to be given by[45] $X/U=x/v_0t=\frac{1}{2}[1+\kappa/(\kappa^2+1)^{\frac{1}{2}}]$, from Eqs. (71). It is easily seen that for nonrecombinative traps real ν implies minority-carrier trapping with positive ν_v and $(\kappa^2+1)^{\frac{1}{2}}$ equal to ζ. From Eqs. (35), (62), and (70), X/U for the maximum and large U accordingly reduces to $\nu_v/(\nu_t+\nu_g)$, which is

$$[\tau_{tn1}-n_0\tau_{gn1}/(p_0-n_0)]/(\tau_{tn1}+\tau_{gn1})$$

for electron trapping or

$$[\tau_{tp1}-p_0\tau_{gp1}/(n_0-p_0)]/(\tau_{tp1}+\tau_{gp1})$$

for hole trapping. Hence this X/U, the factor by which the apparent mobility is smaller than the magnitude of the ambipolar pseudomobility,[3] is in general less than $\tau_t/(\tau_t+\tau_g)$, the fraction of the time minority carriers are free; but this X/U is substantially equal to the free-time fraction[46] under the condition $|n_0-p_0|\gg\mathfrak{N}_1^*$, obtained with the use of Eqs. (35). As $|n_0-p_0|$ approaches $\mathfrak{N}_1^*$ in the nonrecombinative case, X/U approaches zero. Recombination reduces the distance for a maximum at given time, and thus reduces the apparent mobility, since for nonrecombinative traps with recombination of lifetime τ_3 in other centers the distribution of the mobile carriers subject to trapping is simply that for no recombination multiplied by the decay factor $\exp(-x/v_0\tau_3)$. This factor applies because the carriers which arrive at x at whatever time have drifted in the conduction band for time x/v_0.

The decay constant for the straggle effect, or the limiting decay time at fixed x for the tail of the distribution after the maximum has drifted past, is readily evaluated. With the condition $X\ll U$ for the tail of the distribution, this decay constant is clearly the coefficient of t in the exponent of Eqs. (71). It is thus $\frac{1}{2}(\zeta+\kappa)/\tau=\nu_v$, and ν_v has accordingly been named the "straggle constant." It is easily seen from Eqs. (35) and (62) that ν_v for strongly extrinsic material is substantially $\nu_{gn1}+\nu_{gp1}$ plus either ν_{tn1}, for $n_0\gg p_0$, or ν_{tp1}, for $p_0\gg n_0$.

[46] Fan (reference 34) has shown from his solution that for relatively small trap concentration X/U for the maximum approaches the free-time fraction.

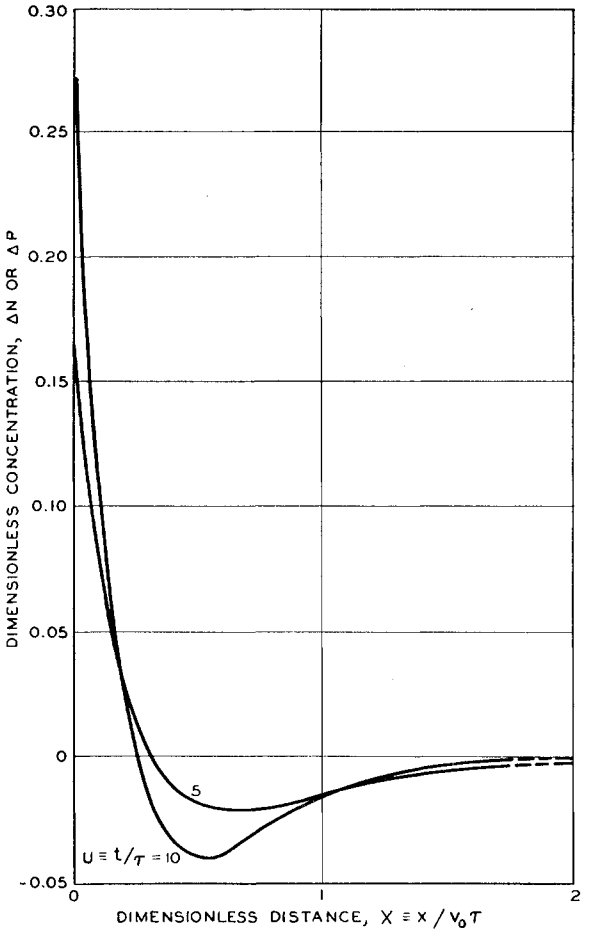

FIG. 2. Continuous concentration distributions at different times of mobile minority carriers from an injected neutral delta pulse for drift with trapping in the reverse-drift range. Equilibrium majority-carrier concentration is taken as $5n_i$ and nonrecombinative trapping is assumed with $\mathfrak{N}_1^*\gg|n_0-p_0|$, whence $0.1\tau_g\gg\tau=2.4\tau_t$ and $\zeta=2.4$, $\kappa=-2.6$ hold, with 0.1 the coefficient of the term in J_0.

Imaginary ν obtains with $\kappa>1$ over the majority-carrier trapping range, and it obtains with $\kappa<-1$ over a range which includes the reverse-drift range, as can be shown[2] from Eqs. (62) and (68). With recombination in the centers, the reverse-drift range applies for the finite range, $\kappa_r<\kappa<-1$. For nonrecombinative trapping, $\kappa<-1$ (with imaginary ν) gives the reverse-drift range. This case is illustrated in Fig. 2, which shows the continuous distribution of mobile minority carriers at differing times for majority-carrier concentration n_0 or p_0 equal to $5n_i$ and $\mathfrak{N}_1^*\gg|n_0-p_0|$, namely $\tau_g\gg 24\tau_t$; the delta pulse of untrapped mobile carriers that leads the distribution is rapidly at-

tenuated by the factor $\exp(-2.5U)$. As the figure shows, the distribution crowds towards the injection point as its maximum excursions both above and below the axis increase with time: There is local carrier depletion, the distribution being negative over part of the drift range after a certain time.[47] The distribution approaches a pulse at the injection point of strength equal, for nonrecombinative trapping, to the initial strength times the free-time fraction. It does not exhibit essentially unidirectional drift: The drift of added carriers, initially in the direction of the ambipolar velocity, is largely in the opposite direction after some trapping has taken place. Numerical estimate of the effect of diffusion indicates that negative added-carrier concentrations can occur over appreciable distances under conditions which can be realized in practice.[48]

For a pulse injected in more strongly extrinsic material under the condition of large $\mathfrak{N}_1^*$ or τ_g which gives reverse drift, smaller concentration changes occur more slowly with U and are appreciable over a smaller fraction of the drift range.[49] For strongly extrinsic material, the exponential factor limits the solution to small X, since κ and ζ are large in magnitude. Approximating the Bessel functions for small values of their argument then gives, for p-type material,

$$\Delta N=[(n_i/2p_0)\exp(n_iU/2p_0)](1-x/v_0\tau_{tn1})\cdot\exp(-x/v_0\tau_{tn1}). \quad (75)$$

This approximate solution bears out the statements made: Because of $\frac{1}{2}(n_i/p_0)\tau_{gn1}\gg\tau=\frac{1}{2}(p_0/n_i-n_i/p_0)\tau_{tn1}$, an increase in p_0 for given τ requires larger τ_{gn1} and, in particular, decreases τ_{tn1} as well as $n_i/2p_0$. A curve from Eq. (75) of ΔN versus $x/v_0\tau_{tn1}$ is qualitatively similar in shape to the curves of Fig. 2: ΔN decreases to a negative relative minimum at $x/v_0\tau_{tn1}=2$ and then asymptotically approaches zero.

The current density ΔI of added carriers provides further interpretations. From Eqs. (12) and (71), this is given by

$$\Delta I=e^2\mu_n\mu_p\sigma_0^{-2}I(\mathcal{P}/L)(n_0\Delta P-p_0\Delta N)$$
$$=ev_0(\mathcal{P}/L)\{\exp[\kappa X-\tfrac{1}{2}(\zeta+\kappa)U]\}\cdot\left\{\delta(U-X)\begin{matrix}+\frac{1}{2}I_1\\-\frac{1}{2}J_1\end{matrix}([X(U-X)]^{\frac{1}{2}})\frac{X}{[X(U-X)]^{\frac{1}{2}}}\right\}\cdot 1[X(U-X)] \quad (76)$$

for the present case of one-dimensional drift with no applied magnetic field. That the Bessel functions of order zero do not occur follows from the easily verified relationship $n_0\xi-p_0\eta=(n_0-p_0)\kappa$. These functions are accordingly associated with carriers that neutralize the charge of trapped carriers or with the trapped carriers themselves, while those of order one are associated with the drift of, in effect, carrier pairs. The direction of drift of a mobile-carrier distribution considered in its entirety depends on the sign of the net ΔI, or ΔI integrated over the drift range. For nonrecombinative trapping and large U, the integral over the drift range of the concentration of mobile carriers that are subject to trapping equals $\mathcal{P}$ times the free-time fraction. The corresponding value of the integral of ΔI with respect to X for electron trapping is therefore $ev_0(\mathcal{P}/L)$, the initial ΔI, times[50] $\nu_v/(\nu_{tn1}+\nu_{gn1})$. Thus, as may be expected, the limiting value of the integral for large U has the sign of v_0 or the opposite sign according to whether ν_v is positive or negative. This result supports the conclusion that, for the reverse-drift case, the distribution from an injected pulse ultimately crowds towards the origin, where $\Delta I=0$ holds.

4. ACKNOWLEDGMENTS

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[47] W. Kaiser (private communication) has suggested that negative added-carrier concentrations which were observed with localized optical injection in silicon under applied field may be accounted for through these results. A theoretical discussion of carrier depletion is included in reference 3.

[48] For $p_0=5n_i$ and electron and hole mobilities of 1500 and 570 cm^2 volt^{-1} sec^{-1}, as for silicon at 300°K, $L\equiv v_0\tau$ is 0.12 cm for $\tau=2.4\tau_{tn1}=10^{-5}$ sec and an applied field of 10 volt cm^{-1}. The diffusion distance $(D_0t)^{\frac{1}{2}}$ for 10τ is 0.06 cm, which is appreciably smaller than the approximate distance $2L=0.24$ cm over which negative added-carrier concentrations occur.

[49] See Fig. 3 of reference 2, which shows a more nearly intrinsic reverse-drift case.

[50] For real ν, which implies positive v_v and p-type material, this factor is the limiting value of X/U for the maximum of the mobile-electron distribution.