Tensor Force Effects in Odd-Odd Spherical Nuclei*

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A study of the ground-state coupling rules in odd-odd spherical nuclei reveals that in almost all cases where Nordheim's weak rule is applicable both particles have spin and orbital angular momentum parallel rather than antiparallel. A semiclassical model is employed to indicate that an attractive *n-p* tensor force will tend to break down Nordheim's weak rule. Examination of the quantum-mechanical formula substantiates this finding.

A RECENT article has surveyed our present experimental and theoretical understanding of the ground-state coupling rules in odd-odd nuclei.[1] As summarized there, studies with zero-range central forces have yielded a qualitative theoretical foundation for Nordheim's coupling rules (we use the form of these rules as given in reference 1). The general feature of the situation is that the ground state of an odd-odd nucleus will be that state which maximizes a combination of (a) overlap of the wave functions of the two particles and (b) the amount of triplet state in the wave function.

Such a physical picture predicts that in a situation where Nordheim's strong rule is applicable ($j_1=l_1\pm\frac{1}{2}$, $j_2=l_2\mp\frac{1}{2}$) both (a) and (b) can be simultaneously maximized and the ground state will be $J=|j_1-j_2|$. (Calculations with zero-range central forces indicate that $J=|j_1-j_2|$ is the lowest in energy and there is a considerable gap between it and the next-higher J states.) In a case where Nordheim's weak rule is applicable ($j_1=l_1\pm\frac{1}{2}$, $j_2=l_2\pm\frac{1}{2}$), the situation is not so clear. We now find that (a) and (b) cannot both be maximized. Detailed studies with short-range central forces reveal that in general there will be *several* J values in close competition for the ground state. A decision as to which of these competing J values will be the ground state in any particular nucleus will require a more detailed study of the wave functions and interactions involved. It is the purpose of the present note to show that an *n-p* tensor force will act in such a way as to contribute to the breakdown of Nordheim's weak rule.

One reason that tensor-force effects have been neglected in studies of the coupling rules in odd-odd nuclei is the complexity of the general matrix-element formula.[2] In contrast to central forces, the tensor-force matrix-element expression does not contain a convenient zero-range limit (at zero range, tensor effects vanish). Going to infinite-range also produces no simplification.

In an attempt to investigate the qualitative features of the tensor-force interaction, a semiclassical (SC) model has been used. This model exploits the similarity under rotations of the tensor interaction

$$V(r)=U_T(r)\left[\frac{3(\boldsymbol{\sigma}_1\cdot\mathbf{r})(\boldsymbol{\sigma}_2\cdot\mathbf{r})}{r^2}-\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\right], \quad (1)$$

and the interaction energy of two magnetic dipoles of magnetic moment $\mathbf{M}_1$ and $\mathbf{M}_2$,

$$V'(r)=\left(-\frac{1}{r^3}\right)\left[\frac{3(\mathbf{M}_1\cdot\mathbf{r})(\mathbf{M}_2\cdot\mathbf{r})}{r^2}-\mathbf{M}_2\cdot\mathbf{M}_2\right]. \quad (2)$$

If one now examines the behavior of Eq. (2) with various particle distributions, it becomes apparent that a definite statement can be made regarding the expectation value of Eq. (1) in certain quantum-mechanical states.

As an example, consider the state

$$|l_1\tfrac{1}{2}j_1=l_1+\tfrac{1}{2},\, l_2\tfrac{1}{2}j_2=l_2+\tfrac{1}{2};\, J=j_1+j_2\rangle = {}^3(L=l_1+l_2)_J. \quad (3)$$

As indicated, this state is also pure in L-S coupling. In a classical interpretation we may say that the two particles move in the same plane and have their intrinsic spins parallel to each other and perpendicular to the plane of their orbits (see discussion in reference 1). In such a case, the first term of Eq. (2) is zero, and the average effect is a raising of the energy of the system by the interaction. Thus, even though $U_T(r)$ is less than zero (as in the deuteron), the tensor-force effect may be to raise the energy of a whole class of quantum-mechanical states. This is, perhaps, not surprising. The quantum-mechanical coupling displayed in Eq. (3) restricts the particles to move in such a way that the interaction (1) is largely repulsive. (A similar physical picture of the probability distributions can be constructed in the case of any *n-p* configuration that is pure in L-S coupling. In what follows we have only applied the model to such *j-j* states.) This semiclassical procedure should at least give the correct sign of the diagonal matrix element and have increasing validity for high-angular-momentum quantum numbers.

In order to investigate the above conclusions, a previously used approximation was employed.[2] This approximation yields an exact answer for the tensor-

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[1] C. J. Gallagher, Jr., and S. A. Moszkowski, Phys. Rev. **111**, 1282 (1958).
[2] N. Newby, Jr., and E. J. Konopinski, Phys. Rev. **115**, 434 (1959).

force diagonal matrix element in the case of a certain long-range force which gives more weight to interactions at large rather than small distances. The SC model indicates, however, that in certain classes of two-particle states the sign of the matrix element is determined by the particle coupling and does not depend on details of well shapes.

Use of this approximation procedure substantiated the prediction of the SC model that an *attractive* tensor force would *raise* the energy level of any state in class (3). The ability to make this statement depends on the fact that the Racah coefficients in the general matrix-element formula all have a "stretched" triangular condition for all states of class (3). Such Racah coefficients are always positive.

Examination of the tables of coupling schemes for odd-odd nuclei reveals several interesting points.[1,3] In situations where the strong rule is applicable, we find a total of 52 cases. The strong rule fails in five cases. In two of these five cases we have evidence that the tensor interaction plays a vital role.[2] In the 47 cases where the strong rule holds, the SC model can be applied to 11 cases. In these 11 cases the model indicates that the tensor effect may be repulsive in the ground state. Actually the situation is not clear, for in five of these cases one of the particles is in an S state. In the great majority of situations where the strong rule holds, the ground state is mixed in L-S coupling, and the tensor-force effect is probably to lower the energy of the level.

One can make a more definitive statement in the cases where the weak rule applies. Out of 23 cases where the weak rule is applicable, we find that 20 cases have $j_1=l_1+\frac{1}{2}$, $j_2=l_2+\frac{1}{2}$ *rather* than spin and orbital momentum antiparallel for both particles. In these 20 cases, the weak rule would predict that the ground state should be given by the coupling in Eq. (3). We find that in these cases the weak rule holds nine times and fails eleven times. Our discussion shows that an attractive n-p tensor force will raise such a state [Eq. (3)] in energy and thus bias against it being the ground state.

Calculations with zero-range central forces generally show a competition between $J=j_1+j_2$ and $J=|j_1-j_2|$ for the ground state in a weak-rule situation. A recent study[4] has shown that in such a coupling situation the observed ground state is often $J=|j_1-j_2|$ rather than $J=j_1+j_2$ (see also reference 1). The SC model described here shows that an attractive n-p tensor force would produce level shifts tending to move $J=|j_1-j_2|$ downward in energy with respect to $J=j_1+j_2$.

[3] K. Way, D. N. Kundu, C. L. McGinnis, and R. van Lieshout, *Annual Review of Nuclear Science* (Annual Reviews, Inc., Palo Alto, California, 1956), Vol. 6, p. 129.

[4] M. H. Brennan and A. M. Bernstein, Bull. Am. Phys. Soc. 5, 20 (1960).

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Photoproton and Photoneutron Production in Aluminum and Copper*

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The ratio of proton to neutron yields from aluminum and copper irradiated with betatron x rays up to 20.8 Mev in energy has been measured. Simultaneous detection of protons and neutrons is accomplished by placing two samples of the same element in series in the x-ray beam. Direct detection methods are used in each case, a shallow proportional counter for protons and a boron-lined detector for neutrons. A photon difference method has been used to reduce yield data to cross-section form. The proton and neutron yields for aluminum are found to be approximately equal at 20 Mev with cross sections of 19 and 21 millibarns, respectively. At 20.8 Mev a yield ratio of one proton to about 6 neutrons is found for copper, with a peak photoproton cross section of 23 millibarns. The results are compared to a calculation based on the assumption that these reactions proceed through the formation of a compound nucleus.

I. INTRODUCTION

ALTHOUGH an abundant literature relating to the subject of photonuclear reactions has been built up in recent years, the preponderance of the available material is concerned with neutron emission, owing to the difficulties involved in the direct detection of photoprotons.[1]

Further information concerning the characteristics of charged particle reactions would be of use for example in investigating the applicability of the compound nucleus concept to photonuclear reactions. The proton energy spectrum and the size of the photoproton cross section relative to the photoneutron cross section

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[1] For a concise summary of earlier work in this field, see G. R. Bishop and R. Wilson, in *Handbuch der Physik*, edited by S. Flügge (Springer-Verlag, Berlin, 1957), Vol. XLII, p. 332.