Proposal for Determination of $\Sigma\Lambda$ Relative Parity from Chew and Low Analysis of Reactions of the Form $A+B \rightarrow C+D+E+\cdots$*

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We use the Chew and Low analysis of reactions of the form $A+B \rightarrow C+D+E+\cdots$, which makes use of the existence of a pole in the S matrix, to propose an experiment to determine the $\Sigma\Lambda$ relative parity and the coupling constant for the $\Sigma\Lambda\pi$ interaction. It is found that the sign of the extrapolated cross section for the reaction $\Sigma^+ + d \rightarrow \Lambda^0 + p + p$ is different for the two parity cases. Other applications of the Chew and Low method to strange particle reactions are briefly looked into.

I

CHEW and Low[1] have proposed a method of analyzing reactions of the general type:

$$A+B \rightarrow C+D+E+\cdots \tag{1}$$

where the final state has three or more particles.

Their method depends on the existence of a pole in the S matrix at the position where Δ^2, the four momentum transfer at vertex "a" squared (see Fig. 1), is equal to minus the square of the mass of a virtual particle which has the same properties (spin, charge, etc.) as particles $A+B$ and as $C+D+E+\cdots$. The existence of this pole, although a conjecture, is made plausible by arguments of Chew[2] and CL.

At the pole the renormalized[3] Born approximation is applicable. The differential cross section valid at the pole obtained in CL as applied to

$$\Sigma^+ + d \rightarrow \Lambda^0 + p + p \tag{2}$$

reads[4]

$$\frac{\partial^2\sigma}{\partial\Delta^2\partial\omega^2} \xrightarrow[\Delta^2=-M_\pi^2]{} \frac{\Gamma^2}{2\pi}\frac{M_\Lambda}{M_\Sigma} \times \frac{\left[\frac{1}{4}\omega^4 - \frac{1}{2}\omega^2(M_d^2+M_\pi^2) + \frac{1}{4}(M_d^2-M_\pi^2)^2\right]}{\left[(W^2-M_\Sigma^2-M_d^2)/2M_\Sigma\right]^2 - M_d^2} \times \frac{\sigma_{\pi d}(\omega)}{(\Delta^2+M_\pi^2)^2}, \tag{3}$$

where

$$\omega^2 = -(p_\Sigma + p_d - p_\Lambda)^2 \tag{4}$$

the p's being 4-momenta. Thus ω^2 is the square of the total energy of the two protons in their barycentric system and, by conservation of energy, equals the total barycentric energy of the pion and deuteron.

$$W^2 = -(p_\Sigma + p_d)^2, \tag{5}$$

that is, W^2 is the total energy in the barycentric system of Σ and deuteron. $\sigma_{\pi d}(\omega)$ is the cross section for the process

$$\pi^+ + d \rightarrow p + p. \tag{6}$$

We assume an unpolarized initial state and no measurement is made of the final state. $4\pi\Gamma^2$ is the square of the Dirac matrix element at vertex "a" averaged over initial spins and summed over final spins.

Evaluating $\Delta^2 = (p_\Sigma - p_\Lambda)^2$ in the Σ^+ rest system, we get

$$\Delta^2 = 2M_\Sigma T_{\Lambda L} - (M_\Sigma - M_\Lambda)^2. \tag{7}$$

$T_{\Lambda L}$ is the recoil kinetic energy of the Λ^0 in the Σ^+ rest system. At the pole

$$T_{\Lambda L}^0 = -\left[M_\pi^2 - (M_\Sigma - M_\Lambda)^2\right]/2M_\Sigma. \tag{8}$$

$T_{\Lambda L}^0$ comes out to be -6 Mev which is unphysical but not unreasonable for extrapolation purposes.

The recipe to be followed is to extrapolate $(\Delta^2+M_\pi^2)^2$ times the experimental differential cross section, $\partial^2\sigma/\partial\Delta^2\partial\omega^2$, for the reaction $\Sigma^+ + d \rightarrow \Lambda^0 + p + p$ for fixed W^2 and fixed ω^2 to the pole at $\Delta^2 = -M_\pi^2$; use experimental data for the cross section $\sigma_{\pi d}(\omega)$; then, by means of Eq. (3), we can solve for Γ^2 and thus infer properties of the reaction at vertex "a," i.e., the $\Sigma\Lambda$ relative parity and the strength of the coupling for the $\Sigma\Lambda\pi$ interaction.

The relative parity has been assumed to be positive for several models of strange particle interactions, for instance, global[5] and cosmic[6] symmetry. Although of importance for these schemes and for our general understanding of strange particles, the $\Sigma\Lambda$ parity is not yet known. Several proposals have thus far been suggested in the literature for its determination.[7–11]

In addition we also propose experiments for the determination of $\Sigma\Sigma\pi$ and $\Xi\Xi\pi$ coupling constants although the reactions needed here are slightly more

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[1] G. F. Chew and F. E. Low, Phys. Rev. **113**, 1640 (1959), denoted as CL throughout the rest of the paper.

[2] G. F. Chew, Phys. Rev. **112**, 1380 (1958).

[3] The corrections to the vertex function and propagation function can entirely be absorbed in the coupling constant and in the mass of the virtual particle.

[4] We use units where $\hbar = c = 1$.

[5] M. Gell-Mann, Phys. Rev. **106**, 1296 (1957).

[6] J. J. Sakurai, Phys. Rev. **113**, 1679 (1959).

[7] D. Amati, Phys. Rev. **113**, 1692 (1959).

[8] A. Pais and S. B. Treiman, Phys. Rev. **109**, 1759 (1958).

[9] S. Barshay, Phys. Rev. Letters **1**, 177 (1958).

[10] S. Barshay and S. L. Glashow, Phys. Rev. Letters **2**, 371 (1959).

[11] J. G. Taylor, Nuclear Phys. **9**, 357 (1958–1959). This list of references on $\Sigma\Lambda$ parity proposals is not complete.

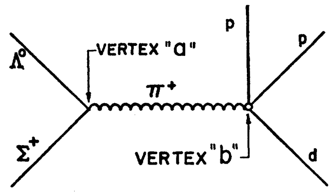

FIG. 1. Example of reaction where Chew and Low method is applicable. We will discuss the reaction $\Sigma^+ + d \rightarrow \Lambda^0 + p + p$ in this paper.

"unphysical" (in the sense that at the pole the recoil kinetic energy of the particle at vertex "a" [see Eq. (8)] will be more negative than in $\Sigma\Lambda\pi$ situation).

This information will serve to determine the correctness of global symmetry. In global symmetry model the pion baryon coupling constants are assumed to be the same.

II

Consider the reaction $\Sigma^+ + d \rightarrow \Lambda^0 + p + p$. Let Σ^+ be the incident particle and the deuteron be the target since this is experimentally the more likely situation. Then Δ^2 and ω^2 and W^2 can be determined from experimental data by means of the equations

$$\begin{aligned} \Delta^2 &= -M_\Lambda^2 - M_\Sigma^2 - 2p_{\Lambda L} p_{\Sigma L} \cos\theta_{\Sigma\Lambda} + 2E_{\Lambda L} E_{\Sigma L}, \\ \omega^2 &= -\Delta^2 + M_d^2 + 2E_{\Sigma L} M_d - 2E_{\Lambda L} M_d, \\ W^2 &= M_\Sigma^2 + M_d^2 + 2M_d E_{\Sigma L}. \end{aligned} \tag{9}$$

The Dirac part of the matrix element for the $\Sigma\Lambda\pi$ interaction for the case of even relative parity between the Σ and Λ is, in analogy with the nucleon-nucleon pion interaction,

$$i g_{\Sigma\Lambda} \bar{u}_\Lambda(p_\Lambda) \gamma_5 u_\Sigma(p_\Sigma), \tag{10}$$

where the u's are Dirac spinors.

If the relative parity between the Σ and Λ is odd, we have an additional γ_5,

$$g_{\Sigma\Lambda} \bar{u}_\Lambda(p_\Lambda) u_\Sigma(p_\Sigma). \tag{11}$$

Thus

$$\begin{aligned} \Gamma_{\text{odd}}^2 &= \tfrac{1}{2} g_{\Sigma\Lambda}^2 \sum_{\text{spins}} |\bar{u}_\Lambda(p_\Lambda) u_\Sigma(p_\Sigma)|^2, \\ \Gamma_{\text{even}}^2 &= \tfrac{1}{2} g_{\Sigma\Lambda}^2 \sum_{\text{spins}} |\bar{u}_\Lambda(p_\Lambda) \gamma_5 u_\Sigma(p_\Sigma)|^2. \end{aligned} \tag{12}$$

By the usual methods of inserting projection operators we evaluate Γ^2

$$\begin{aligned} \Gamma_{\text{odd}}^2 &= \frac{g_{\Sigma\Lambda}^2}{2M_\Sigma M_\Lambda}\left(+M_\Sigma M_\Lambda + \frac{\Delta^2 + M_\Sigma^2 + M_\Lambda^2}{2} \right), \\ \Gamma_{\text{even}}^2 &= \frac{g_{\Sigma\Lambda}^2}{2M_\Sigma M_\Lambda}\left(-M_\Sigma M_\Lambda + \frac{\Delta^2 + M_\Sigma^2 + M_\Lambda^2}{2} \right). \end{aligned} \tag{13}$$

At the pole we insert $\Delta^2 = -M_\pi^2$. Using[12]

$$M_\Sigma = 1190 \text{ Mev}, \quad M_\Lambda = 1115 \text{ Mev}, \quad M_\pi = 139.6 \text{ Mev},$$

we obtain that at the pole for a given value of the coupling constant

$$\Gamma_{\text{odd}}^2 / \Gamma_{\text{even}}^2 = 2647/(-6.93) = -382. \tag{14}$$

The presence of the minus sign will enable us to determine both the relative parity of the $\Sigma\Lambda$ system and the coupling constant for the $\Sigma\Lambda\pi$ interaction. For odd parity $(\Delta + M_\pi^2)^2 \partial^2\sigma / \partial\Delta^2 \partial\omega^2$ will be positive at the pole; for even parity it will be negative [the remaining factors in Eq. (3) are always positive for ω corresponding to a real physical process $\pi^+ + d \rightarrow p + p$]. Once the relative parity has been determined by observing the sign, Eq. (3) will give us the coupling constant. Assuming that odd and even coupling constants are not too much different, we note there is a large magnitude difference between the two parity cases. Hence even fairly crude data could give us a good indication of the $\Sigma\Lambda$ parity.

The amount of extrapolation depends on which ω^2 and which W^2 is used. An estimate of the "unphysicalness" of the situation is given by Eq. (8), $T_{\Lambda L}^0 \sim -6$ Mev.

A summary of experimental data with a graph of the total cross section for the process $p + p \rightarrow d + \pi^+$ plotted against pion momentum in barycentric system is given by Sachs et al.[13] The cross section we need may be obtained by detailed balance. Also a semi-empirical formula has been derived for $p + p \rightarrow d + \pi^+$ cross section near threshold.[14,15]

Using the best fit to the experimental data for $p + p \rightarrow d + \pi^+$ we have drawn a graph for the quantity $(\Delta^2 + M_\pi^2)^2 \partial^2\sigma / \partial\Delta^2 \partial\omega^2$ versus ω^2 at the pole for odd parity assuming the coupling constant squared is equal to unity in the approximation of neglecting the recoil motion of the Λ^0 in the Σ^+ rest system (see Fig. 2). This approximation follows from Eq. (8) where we see

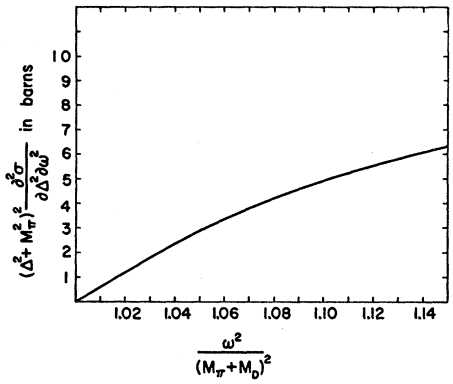

FIG. 2. $(\Delta^2 + M_\pi^2)^2 \partial^2\sigma / \partial\Delta^2 \partial\omega^2$ versus $\omega^2/(M_\pi + M_d)^2$ at the pole for odd $\Sigma\Lambda$ parity assuming $g_{\Sigma\Lambda}^2 = 1$ in the approximation of neglecting the recoil motion of the Λ in the Σ^+ rest system.

[12] J. D. Jackson, *The Physics of Elementary Particles* (Princeton University Press, Princeton, New Jersey, 1958), p. 51.

[13] A. H. Sachs, H. Winick, and B. Wooten, Phys. Rev. **109**, 1733 (1958).

[14] M. Gell-Mann and K. M. Watson, *Annual Review of Nuclear Science* (Annual Reviews, Inc., Palo Alto, California, 1954), Vol. 4, p. 219.

[15] D. B. Lichtenberg, Phys. Rev. **105**, 1084 (1957).

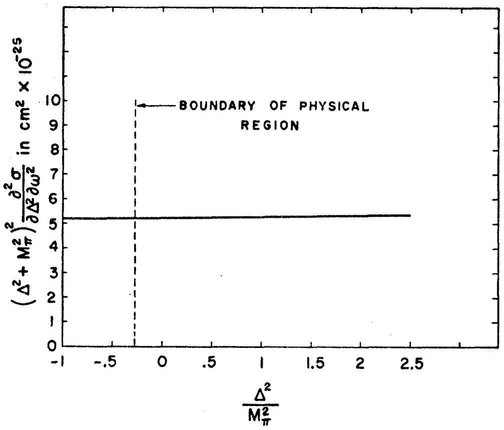

FIG. 3. $(\Delta^2+M_\pi^2)^2\partial^2\sigma/\partial\Delta^2\partial\omega^2$ versus Δ^2/M_π^2 in the Born approximation for odd $\Sigma\Lambda$ parity for $\omega^2/(M_\pi+M_d)^2=1.01$ and $T_{\Sigma L}=1197$ Mev assuming $g_{\Sigma\Lambda}^2=1$.

$T_{\Lambda L}^0 \ll M_\Lambda$. On expanding ω^2 and using $E_{\Sigma L}=E_{dL}M_\Sigma/M_d$, we get in this approximation

$$E_{\Sigma L}\sim\frac{M_\Sigma}{M_d}\frac{(\omega^2-M_\pi^2-M_d^2)}{2(M_\Sigma-M_\Lambda)}. \tag{15}$$

The graph for even parity will give the same curve times $-1/382$. These results can be compared with the experimental extrapolated quantity for the determination of the coupling constant. [Note: This approximation requires that the experiments must be done with $T_{\Sigma L}\gtrsim 1025$ Mev in order that $\sigma_{\pi d}(\omega)$ describe a physical process.[16]]

In Figs. 3 and 4, we have graphed $(\Delta^2+M_\pi^2)^2\partial^2\sigma/\partial\Delta^2\partial\omega^2$ against Δ^2 using the Born approximation[17] for odd and even parity for the specific case $T_{\Sigma L}=1197$ Mev and $\omega^2=1.01(M_\pi+M_d)^2$ assuming $g_{\Sigma\Lambda}^2=1$. This may be useful in showing a possible trend for the experimental data.

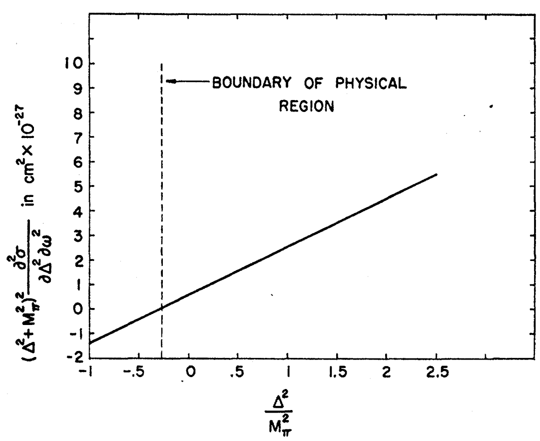

FIG. 4. $(\Delta^2+M_\pi^2)^2\partial^2\sigma/\partial\Delta^2\partial\omega^2$ versus Δ^2/M_π^2 in the Born approximation for even $\Sigma\Lambda$ parity for $\omega^2/(M_\pi+M_d)^2=1.01$ and $T_{\Sigma\Lambda}=1197$ Mev assuming $g_{\Sigma\Lambda}^2=1$.

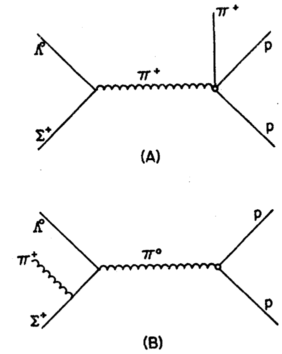

FIG. 5. $\Sigma^+ + p \to \Lambda^0 + p + \pi^+$.

Another possible process from which we might hope to attain the $\Sigma\Lambda$ parity is

$$\Sigma^+ + p \to \Lambda^0 + p + \pi^+. \tag{16}$$

This has the advantage that $\sigma(\omega)$ at vertex "b" now describes pion proton scattering $p+\pi^+ \to p+\pi^+$ which has been studied more fully experimentally than $\pi^+ + d \to p+p$. However, now we have the added complication of a second diagram that interferes with the first having a pole in the same neighborhood as the first (see Fig. 5). Therefore we conclude that $\Sigma^+ + d \to \Lambda^0 + p + p$ would be more reliable in determining the $\Sigma\Lambda$ parity than $\Sigma^+ + p \to p + \pi^+ + \Lambda^0$.

We may determine the $\Sigma\Sigma\pi$ and $\Xi\Xi\pi$ coupling constants by using the following diagrams (see Fig. 6). Equation (8) (applied to these cases gives about -8 Mev for Fig. 6(A) and about -7.5 Mev for Fig. 6(B) for extrapolation in recoil kinetic energy.

Within the framework of our approach, we note also that the reaction $K^+ + d \to K^0 + p + p$ can be used to determine the K^+K^0 relative parity. (As Pais[18] remarked,

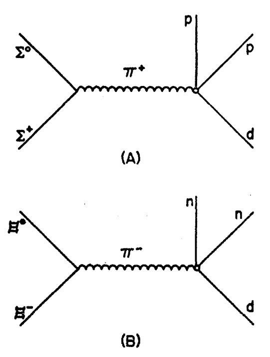

FIG. 6. (A) $\Sigma^+ + d \to \Sigma^0 + p + p$; (B) $\Xi^- + d \to \Xi^0 + n + n$.

[16] For $T_\Sigma \lesssim 1025$ Mev we may still hope to get an indication of the $\Sigma\Lambda$ relative parity from the sign of the extrapolated quantity as before. (This would not be so if there were any zero, of $\sigma_{\pi d}(\omega)$ in the extrapolated region.) We could not, however, calculate $g_{\Sigma\Lambda}^2$ since we cannot estimate the magnitude of $\sigma_{\pi d}(\omega)$ for ω corresponding to a nonphysical value of $\pi^+ + d \to p + p$.

[17] The pion is off the mass shell away from the pole. In the lowest order, when not too far from the pole we take the pion to be real and use the experimental cross sections in reference 13.

[18] A. Pais, Phys. Rev. **112**, 624 (1958).

this is not necessarily even.) For even parity the extrapolated quantity in the case of a virtual pion should be identically zero, since the vertex $KK\pi$ is forbidden. (We are, in effect, extrapolating to a pole where no pole exists; hence, the extrapolated quantity is zero.) For odd parity the $KK\pi$ vertex is allowed but since odd relative parity implies charge independence is violated, the $g_{KK\pi}$ coupling constant is small. However experimental methods are not precise enough to deal with such a small effect.

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Commutation Relations of Quantum Mechanics*

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The mathematical and physical meaning of the commutation relations of nonrelativistic quantum mechanics is discussed in terms of the representation of translations, Galilean transformations, and rotations of the coordinate system by unitary transformations acting on the unitary vector space of quantum states.

INTRODUCTION

THE discussion of this paper is confined to statements concerning part of the conceptual structure of the nonrelativistic quantum mechanics of particles, even though the arguments may be extended to the discussion of relativistic quantum field theories. This restriction makes it possible to study the essential points that are involved without the use of cumbersome formulas.

Most treatises on quantum mechanics include among the various postulates of the theory a statement of the fundamental commutation relations between the Cartesian components of the coordinate and the canonical momentum of a particle:

$$(x_i,p_j)=i\hbar\delta_{ij}. \qquad (1)$$

Quite naturally, a great deal of attention is paid to the physical consequences of these relations as expressed by the Heisenberg uncertainty principle. However, with few exceptions,[1,2] there is little discussion of the mathematical and physical ideas which underlie them. These ideas are concerned with the representation of translations, Galilean transformations, and rotations of the coordinate system by unitary transformations acting on the unitary vector space of quantum states.

The author has discussed the commutation relations with many physicists during the past few years and has found that only the most sophisticated among them are familiar with the ideas involved. The present article is concerned with an attempt to present them in a simple and concise fashion to a wider audience. It should be remarked here that this situation has been clearly recognized by Schwinger,[3] who has given a concise and complete statement of the laws of quantum physics in terms of his general dynamical principle, the quantum analog of Hamilton's principle. His discussion has not appeared in textbook form, however. Furthermore, Schwinger deals with the most general situation appropriate to relativistic, localizable field theories. Consequently, it is not easy to divide his arguments into their various parts in order to clearly recognize the concepts that are involved because the generality of the problem that he attacks requires the use of elaborate mathematical techniques, which are not necessary for the analysis of the simpler problem to be discussed here.

RELATION BETWEEN THE COORDINATE SYSTEM AND UNITARY VECTOR SPACE OF QUANTUM STATES

The basic postulates of quantum mechanics assert that a physical system is described by a vector which is an element of a linear unitary vector space and that observables are represented by Hermitian operators whose eigenvectors may be used to define a coordinate system in this space. They also assert that if $|A'\rangle$ is an eigenvector corresponding to the eigenvalue A' of an observable A, then the probability that a measurement of A will lead to A' when the system is in the state $|\psi\rangle$ is the absolute square of the scalar product $\langle A'|\psi\rangle$. This leads to the requirement that $\langle\psi|\psi\rangle$ be unity and is, in fact, the reason why the transformations of

* This work was done under the auspices of the U. S. Atomic Energy Commission.

[1] Hermann Weyl, *The Theory of Groups and Quantum Mechanics* (Dover Publications, New York, 1931), p. 175 and p. 272. Translated from second revised German edition by H. P. Robertson.

[2] P. A. M. Dirac, *The pri iciples of Quantum Mechanics* (Oxford University Press, Oxford, 1947), 3rd ed., p. 89 and p. 99.

[3] Julian S. Schwinger, Phys. Rev. **82**, 914 (1957).