K^--Deuteron Scattering and the K^--Nucleon Scattering Lengths*

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Cross sections for K^--d reactions have been calculated in the low-momentum region for several possible values of the elementary $\bar{K}$-nucleon scattering amplitudes. Multiple-scattering effects have been included in an approximate way. A comparison of the results for the sum of the elastic plus breakup cross sections with the preliminary measurements available is presented.

DALITZ and Tuan have recently[1] re-analyzed K^--proton scattering in the light of the data presented at the 1959 Kiev Conference.[2] They have incorporated the effects of the $K^- - \bar{K}^0$ mass difference, and find that within the zero effective range approximations,[3] the data, for momenta less than 300 Mev/c,[4] can be fitted approximately with four solutions:

	A_0(fermi)	A_1(fermi)
$(a+)$	$0.2+0.8\,i$	$1.6\,+0.4\,i$
$(a-)$	$-0.3+1.6\,i$	$-1.0\,+0.18i$
$(b+)$	$1.6+1.6\,i$	$0.7\,+0.22i$
$(b-)$	$-1.8+0.6\,i$	$-0.33+0.5\,i.$

(1)

The complex scattering lengths A_0 and A_1 for the K^--nucleon scattering in isotopic spin states $I=0$ and $I=1$, respectively, are related to the complex phase shifts by

$$k\cot\delta_I = 1/A_I. \tag{2}$$

The evidence from the Coulomb interference in the K^--p elastic scattering at 175 Mev/c favors the $(+)$ solutions.[2] Dalitz and Tuan point out that the preliminary low-energy data[5] on the Σ^-/Σ^+ production ratio seems to favor the $(a+)$ solution.

We would like to point out in this paper that a calculation of the nonabsorptive scattering of K^- mesons from deuterons, using the new solutions of Eq. (1), when compared to the preliminary deuterium data at K^- laboratory momentum $\sim$200 Mev/c given at Kiev[2] seems to favor solution $(b+)$.

Since the complex scattering lengths, Eq. (1), are large, an impulse approximation calculation of K^--d scattering reactions is expected to give quite unreliable results.[6] We have made approximate calculations, including multiple scattering corrections,[7] of the K^--d nonabsorptive scattering cross sections, and of the total cross sections.

Since it is difficult experimentally[2] to distinguish between elastic scatterings of the K^- from deuterons, and those scatterings which are inelastic and accompanied by breakup of the deuteron, we have computed the sum of the differential cross sections for these processes. Using the closure approximation, we can write for this sum of differential cross sections[8] (in the K^--d center-of-mass system)

$$\frac{d\sigma_{i+e}{}^d}{d\Omega} = (k'/k)_{\rm av}\int d\mathbf{R}\,|u_d(\mathbf{R})|^2|N(\mathbf{R})/D(\mathbf{R})|^2. \tag{3}$$

In Eq. (3), k is the incident K^- momentum in the K^--d center-of-mass system; $k_{\rm av}'$ is an average final momentum[9] of the K^- meson which depends on the details of the final two-nucleon interaction in the breakup process, and is taken here as $\frac{2}{3}k$; $u_d(\mathbf{R})$ is the Hulthén deuteron wave function; and

$$N(\mathbf{R}) = \eta_n e^{i\mathbf{q}\cdot\mathbf{R}/2} + \eta_p e^{-i\mathbf{q}\cdot\mathbf{R}/2} + 2\eta_n\eta_p(i\sin kR/R)\cos(\mathbf{Q}\cdot\mathbf{R}/2), \tag{4}$$

$$D(\mathbf{R}) = 1+\eta_n\eta_p\sin^2 kR/R^2.$$

Here,

$$\mathbf{q}=\mathbf{k}-\mathbf{k}',\quad \mathbf{Q}=\mathbf{k}+\mathbf{k}', \tag{5}$$

where $|\mathbf{k}'|=|\mathbf{k}|$, as for elastic scattering.[8] The η_α $(\alpha=p,n)$ are related to the free K^--nucleon scattering

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[1] R. H. Dalitz and S. F. Tuan, Ann. Phys. (to be published).

[2] Reported by L. Alvarez, *Proceedings of the Ninth Annual International Conference on High-Energy Physics, Kiev*, 1959 (unpublished).

[3] J. D. Jackson, D. G. Ravenhall, and H. W. Wyld, Nuovo cimento **9**, 834 (1958).

[4] At K^- laboratory momenta of 300–400 Mev/c, there is evidence for large p-wave scattering (see reference 2).

[5] Reported by M. F. Kaplon, *Proceedings of the 1958 Annual International Conference on High-Energy Physics*, at CERN, edited by B. Ferretti (CERN Scientific Information Service, Geneva, 1958).

[6] T. B. Day, G. A. Snow, and J. Sucher, Nuovo cimento **14**, 637 (1959).

[7] K. A. Brueckner, Phys. Rev. **89**, 128 (1953); K. A. Brueckner, Phys. Rev. **90**, 715 (1953); T. B. Day and J. Sucher, University of Maryland Technical Report No. 146 (unpublished); T. B. Day, G. A. Snow, and J. Sucher, University of Maryland Technical Report No. 169 (unpublished).

[8] Using the closure approximation, we can write

$$d\sigma_{i+e}/d\Omega = (\text{phase space})_{\rm av}\langle\Psi_d|T_{\mathbf{k}',\mathbf{k}}{}^\dagger T_{\mathbf{k}',\mathbf{k}}|\Psi_d\rangle, \tag{a}$$

where $T_{\mathbf{k}',\mathbf{k}}$ is the matrix element of the complete K^--d transition operator taken between plane-wave states for the K^- meson. This is very similar to the equation for elastic scattering,

$$d\sigma_{\rm elast.}/d\Omega = (\text{phase space})|\langle\Psi_d|T_{\mathbf{k}',\mathbf{k}}|\Psi_d\rangle|^2. \tag{b}$$

If the adiabatic approximation for nucleon motion is made in Eq. (b), Brueckner's point-scatterer model is obtained (reference 7). If the same approximation, with a propagator ($i\sin kR/R$), is made in Eq. (a), then Eq. (3) of the text results. [See S. D. Drell and L. Verlet, Phys. Rev. **99**, 849 (1955) for a discussion of various propagators.]

[9] T. B. Day, G. A. Snow, and J. Sucher, Nuovo cimento **13**, 614 (1959).

amplitudes f_α by

$$\eta_\alpha = (\mu_{Kd}/\mu_{K\alpha}) f_\alpha, \quad (6)$$

with μ_{Kd} and $\mu_{K\alpha}$ the respective reduced masses. The amplitudes f_α may be expressed in terms of the S-wave complex scattering lengths A_I of Eq. (2) by

$$f_p = \tfrac{1}{2}[A_1/(1-ik_0'A_1)] + \tfrac{1}{2}[A_0/(1-ik_0'A_0)],$$
$$f_n = A_1/(1-ik_0'A_1). \quad (7)$$

[In Eq. (7), k_0' is the K^- momentum in the K^--nucleon center-of-mass system, the nucleons being considered at rest in the laboratory.]

Total cross sections may be obtained from the forward elastic scattering amplitude, by use of the optical theorem, and are given by

$$\sigma_{\text{total}}^d = (4\pi/k)\,\text{Im}\left\{\int d\mathbf{R}\,|u_d(\mathbf{R})|^2 D(\mathbf{R})^{-1} \times [\eta_n + \eta_p + 2i\eta_n\eta_p(\sin^2 kR/kR^2)]\right\}. \quad (8)$$

The deuteron charge-exchange cross section has also been calculated. In the impulse approximation it is small. Multiple scattering corrections may be estimated in a similar way[10] as for the nonexchange cross section. These corrections do not change the order of magnitude of the impulse approximation estimates.

To facilitate numerical evaluation of Eqs. (3), (8), and the charge-exchange cross section, the multiple scattering denominators (or their absolute squares, as the case may be) were replaced by their average values [averaged over $|u_d(\mathbf{R})|^2$]. This approximation, which allows the remaining integrals to be done analytically, is expected to underestimate the cross section.[6] The results of the calculations for a K^- laboratory momentum of $\sim$200 Mev/c are given in Table I.[11] Also included in Table I are the cross sections for absorption of K^- mesons by deuterons σ_{abs}^d, derived from the computed values of σ_{i+e}^d, σ_{cx}^d and σ_{total}^d. For comparison, the sum of the absorption cross sections for K^- mesons on free protons and neutrons, derived from the curves of Dalitz and Tuan,[1] are also given.

It is clear from Table I that the multiple-scattering corrections to σ_{i+e}^d are appreciable. The corrections to σ_{cx}^d and the derived σ_{abs}^d are not nearly so great, except for solution $(a+)$.

Table I. Results of the calculations, including multiple-scattering corrections, of the K^--d cross sections for the four Dalitz and Tuan pairs of K^--nucleon scattering lengths, $(a\pm)$, $(b\pm)$. Given are the cross sections for elastic plus breakup scattering, σ_{i+e}^d; for charge-exchange scattering, σ_{cx}^d; the total cross sections, σ_{total}^d; and the cross sections for absorption of the K^- by deuterons derived from the previous three. The impulse approximation results are given for comparison, where appropriate, as is the sum of the cross sections for absorption by free protons and neutrons, $\sigma_{\text{abs}}^p + \sigma_{\text{abs}}^n$. The initial K^- laboratory momentum is 200 Mev/c, and all cross sections are in units of millibarns.

	$(a+)$	$(a-)$	$(b+)$	$(b-)$
σ_{i+e}^d	197	139	117	95
σ_{i+e}^d(impulse)	294	206	166	132
σ_{cx}^d	8.1	6.4	5.5	5.9
σ_{cx}^d(impulse)	5.7	6.0	5.7	5.9
σ_{abs}^d	45	50	58	89
$\sigma_{\text{abs}}^p + \sigma_{\text{abs}}^n$	67	54	61	82
σ_{total}^d	250	195	181	190

Preliminary Berkeley data presented at Kiev,[2] for this momentum range gives a value $\sigma_{i+e}^d \sim 100 \pm 25$ mb. If the evidence[1] from Coulomb interference in K^--p elastic scattering at 175 Mev/c is accepted, then the K^--proton force is attractive, and the $(a-)$ and $(b-)$ solutions of Eq. (1) are ruled out (although they would be consistent with the K^--d data alone). Thus, our results, when compared with the data on σ_{i+e}^d, would favor the $(b+)$ solution. The $(a+)$ solution gives a result too large by a factor of two, well outside the errors quoted.[12] For the $(a+)$ solution to be compatible with the data, the corrections to the impulse approximation would have to be about twice as large as the multiple scattering corrections calculated here.

The only evidence cited by Dalitz and Tuan[2] in favor of the $(a+)$ solution is the apparent upward trend of the Σ^-/Σ^+ production ratio from K^--p reactions, as a function of increasing energy, in the energy interval below the $\bar{K}^0$ production threshold (90 Mev/c). The experiments are quite difficult to do in this energy region, and the existing data is quite meager.[5] Hence it would appear that the curve given by Dalitz and Tuan for the Σ^-/Σ^+ ratio for solution $(b+)$ cannot be ruled out. The energy region around 200 Mev/c is more amenable to experiment, and since the predicted difference between σ_{i+e}^d for $(a+)$ and $(b+)$ is so large, we feel that the stronger experimental evidence favors solution $(b+)$.

[10] See the last reference cited in footnote 7 above for the formula. In a similar manner, one can consider the charge-exchange corrections in the multiple-scattering model for the elastic or breakup reactions; here, however, the corrections themselves are of the same order as the ratio of charge-exchange cross section to total elastic cross section, $\sim$10%.

[11] The impulse approximation results if only the first two terms in $N(\mathbf{R})$, Eq. (4) are included when using Eq. (3) for $d\sigma_{i+e}/d\Omega$. Similarly for the charge-exchange process, only the single scattering from the proton is considered, and the only effect of the neutron's presence is to make the cross section go to zero in the forward direction, due to the restriction of the Pauli principle on the two neutrons. [See E. M. Ferreira, Phys. Rev. **115**, 1727 (1959) for the corresponding impulse and closure approximations formula for K^+-d scattering.]

[12] For the impulse approximation, the $(a+)$ solution is expected to give a considerably larger cross section than the $(b+)$ solution, since the difference comes mainly from K^--n scattering. [The four solutions Eq. (1) are all such that they agree with the K^--p scattering at 175 Mev/c.] This is a pure $I=1$ state, and Eq. (1) shows that for A_1, the $(a+)$ solution is much larger than the $(b+)$ solution. It should be emphasized that the conclusion that the (b) type of solution is favored over the (a) is not very sensitive to the exact values of the Dalitz amplitudes.

It would be desirable to have more accurate measurements of the low-momentum ($\sim$100 Mev/c) Σ^-/Σ^+ production ratio from K^--p reactions, as well as of the K^--d nonabsorptive scattering at higher momenta ($\sim$200 Mev/c). More refined data would require, in turn, a more detailed and careful analysis of the corrections to the impulse approximation for K^--d reactions.

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Gauge Invariance and the Lorentz Pondermotive Force

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In this paper it is shown that if one introduces into the Weyl theory of gauge invariance the two additional conditions that gauge (and therefore length), except for an arbitrary phase factor, be integrable along the path of a particle, and that the change in dimensions of a particle be a minimum, one immediately obtains the Lorentz pondermotive force for a charged particle in an electromagnetic field and the Bohr-Sommerfeld quantum integral.

INTRODUCTION

In a previous paper[1] an attempt was made to obtain the Maxwell-Lorentz equations for an electron by starting out from a generalized Lagrangian which was derived from the contracted Riemann-Christoffel tensor $G_{\mu\nu}$ by imposing the condition that the Lagrangian be gauge invariant in Weyl's sense of the term. This led to the second rank in-tensor as defined by Eddington[2]

$$^*G_{\mu\nu}=G_{\mu\nu}-(\kappa_\alpha{}^\alpha-2\kappa_\alpha\kappa^\alpha)g_{\mu\nu}-2\kappa_\mu\kappa_\nu+(\kappa_{\mu,\nu}+\kappa_{\nu,\mu})-2\mathfrak{F}_{\mu\nu}, \quad (1)$$

where the $g_{\mu\nu}$ are the components of the metric tensor, and $\kappa_{\mu,\nu}=\partial\kappa_\mu/\partial X_\nu$, and

$$\mathfrak{F}_{\mu\nu}=\frac{\partial\kappa_\mu}{\partial X_\nu}-\frac{\partial\kappa_\nu}{\partial X_\mu}. \quad (2)$$

We shall see later that the 4-vector κ_μ may be chosen proportional to the electromagnetic 4-vector potential A_μ so that the $\mathfrak{F}_{\mu\nu}$ are essentially the electromagnetic field strengths.

In this note we shall show first that we can derive the Lorentz expression for the pondermotive force on a charge in an electromagnetic field if we impose upon the Weyl theory the condition that the charge must move in such a way that the change in its dimensions along a given path (resulting from the change of gauge along this path) must be a minimum. Secondly, we shall show that the Bohr-Sommerfeld quantum integral follows directly if we impose the condition that the dimensions of a charged particle, except for a possible change of phase, must return to their initial values when the particle moves around a closed orbit. We shall see that this condition will eliminate one of the most serious objections to the Weyl theory arising from the nonintegrability of length that is a consequence of the theory.

LORENTZ PONDERMOTIVE FORCE

The Weyl theory of gauge invariance arose out of the concept that lengths at different places cannot be compared because of the change of gauge that takes place as one moves from point to point in a space-time continuum. Since the gauge was assumed to be determined by a vector field κ_μ, comparison of lengths at different places would be ambiguous because the result of the comparison would depend on the path taken in going from one point to the other. Although this theory introduced a four vector (to be identified with the electromagnetic vector potential) into the description of the world quite naturally, the nonintegrability of length which it brought with it led to apparently insurmountable difficulties concerning the structure of atoms.

Thus the objection was raised that according to the Weyl theory the natural frequency of an atom at a point in space-time should depend on the path the atom took to reach that point. This objection was met by introducing the assumption that although lengths and frequencies depend on the path taken, the effect is much too small to be measurable in actual physical phenomena. This, however, is not a satisfactory way out of the difficulty since the ambiguity is still present in the theory. It is possible to eliminate this ambiguity without destroying the content of the Weyl theory by imposing the condition that the measurable physical dimensions of a particle shall be integrable along the path of its motion. We must note that this is not the same thing as imposing the condition that the gauge

[1] L. Motz, Phys. Rev. **89**, 60 (1953).
[2] A. S. Eddington, *The Mathematical Theory of Relativity* (Cambridge University Press, Cambridge, 1923), p. 204.