happens that the best least squares fit to the data on τ decay given by McKenna et al.[2] gives just about the ratio implied above, though the errors are large and this agreement must not be taken too seriously. The experimental indication is that the π^- spectrum (relative to statistical) is an increasing function of the energy the variation amounting to 50% from one end of the spectrum to the other. With $a_2-a_0\approx 0.8$, Eq. (43) would reproduce the least squares fit to the data of McKenna et al.

Most of the S-wave dominant solutions of the pion-pion integral equations obtained by Chew, Mandelstam, and Noyes[5] have the following general properties: (i) a_2 and a_0 have the same sign; (ii) the ratio a_0/a_2 is of the order $\frac{5}{2}$. If one accepts these properties, then our results will lead to the conclusion that both a_0 and a_2 are negative, hence a repulsive S-wave pion-pion interaction. The values $a_2\approx -0.3$ and $a_0\approx -1$ will give agreement with the data and correspond to $\lambda\approx 0.15$, where λ is the Chew-Mandelstam pion-pion coupling constant.

Finally, we point out that a very useful test of the results of this paper would be provided by looking at the π^+ spectrum in the τ'-decay mode.[10]

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Unstable Particles in a General Field Theory

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The problem of unstable particles in quantum field theory is treated as one of the interpretation of complex singularities appearing in the analytic continuation of scattering amplitudes into unphysical sheets of their Lorentz invariant variables. Suitable continuations are shown to hold under certain restrictive assumptions in a general field theory, making use of unitarity and causality of the S matrix. The extra singularities appearing in the continuation are fixed isolated poles, in accordance with a conjecture of Peierls.

1. INTRODUCTION

THE problem of the definition of unstable particles in quantum field theory has received much attention recently.[1] The difficulty met with in framing such a definition lies in the fact that the asymptotic "in" and "out" states containing such particles do not exist and so the usual methods of field theory based on these asymptotic states do not apply. Two main methods of approach to this problem have appeared. One of these[2] is to define the mass and lifetime of an unstable particle in terms of the average mass and mass spread of a certain mass distribution appearing in the spectral representation of the propagator. A limitation is imposed on the high-energy behavior of the vertex function in order that the mass and lifetime defined in this manner exist. This limitation is not satisfied by the Lee model with a certain cutoff[1] and may not be satisfied in a realistic field theory. For this reason we follow here the suggestion of Peierls[3] that unstable particles may be associated with poles appearing in the unphysical sheets of the analytic continuation of the particle propagator in momentum space. The work of Lévy[1] and others shows that the Lee model provides a satisfactory demonstration of this suggestion. In this paper, we attempt to extend the methods developed by Lévy to a more general field theory. However, it is not yet possible to give a satisfactory definition of the local field to be associated with an unstable particle in a general field theory satisfying the usual axioms of causality, Lorentz invariance, and an asymptotic condition. In consequence, we consider mainly the interpretation of poles appearing in the scattering amplitudes and then show that the same poles should occur in the propagator if it exists. We do not consider here the decay properties of unstable particles as a function of time but only how they manifest themselves as resonances in scattering or production processes.

In Sec. 2, we discuss the continuation of the two-particle scattering amplitude into the first unphysical

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1 M. Lévy, Nuovo cimento 13, 115 (1959). This paper contains further references.

2 P. T. Matthews and A. Salam, *1958 Annual International Conference on High-Energy Physics, Cern*, edited by B. Ferretti (CERN Scientific Information Service, Geneva, 1958). Also Phys. Rev. 112, 283 (1958).

3 R. E. Peierls, *Proceedings of the Glasgow Conference on Nuclear and Meson Physics* (Pergamon Press, New York, 1954), p. 296.

sheet, making use of the unitarity condition expressed in the angular momentum representation. From an assumed analyticity of the partial wave amplitude in the cut energy plane, we show that these amplitudes are analytic also in the first unphysical sheet, except for isolated poles. The contributions that they make to cross sections are of the usual resonance type. In Sec. 3 we show that a Mandelstam type representation, if true on the physical sheet, implies analyticity in both variables inside a certain domain on the first unphysical sheet with additional terms arising from the complex poles. These poles are shown to occur also in the vertex function and propagator for the corresponding unstable particles in Sec. 4. The general problem of continuing many particle amplitudes into higher unphysical sheets is discussed in Sec. 5. We conclude with a brief discussion of our results.

2. TWO-PARTICLE SCATTERING AMPLITUDES ON THE FIRST UNPHYSICAL SHEET

For convenience, we consider the scattering amplitude for two neutral scalar bosons of masses m and μ, called "nucleon" and "meson." We assume that no other stable particles occur in the theory and that production of an arbitrary number of mesons can occur at sufficiently high energy. In order to facilitate the application of the unitarity conditions we expand the total amplitude $M(W^2,\Delta^2)$ into partial wave components:

$$\begin{aligned} M(W^2,\Delta^2)&=(8p_0'k_0'p_0)^{\frac{1}{2}}\langle p',k'|j(0)|p\rangle \\ &=8\pi\frac{W}{K}\sum_{l=0}^{\infty}(2l+1)C_l(W^2)P_l(\cos\theta), \end{aligned} \tag{2.1}$$

where $\cos\theta$ is the cosine of the nucleon scattering angle in the center-of-mass coordinate system, W is the total invariant energy, and Δ the invariant momentum transfer, given by

$$\begin{aligned} W^2&=[(K^2+m^2)^{\frac{1}{2}}+(K^2+\mu^2)^{\frac{1}{2}}]^2, \\ \Delta^2&=-2K^2(1-\cos\theta), \end{aligned} \tag{2.2}$$

where K is the center-of-mass momentum. We must now assume suitable analytic properties in W^2 for the particle wave amplitudes:

$$C_l(W^2)=\frac{1}{16\pi}\frac{K}{W}\int_{-1}^{+1}d(\cos\theta)M(W^2,\cos\theta)P_l(\cos\theta). \tag{2.3}$$

The obvious choice is a cut plane of analyticity, where for Re $(W^2)>m^2$, the cut is along the real axis from $(m+\mu)^2$ to ∞. This property holds to fourth order in perturbation theory. We assume that this holds generally. It is possible to obtain results analogous to the following ones for smaller regions of analyticity bounded by suitable intervals of the real axis.

We have branch points at $W^2=(m+n\mu)^2$ for $n=1, 2, \cdots$ (Fig. 1). The amplitude satisfies $C_l(W^2)=C_l^*(W^{*2})$ and the physical retarded amplitude is obtained by taking the improper limit as $C_l(W^2)$ tends to the real axis on the top side of the cut. The continuation into the first unphysical sheet exposed by a clockwise rotation of the cut starting from the branch point $W^2=(m+\mu)^2$ can be written formally as

$$C_l^1(W^2)=C_l(W^2)+2iA_{l2}(W^2), \tag{2.4}$$

where A_{l2} is the contribution to the absorptive part of the amplitude arising from the two-particle intermediate state and $C_l(W^2)$ is the advanced function in the physical sheet. For real $W^2\geqslant(m+\mu)^2$, unitarity gives the relation:

$$A_{l2}(W^2)=C_l^*(W^2)C_l(W^2), \tag{2.5a}$$

where $C_l(W^2)$ is the retarded amplitude. We may continue (2.5a) in W^2 to the lower half plane by replacing $C_l(W^2)$ by its continuation $C_l^1(W^2)$ there. So in the lower half plane, (2.5a) becomes

$$A_{l2}(W^2)=C_l(W^2)C_l^1(W^2). \tag{2.5b}$$

Substituting in (2.4) we get:

$$C_l^1(W^2)=C_l(W^2)/[1-2iC_l(W^2)]. \tag{2.6}$$

Now $C_l^*(W^2)$ and $C_l(W^2)$ are both analytic in the same cut plane, so we deduce that the continued amplitude is analytic in a similar region unless the denominator vanishes at some point, when we obtain an additional pole. The two functions $C_l(W^2)$ and $C_l^1(W^2)$ are true continuations of each other, as $C_l(W^2)$ is a tempered distribution for W^2 real and a known theorem applies.[4] We conjecture that such a pole in $C_l^1(W^2)$ corresponds to an "unstable particle" intermediate state or "resonance" state of spin l. In Sec. 4 we show that the same poles will occur in the propagators constructed for each unstable particle in the theory, assuming that these can be well defined.

The unitarity relation (2.5) leads to the possibility of expressing the partial wave amplitudes in terms of a phase shift $\delta_l(W^2)$:

$$C_l(W^2)=\exp[i\delta_l(W^2)]\sin\delta_l(W^2), \tag{2.7}$$

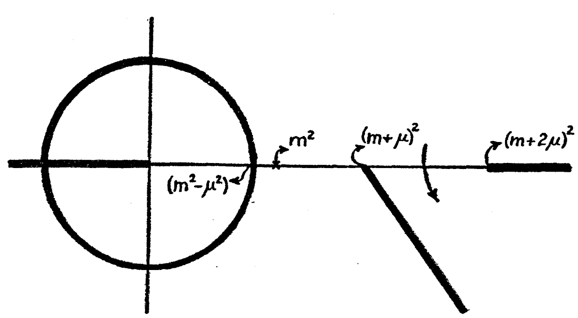

FIG. 1. Singularities of $C_l(W^2)$. The cuts and poles correspond to those given by the Mandelstam representation for a particular choice of mass spectrum. A part of the first unphysical sheet is exposed by rotation of the appropriate branch cut.

[4] J. G. Taylor, Ann. Phys. 5, 391 (1958), theorem 2.

where $\delta_l(W^2)$ is a real function of W^2 on the real axis for $W^2 \geqslant (m+\mu)^2$. We can now write (2.6) in the simple form:

$$C_l^1(W^2)=C_l(W^2)\exp[2i\delta_l(W^2)]. \tag{2.8}$$

This agrees with a result derived by Fubini, Nambu, and Wataghin[5] for W^2 real, and more recently by Omnès.[6] The factor $e^{2i\delta}$ for real W^2 is just the element of the S matrix corresponding to the lth partial wave, as it is diagonal in the angular momentum representation. We note that at a pole of $C_l^1(W^2)$ we have $\operatorname{Im}\delta(W^2)=-\infty$, so that the S-matrix element $S_l(W^{*2})$ at the conjugate point vanishes.

The continuation into the next higher sheet can be written as

$$C_l^2(W^2)=C_l^1(W^2)+2iA_{l3}(W^2), \tag{2.9}$$

where $A_{l3}(W^2)$ is the three particle contribution to the absorptive part for physical W^2. The problem of proving the continuation is deferred until Sec. 5, but it is clear that the poles already present in the first sheet will be present in all the sheets obtained by continuing around the higher branch points.

We now use the analytic properties of the absorptive part as expressed in (2.5) to show how the contour of integration in a dispersion relation for $C_l(W^2)$ can be deformed so as to include the contribution from the poles in the unphysical sheets. We have the dispersion relation, neglecting subtractions,

$$C_l(W^2)=\frac{1}{\pi}\int_{-\infty}^{+\infty}\frac{A_l(W'^2)}{W'^2-W^2}dW'^2. \tag{2.10}$$

Let $z=W'^2$ where $A_l(W^2)$ is the complete absorptive part. Writing $A_l=A_{l2}+A_l^{(r)}$ we may transform the dispersion integral involving A_{l2} for $z\geqslant(m+\mu)^2$ to a contour just below the cut. We are here using the fact that $A_{l2}(z)$ has an analytic continuation from the cut $z\geqslant(m+\mu)^2$ into the lower half plane, given by (2.5).

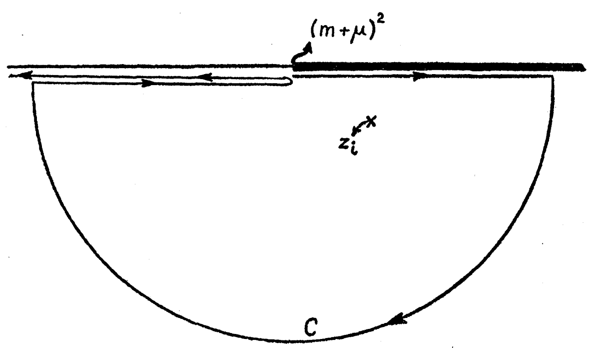

FIG. 2. Deformation of contour of integration in z plane for (2.11). The limit as the radius of the semicircle C tends to infinity is understood.

[5] S. Fubini, Y. Nambu, and V. Wataghin, Phys. Rev. **111**, 329 (1958).

[6] R. Omnès preprint (to be published).

We may close the contour of integration in the lower half z plane by a large semicircle C (Fig. 2) and a line just below the real axis from $-\infty$ to $(m+\mu)^2$. Assuming no contribution from the integral round C, we have

$$C_l(W^2)=\frac{1}{\pi}\int_{-\infty}^{+\infty}\frac{A_l^{(r)}(z)}{(z-W^2)}dz+\frac{1}{\pi}\int_{-\infty}^{(m+\mu)^2}\frac{A_{l2}(z)}{(z-W^2)}dz$$
$$+\frac{1}{\pi}\int_{(m+\mu)^2}^{-\infty}\frac{A_{l2}^{(c)}(z)}{(z-W^2)}dz+2i\sum_i\frac{R(z_i)}{z_i-W^2}. \tag{2.11}$$

$R(z_i)$ is the residue of $A_{l2}^{(c)}(z)$ at the pole z_i in the lower half z plane and $A_{l2}^{(c)}(z)$ is the continuation of $A_{l2}(z)$ from the cut $(m+\mu)^2$ to ∞. For a pole z_i close to the real axis, the most rapidly varying term in $C_l(W^2)$ for W^2 near $\operatorname{Re}(z_i)$ will come from the last term of (2.11). This term gives a Breit-Wigner resonance type contribution to scattering cross sections. By a similar procedure, we may show that poles in higher unphysical sheets give the same resonance behavior, and also in the case when we must use the subtracted form of the dispersion relation.

3. ANALYTICITY IN MOMENTUM TRANSFER

In the previous section we considered only partial wave amplitudes. The analyticity assumed for them is implied by the Mandelstam representation for $M(W^2,\Delta^2)$. If we now assume that this representation holds for our theory, the partial wave expansion (2.1) is uniformly convergent inside a certain domain in the space of the two complex variables W^2 and Δ^2. This domain intersects the planes $W^2=$constant in the usual ellipse of convergence of the Legendre series. The corresponding series in the first unphysical sheet,

$$M^1(W^2,\Delta^2)=8\pi\frac{W}{K}\sum_{l=0}^{\infty}(2l+1)$$
$$\times\frac{C_l(W^2)}{1-2iC_l(W^2)}P_l(\cos\theta), \tag{3.1}$$

can be proved to be uniformly convergent in the same domain by direct comparison with the series (2.1). This makes use of the inequality:

$$\left|\frac{C_l(W^2)}{1-2iC_l(W^2)}\right|<2|C_l(W^2)| \quad\text{or}\quad |C_l(W^2)|<\tfrac{1}{4}, \tag{3.2}$$

for sufficiently large l and fixed W^2. Continuation to the whole of the usual domain of the Mandelstam representation except for the extra poles is not possible. This will be discussed elsewhere.

4. VERTEX FUNCTION AND PROPAGATOR

We assume that a local field $\Phi_\mu(x)$ and current $J_\mu(x)$ can be constructed to describe an unstable particle of spin l. The field is in general multicomponent and transforms as the spin l representation of the homogeneous Lorentz group. With suitable conditions on the

spectrum of masses to which the particles are coupled, the vertex function $\Gamma_\mu(W^2)=\langle p',k'|J_\mu(0)|0\rangle$ is analytic in the W^2 plane with a cut from $(m+\mu)^2$ to ∞. We can now continue into the first unphysical sheet by the methods of Sec. 2. The equations corresponding to (2.4), (2.5), and (2.6) are

$$\Gamma_\mu{}^1(W^2)=\Gamma_\mu(W^2)+2iG_\mu(W^2), \tag{4.1}$$

$$G_\mu(W^2)=C_l{}^*(W^2)\Gamma_\mu{}^1(W^2) \quad [W^2\geqslant(m+\mu)^2], \tag{4.2}$$

$$\Gamma_\mu{}^1(W^2)=\Gamma_\mu(W^2)/[1-2iC_l(W^2)]. \tag{4.3}$$

Moreover, the particle propagator has the same singularities in the unphysical sheet, as the two particle contribution to the weight function in the Lehmann spectral representation possesses analytic properties. This can be seen from the relation

$$\rho_2(W^2)=\sum_{(p+k)^2=W^2}\langle 0|\Phi(0)|p,k\rangle\langle p,k|\Phi(0)|0\rangle \tag{4.4}$$

in which both terms of the product can be continued into the lower half plane, one of them containing the correct factor $[1-2iC_l(W^2)]^{-1}$, the other being analytic there.

5. CONTINUATION TO HIGHER UNPHYSICAL SHEETS

The problem of continuing the two-particle scattering amplitude into higher sheets differs from the problem of the first unphysical sheet in that the unitarity condition involves higher order amplitudes. This requires that we treat it as part of the more general problem; the continuation of all possible scattering amplitudes into unphysical sheets, i.e., amplitudes describing processes with an arbitrary number of incoming and outgoing particles. This implies that we already have suitable analytic properties for these amplitudes on the physical sheets; at least analyticity in the cut W^2 (total center of mass energy square) plane, when we hold all the other independent variables fixed at physically possible values.

The formal expression for continuation into the pth unphysical sheet (denoted by an index p) of the amplitude for m incoming and n outgoing particles is

$$\begin{aligned} M^p(q_1',\cdots q_n';q_1,\cdots q_m) &= M^{p-1}(q_1',\cdots q_n';q_1,\cdots q_m)\\ &\quad +2iA_{(p+1)}(q_1',\cdots q_n';q_1,\cdots q_m), \end{aligned} \tag{5.1}$$

where $A_{(p+1)}$ is the $(p+1)$ particle contribution to the absorptive part of the total amplitude for $W^2=(\sum_i q_i)^2=(\sum_i q_i')^2$ real and above threshold. It can be written

$$\begin{aligned} A_{(p+1)}(q_1'\cdots q_n';q_1\cdots q_m) &= \int d^3k_1\cdots d^3k_{p+1}\,\delta^{(4)}(\sum_i q_i-\sum_j k_j)\\ &\quad\times M(q_1',\cdots q_n';k_1,\cdots k_{p+1})\\ &\quad\times M^p(k_1,\cdots k_{p+1};q_1\cdots q_m), \end{aligned} \tag{5.2}$$

which could be continued immediately into the lower half W^2 plane if we knew the continuation $M^p(k_1\cdots k_{p+1};q_1\cdots q_m)$ of the m particle to $(p+1)$ particle scattering amplitude. The other factor $M(q_1'\cdots q_n';k_1\cdots k_{p+1})$ is just the advanced function for $p+1$ particles to n particles, which has by assumption a continuation into the lower half plane. Thus we have reduced the problem to one of the continuation of $M^p(k_1\cdots k_{p+1};q_1\cdots q_m)$ into the pth unphysical sheet. For this amplitude we have

$$\begin{aligned} M^p(k_1\cdots k_{p+1};q_1\cdots q_m) &= M^{p-1}(k_1\cdots k_{p+1};q_1\cdots q_m)+2i\int d^3r_1\cdots d^3r_{p+1}\\ &\quad\times\delta^{(4)}(\sum_i q_i-\sum_j r_j)M(k_1,\cdots k_{p+1};r_1\cdots r_{p+1})\\ &\quad\times M^p(r_1\cdots r_{p+1};q_1\cdots q_m). \end{aligned} \tag{5.3}$$

This is an integral equation for the unknown function M^p which cannot be solved immediately. If, however, we restrict the range of W^2 temporarily to those values on the real axis where the physical sheet of M joins on to the pth unphysical sheet, we can make certain observations.

Firstly, the matrix elements can be expressed as functions of $3N-10$ independent Lorentz invariants which can be formed from the $(N-1)$ independent 4 momenta of an amplitude involving a total of N particles. We initially consider only the case where we have at least 4 particles on each side. As a typical case, consider a matrix element involving m incoming momenta $q_1, \cdots, q_m$ and $(p+1)$ outgoing momenta $r_1\cdots r_{p+1}$. The integration in (5.3) can then be transformed to one over the "physical" domain in the space of $3p-1$ real invariants formed from the internal and external momenta. The invariants can be chosen as follows, where we work in the center-of-mass system for which $\mathbf{r}_i$ denotes the 3-momentum part of the 4 vector r, $(p-1)$ lengths $\mathbf{r}_1^2$, $\mathbf{r}_2^2\cdots\mathbf{r}_{(p-1)}^2$, $(2p-3)$ angles, consisting of the angle between $\mathbf{r}_1$ and $\mathbf{r}_2$ and $(2p-2)$ angles θ_i, ϕ_i, specifying the directions of $\mathbf{r}_3, \cdots \mathbf{r}_p$ in the spherical polar coordinate system with $\mathbf{r}_1$ as axis and azimuthal angle measured from the $(\mathbf{r}_1,\mathbf{r}_2)$ plane. We denote these $(3p-4)$ invariants by a_i', and corresponding sets formed from the $\mathbf{k}_i$ and $\mathbf{q}_j$ by a_i and c_j, respectively. We note that $\mathbf{r}_p^2$ can be expressed in terms of the a_i and W^2.

The remaining 3 invariants connect incoming and outgoing momenta. Suitable ones are obtained by considering the orthogonal transformation between the coordinate systems constructed above for the $\mathbf{q}_i$ and $\mathbf{r}_j$. In particular we may regard this transformation as a

rotation through an angle Ψ about an axis with coordinates θ, ϕ in the coordinate system of the $\mathbf{q}_i$. We denote these invariants by b', and the corresponding sets formed from the $\mathbf{k}_i$ and $\mathbf{q}_j$, $\mathbf{k}_i$ and $\mathbf{r}_j$, by b and d.

The integration over intermediate momenta in (5.3) is now over the a_i' and b_j', the Jacobian of the change of variables being a function $J(W^2,a_i')$ of W^2 and a_i' only. (5.3) now becomes

$$M^p(W^2,a,b,c)=M^{p-1}(W^2,a,b,c)+2i\int da'db'J(W^2,a')\times\bar{M}(W^2,a,a',d)M^p(W^2,a',b'c). \quad (5.4)$$

Regarding the d_i as functions of the b_j and b_k', we obtain

$$M^p(W^2,a,b,c)=M^{p-1}(W^2,a,b,c)+2i\int da'db'\times K(W^2,a,b,a',b')M(W^2,a',b',c), \quad (5.5)$$

where

$$K(W^2,a,b,a',b')=J(W^2,a')\bar{M}(W^2,a,a',d(b,b')).$$

This is in the form of a Fredholm equation in several real variables, for which the solution can be written down immediately if we know that the kernel K has suitable properties of uniform continuity in the variables a, b, a', b'.

Secondly, we can show that the kernel is uniformly continuous if we use an inductive process of continuation. In effect, we assume that we have continued separately into all the unphysical sheets up to the $(p-1)$th in each of the $(2^{N-1}-N-1)$ "energy" variables $(\sum_i p_i)^2$ of the general amplitude involving N particles. As we restricted W^2 in (5.3) to an interval just above the $(p+1)$ particle branch point, the matrix element $\bar{M}(W^2,a,a',d)$ is already a continuous function of the other invariants in view of the analytic properties on the real axis already proved, if we restrict the range of these variables to the physical phase space over which we integrate. This holds because the a, b, a', b' can be expressed as continuous functions of the "total energy variables $(\sum r_i)^2$, etc., none of which can exceed the value of W^2 at the $(p+1)$ particle branch point. In addition, the branch points introduce discontinuities only into the derivatives of this kernel.

The general solution of (5.7) can now be expressed,

$$M^p(W^2,a,b,c)=M^{p-1}(W^2,a,b,c)+\int da'db'\frac{D(W^2,a,b,a',b')}{\Delta(W^2)}\times M^{p-1}(W^2,a',b',c), \quad (5.6)$$

where Δ, D, are the Fredholm determinant and the first minor, respectively. This completes one step in the inductive process. We have shown earlier in the paper how to continue into the first unphysical sheet of the total two-particle scattering amplitude under certain assumptions; this generalizes readily to the first unphysical sheets of the general amplitude. The above argument has to be modified slightly when fewer than four particles is involved in one or more of the incoming, intermediate, or outgoing states.

This shows that for fixed a, b, a', b', $D(W^2)$ and $\Delta(W^2)$ will each be analytic in a cut W^2 plane, with cuts determined by those of $\bar{M}(W^2)$ and $J(W^2)$. Hence $M^p(W^2)$ will be analytic in a cut plane except for isolated poles arising from possible zeros of $\Delta(W^2)$ and poles already present in $M^{p-1}(W^2)$. These poles we interpret as in section 2. The cuts in $J(W^2)$ are on the real axis for unphysical values of W^2, being purely kinematic.

6. CONCLUSION

We have not discussed the important question of continuation of the scattering amplitudes simultaneously in all the possible invariants to unphysical sheets. We can conjecture on the basis of suitable analyticity in the invariants simultaneously and unitarity that we may continue on to the complete Riemann surface, meeting only isolated poles on the way there, in addition to the expected cuts. This will be discussed elsewhere.

We interpret all these complex poles as being associated with unstable particles or resonances. A discussion of an extended Lee model[1] seemed to indicate that one of these poles cannot be so interpreted. A more detailed analysis of this model shows that this spurious pole coincides with the expected unstable particle pole.

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