angular momentum of the two-pion system is $J=0$ (S wave), or at least that the S wave is dominant. If we had an appreciable amount of $|\Delta\mathbf{I}|=\frac{3}{2}$ part in the transition amplitude, the system of final pions could reach the state $I=2$, $J=0$. The enhancement factor (E.F.) in this state as obtained by the Fermi method and given in reference 4 is

$$\text{E.F.}=\frac{\cot^2\delta}{1+\cot^2\delta}\left[1+\frac{1}{\alpha k\cot\delta}\right]^2;\quad k\cot\delta=\frac{1}{a_{20}},$$

where δ is the pion-pion phase shift in the $I=2$, $J=0$ state (a_{20} being the corresponding scattering length), α is the radius of interaction of the outgoing particles, and k the relative momentum of the pions. With the same value of parameters chosen in reference 4, this formula leads to

$$75\leqslant \text{E.F.}\leqslant 225,$$
$$k_{\max}\leftarrow k\rightarrow 0,$$

where $k_{\max}=m_\pi c/0.7\hbar$. Perhaps it is worth observing that the E.F. is larger at low momenta, when the S wave is more likely to be dominant, than at high momenta. Combining these results with the pion spectrum given by Mathur,[5] we see that the ratio (2) may be enhanced by two orders of magnitude leading again to a value $\sim 10^{-1}$. Therefore, the fact that the K_{e4} decay has not yet been observed implies in this case that the $|\Delta\mathbf{I}|=\frac{3}{2}$ part plays an insignificant role in K_{e4} modes. So if we insist on the standpoint (II), the leptonic decay modes of the K meson must satisfy the approximate, if not strict, $|\Delta\mathbf{I}|=\frac{1}{2}$ rule.

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Some Remarks on the Vertex Function

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It is noted that the integral representations for matrix elements derived in three papers, (1) Dyson, (2) Deser, Gilbert, and Sudarshan, and (3) Faĭnberg, do not give the most general functions satisfying the conditions stated. It is shown that in (1) and (2) the convergence of certain integrals was not treated rigorously, while in (3) there was a misunderstanding of the Jost-Lehmann-Dyson representation.

WE wish to point out that the integral representations for matrix elements derived in three papers[1–3] do not give the most general functions satisfying the conditions stated.

Dyson's representation for the double commutator is

$$\langle 0|[A(x),[B(0),C(-y)]]|0\rangle$$
$$=\int_0^\infty ds\int_0^\infty dt\int_0^1 d\lambda\,\phi(s,t,\lambda)\delta(y^2-t^2)\epsilon(y_0)$$
$$\times\delta((x+\lambda y)^2-s^2)\epsilon(x+\lambda y).\quad(1)$$

This is not the most general form for a function with the properties of the double commutator. For, Eq. (1) implies

$$\langle 0|A(x)[B(0),C(-y)]|0\rangle$$
$$=\int_0^\infty ds\int_0^\infty dt\int_0^1 d\lambda\frac{\delta(y^2-t^2)\epsilon(y_0)\phi(s,t,\lambda)}{(x+\lambda y)^2-s^2},\quad(2)$$

showing this to be, for fixed $y^2>0$, a regular function of x^2, $(x+y)^2$ except "to the right" of the S curve, as shown with horizontal shading in Fig. 1. But the most general form[4] for (2) is

$$\langle 0|A(x)[B(0),C(-y)]|0\rangle=\int_0^\infty\psi(s,t,\lambda;k)$$
$$\times\Delta_k(y)\Delta^{(+)}(x,x+y;s,t,\lambda)dsdtd\lambda dk,$$

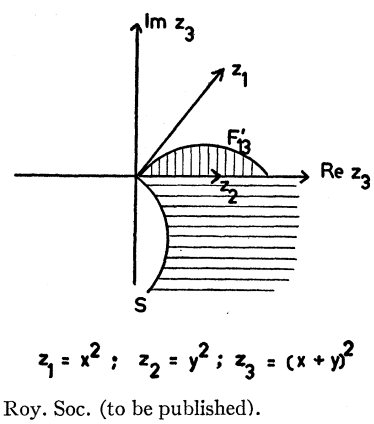

FIG. 1. Diagram showing singularities of the function $\langle 0|A(x)[B(0),C(-y)]|0\rangle$.

[1] F. J. Dyson, Phys. Rev. 111, 1717 (1958).
[2] S. Deser, W. Gilbert, and E. Sudarshan, Phys. Rev. 115, 731 (1959).
[3] V. Ya. Faĭnberg, Zhur. Eksp. i Teoret. Fiz. 36, 1503 (1959) [Translation: Soviet Phys.—JETP 36(9), 1066 (1959)].
[4] R. F. Streater, Proc. Roy. Soc. (to be published).

which has singularities also in the upper half-plane (vertical shading). The reason why (1) does not include these singularities (which come, by the way, from perturbation theory) is that, in going from Eq. (17) to Eq. (6) of reference 1, Dyson writes

$$\nu(y^2,t,\lambda)=\int \Delta_s(y)\phi(s,t,\lambda)ds, \tag{3}$$

which is not always valid. The weight function $\nu(y^2,t,\lambda)$ of reference 1 was obtained by the transformation (16),[1] and $\nu(y^2,t,\lambda)$ will behave as $\exp[t(y^2)^{\frac{1}{2}}]$ for large y^2 or t, unless the Jost-Lehmann weight functions Φ_1 and Φ_2 from which ν was obtained satisfy special conditions. These conditions have no physical meaning, since they are not satisfied in perturbation theory. This was shown by Symanzik,[5] who computed $\nu(y)$ and found it had exponential dependence on y at ∞. The Hankel transform (3) will therefore not exist in general.

In the appendix to their paper, Deser, Gilbert, and Sudarshan make use of the same transformation (16) of reference 1, and then perform a Hankel transform with respect to the t variable. They themselves remark that it is not always valid. The integral representation they get for the retarded vertex function is equivalent to

$$\int \langle 0|\theta(x_0)[\bar{\psi}(x/2),\psi(-x/2)]|p\rangle e^{iqx}d^4x = \int_0^\infty dk\int_{-\frac{1}{2}}^{+\frac{1}{2}} d\alpha \frac{\rho(p^2,\alpha,k)}{(p-\alpha q)^2-k^2}, \tag{4}$$

which has fewer singularities than the most general function. For, if $p^2=\sigma^2$, $p=(\sigma,0,0,0)$, $q_0=0$, $\mathbf{q}^2=z$, the functions $f(z)$ of Eq. (4) have singularities if $z\geqslant -\frac{1}{4}\sigma^2$, real. Now, by considering rotational invariance, the Jost-Lehmann-Dyson representation can be put in the form[6]

$$f(z)=\int_0^{\frac{1}{2}\sigma} d\alpha \int_{\alpha-\frac{1}{2}\sigma}^{\frac{1}{2}\sigma-\alpha} d\beta \int_0^\infty dk \frac{\chi(\alpha,\beta,k)}{(k^2+z+\alpha^2-\beta^2)^2-4\alpha^2 z},$$

which has singularities also off the real axis. To complete the proof, we must show that an arbitrary weight function χ leads to a tempered distribution for the retarded function. This follows from Gårding's[7] proof of the Jost-Lehmann-Dyson representation.

Faĭnberg derives Eq. (4), but his proof is based on a misunderstanding of the "uniqueness" part of the Jost-Lehmann-Dyson[7] theorem. The integration over α in (4), going from $-\infty$ to $+\infty$ in the first place, cannot be restricted to go from $-\frac{1}{2}$ to $+\frac{1}{2}$, since we cannot be sure that the weight function is nonzero only on "admissable hyperboloids." Faĭnberg's footnote criticizing Dyson's proof is wrong. The theorem only says that *there exists* a weight function using only admissible hyperboloids, not that *every* possible weight function has this property. When the limits of α integration are $-\infty$ and $+\infty$ in (4), we cannot conclude that $f(z)$ is regular everywhere except for the real z axis, as would appear at first sight from (4).

The retarded function (4) can be analytically continued into the wedge $\operatorname{Im} q_0>|\operatorname{Im}\mathbf{q}|$, and if $\operatorname{Im} q_0=0$ no point is a regular point of $f(z)$. Though correct, in this form it does not contain the information about the positive mass spectrum; only the causality condition is incorporated.

Applying his method to the scattering function, Faĭnberg "proves" regularity as a function of Δ in the cut plane. This is too large a region (on the information used), since the Lehmann ellipse is the best possible result.

Faĭnberg's technique of taking Fourier transforms with respect to the invariants q^2, $q\cdot x$, $\cdots$ does not lead to wrong results in itself. But it has been rigorously justified so far only for Lorentz-invariant functions of one four-vector.[8]

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Note.—In two further papers,[9] Deser, Gilbert and Sudarshan propose spectral representations for scattering functions similar to those for the vertex function in reference 2. By analogy we think that they are not true in perturbation theory, but have not found a counter example.

[5] K. Symanzik (private communication).
[6] R. Oehme, Phys. Rev. **117**, 1151 (1960).
[7] R. Jost and H. Lehmann, Nuovo cimento **5**, 1598 (1957); F. J. Dyson, Phys. Rev. **110**, 1460 (1958); L. Gårding, seminar at Varenna, Italy, summer, 1958 (unpublished); R. F. Streater, CERN Report February, 1960 (unpublished).
[8] L. Gårding and J. L. Lions, Suppl. Nuovo cimento **14**, 9 (1959). See, however, P. Methée, Comm. Math. Helv. **28**, 225 (1954); **32**, 153 (1957), where Lorentz-invariant distributions in any number of variables are considered.
[9] Phys. Rev. **117**, 266; 273 (1960).