Ferrimagnetic Resonance in Three-Sublattice Systems

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Two classes of three-sublattice systems are considered: antiparallel and triangular. The general solution of the undamped equations of motion is obtained and approximated to the highest order of the molecular field coefficients of interest for each case. For the antiparallel case, the components of the susceptibility tensor are obtained up to the first correction term to the usual expressions. For the triangular case, the leading terms needed for the susceptibility tensor are obtained and the effective gyromagnetic ratio evaluated; the latter is quite different from the form usually assumed. A new effect is also predicted for the triangular case when all sublattice gyromagnetic ratios are different; this effect consists of the production of an oscillating magnetization component perpendicular to the oscillating field and of the same frequency and is not a consequence of nonlinear terms in the equations of motion.

INTRODUCTION

THREE-SUBLATTICE ferrimagnetic systems which are of interest can be divided into two broad classes. The first is the usual antiparallel type in which the different magnetic sublattices are oriented either parallel or antiparallel with respect to each other. This is illustrated in Fig. 1(a). The second is the triangular configuration in which two of the sublattices make an angle with each other and with the direction of the net magnetization.[1] This is illustrated in Fig. 1(b). The first type is probably of most interest with respect to the properties of the ferrimagnetic garnets which have recently been quite intensively studied. Although the evidence for the existence of materials with triangular arrangements is not completely conclusive, it is enough to indicate that some materials are very likely of this type,[2] so that the resonance properties of these systems deserve some study. As we shall see, it is possible to some extent to discuss both cases of Fig. 1 simultaneously.

For the antiparallel three-sublattice case, only the first approximation to its behavior has been found as a special case of an arbitrary number of sublattices.[3] In the limit of large molecular fields and no damping, it was found that the macroscopic equations of motion are exactly like those in the ferromagnetic case provided that one uses an effective gyromagnetic ratio equal to the ratio of the total magnetization to the total angular momentum of the system. Thus, to this approximation, its properties would be just the well-known ones of ferromagnetic resonance.

The resonance properties of the triangular system have been studied in detail only for the case in which the only external magnetic field was a steady one and for which sublattices two and three had equal gyromagnetic ratios.[4] Then it was found that the whole system is equivalent to a two-sublattice system in spite of the constant components of magnetization transverse to the direction of net magnetization and applied field.

In what follows, we shall find the general solution for the net time dependent magnetization of the three-sublattice system. The equations of motion of this coupled system which we use will not include damping effects for simplicity, but will include an external magnetic field composed of a constant component parallel to the net magnetization and a small transverse oscillating component. The general solution can be obtained for both cases at one time; we then look at the antiparallel and triangular cases separately in order to find the highest order approximation of interest. In particular, for the general triangular case, we shall show that the effective gyromagnetic ratio has quite a complicated form, and that our results predict an entirely new effect, namely, the production of an oscillating magnetization which is transverse to the applied oscillating field and of the same frequency.

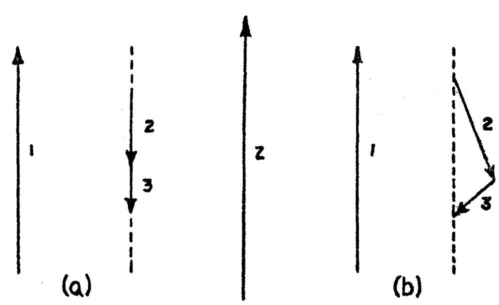

FIG. 1. The two classes of three sublattice systems considered: (a) antiparallel; (b) triangular.

GENERAL SOLUTION

We neglect damping and anisotropy fields; the total field on the ith sublattice is then due only to external

[1] Y. Yafet and C. Kittel, Phys. Rev. **87**, 290 (1952).
[2] F. K. Lotgering, Philips Research Repts. **11**, 190, 337 (1956); P. L. Edwards, Phys. Rev. **116**, 294 (1959); I. S. Jacobs, J. Phys. Chem. Solids **11**, 1 (1959); General Electric Research Laboratory Report MB-39, October, 1959 (unpublished).
[3] R. K. Wangsness, Phys. Rev. **98**, 1200 (1955).
[4] A. Eskowitz and R. K. Wangsness, Phys. Rev. **107**, 379 (1957).

and molecular fields and is given by

$$\mathbf{H}_i=\mathbf{H}+\sum_j \lambda_{ij}\mathbf{M}_j, \tag{1}$$

where $\mathbf{H}$ is the external field, $\mathbf{M}_j$ is the magnetization of the jth sublattice, and $\lambda_{ij}=\lambda_{ji}$ are the molecular field coefficients.

In the triangular case, the requirement that the local field must be parallel to the corresponding sublattice magnetization leads to extra conditions to be satisfied. If we take $\mathbf{H}=0$, choose the z axis as the common direction of the net magnetization and of sublattice 1, and put $M_{iz}=M_i$, we find that we must have[4]

$$\lambda_{12}=\lambda_{13}=\lambda, \tag{2}$$

$$M_1+\beta(M_2+M_3)=0, \tag{3}$$

$$\beta=\lambda_{23}/\lambda, \tag{4}$$

$$(M_{2x}+M_{3x})_{\text{static}}=(M_{2y}+M_{3y})_{\text{static}}=0. \tag{5}$$

Because of (3), we can write the total magnetization as

$$M=M_1+M_2+M_3=(1-\beta^{-1})M_1. \tag{6}$$

These requirements apply only to the triangular case; in the antiparallel case, the local fields are all automatically parallel to the sublattice magnetizations.

The sublattice equations of motion are

$$d\mathbf{M}_i/dt=\gamma_i\mathbf{M}_i\times\mathbf{H}_i, \tag{7}$$

where γ_i is the gyromagnetic ratio of the ith sublattice. The components of the external field $\mathbf{H}$ are taken to be h_x, h_y, $H_z=H=\text{const.}$, where h_x and h_y are small oscillating components each proportional to $e^{i\omega t}$. For simplicity, we shall also set

$$\lambda_{12}=\lambda_2, \quad \lambda_{13}=\lambda_3, \quad \lambda_{23}=\lambda_0. \tag{8}$$

We now write the sublattice magnetization components as the sum of static components plus a small induced component as follows:

$$\begin{aligned} M_{1x}&=m_{1x}, & M_{1y}&=m_{1y}, & M_{1z}&=M_1+m_{1z}', \\ M_{2x}&=\bar{M}_x+m_{2x}, & M_{2y}&=\bar{M}_y+m_{2y}, & M_{2z}&=M_2+m_{2z}', \\ M_{3x}&=-\bar{M}_x+m_{3x}, & M_{3y}&=-\bar{M}_y+m_{3y}, & M_{3z}&=M_3+m_{3z}'. \end{aligned} \tag{9}$$

These can be used for both cases: for triangles, they satisfy (5), and (3) can be used as a condition on the z components; for the nontriangular case we can simply set $\bar{M}_x=\bar{M}_y=0$.

We can now substitute (9) into the set of equations of motion (7), and drop all terms which are second order in the m's. Some of the terms will also vanish for both the antiparallel and triangular cases because $\bar{M}_{x,y}=0$ *or* because of (2) and (3).

Since the M_i are the values of the z components of the magnetization when $H=0$, we also set $m_{iz}'=m_{iz}^0+m_{iz}$ where the m_{iz}^0 are small static components which arise when $H\neq 0$, and m_{iz} are the oscillating components. It is necessary to introduce the m_{iz}^0 in order that the oscillating terms $m_{ij}(j=x,y,z)$ will vanish when $h_x=h_y=0$. This situation arises only in the triangular case, and we find that the m_{iz}^0 satisfy the condition

$$\lambda m_{iz}^0+\lambda_0(m_{2z}^0+m_{3z}^0)+H=0. \tag{10}$$

We can then assume that the m_{ij} are also proportional to $e^{i\omega t}$ so that, for all components,

$$dM_{ij}/dt=dm_{ij}/dt=i\omega m_{ij}. \tag{11}$$

If we now substitute (9) into (7), take into account (8) and (10)–(11), we find that $m_{1z}=0$, and, if we set

$$m_z=m_{2z}+m_{3z}, \tag{12}$$

we find that the rest of the equations become

$$\begin{aligned} A_1m_{1x}-H_1m_{1y}+Bm_{2y}+Cm_{3y}&=-M_1h_y, \\ A_2m_{2x}+Dm_{1y}-H_2m_{2y}+Em_{3y}-J_2m_z&=-M_2h_y, \\ A_3m_{3x}+Fm_{1y}+Gm_{2y}-H_3m_{3y}+J_2m_z&=-M_3h_y, \\ A_1m_{1y}+H_1m_{1x}-Bm_{2x}-Cm_{3x}&=M_1h_x, \\ A_2m_{2y}-Dm_{1x}+H_2m_{2x}-Em_{3x}+J_1m_z&=M_2h_x, \\ A_3m_{3y}-Fm_{1x}-Gm_{2x}+H_3m_{3x}-J_1m_z&=M_3h_x, \\ m_z=-K_2m_{1x}-L_2(m_{2x}+m_{3x})+K_1m_{1y} & \\ +L_1(m_{2y}+m_{3y})-N_2h_x+N_1h_y, & \end{aligned} \tag{13a–g}$$

where

$$A_{1,2,3}=i\omega/\gamma_{1,2,3} \tag{14}$$

$$B=\lambda_2M_1, \quad C=\lambda_3M_1, \quad D=\lambda_2M_2, \tag{15a}$$

$$E=\lambda_0M_2, \quad F=\lambda_3M_3, \quad G=\lambda_0M_3, \tag{15b}$$

$$H_1=H+D+F, \quad H_2=H+B+G, \quad H_3=H+C+E, \tag{16}$$

$$J_1=\lambda_0\bar{M}_x, \quad J_2=\lambda_0\bar{M}_y, \tag{17}$$

$$K_1=\bar{M}_x(\gamma_2\lambda_2-\gamma_3\lambda_3)/i\omega, \quad K_2=\bar{M}_y(\gamma_2\lambda_2-\gamma_3\lambda_3)/i\omega, \tag{18}$$

$$L_1=J_1(\gamma_2-\gamma_3)/i\omega, \quad L_2=J_2(\gamma_2-\gamma_3)/i\omega, \tag{19}$$

$$N_1=L_1/\lambda_0, \quad N_2=L_2/\lambda_0. \tag{20}$$

If we eliminate m_{iy} with the use of (13d–f), we finally find that our equations reduce to

$$\mathcal{A}m_{1x}+\mathcal{B}m_{2x}+\mathcal{C}m_{3x}+\mathcal{D}m_z=\mathcal{R}h_x-M_1h_y=\mathcal{V}_1, \tag{21a}$$

$$\mathcal{E}m_{1x}+\mathcal{F}m_{2x}+\mathcal{G}m_{3x}+\mathcal{H}m_z=\mathcal{S}h_x-M_2h_y=\mathcal{V}_2, \tag{21b}$$

$$\mathcal{I}m_{1x}+\mathcal{J}m_{2x}+\mathcal{K}m_{3x}+\mathcal{L}m_z=\mathcal{T}h_x-M_3h_y=\mathcal{V}_3, \tag{21c}$$

$$\mathcal{M}m_{1x}+\mathcal{N}m_{2x}+\mathcal{P}m_{3x}+\mathcal{Q}m_z=\mathcal{U}h_x+N_1h_y=\mathcal{V}_4, \tag{21d}$$

where

$$
\begin{aligned}
\mathcal{A}&=A_1+(H_1^2/A_1)+(BD/A_2)+(CF/A_3),\\
\mathcal{B}&=-(BH_1/A_1)-(BH_2/A_2)+(CG/A_3),\\
\mathcal{C}&=-(CH_1/A_1)+(BE/A_2)-(CH_3/A_3),\\
\mathcal{D}&=-(J_1B/A_2)+(J_1C/A_3),\\
\mathcal{E}&=-(DH_1/A_1)-(DH_2/A_2)+(EF/A_3),\\
\mathcal{F}&=A_2+(BD/A_1)+(H_2^2/A_2)+(EG/A_3),\\
\mathcal{G}&=(CD/A_1)-(EH_2/A_2)-(EH_3/A_3),\\
\mathcal{H}&=-J_2+(J_1H_2/A_2)+(J_1E/A_3),\\
\mathcal{I}&=-(FH_1/A_1)+(DG/A_2)-(FH_3/A_3),\\
\mathcal{J}&=(BF/A_1)-(GH_2/A_2)-(GH_3/A_3),\\
\mathcal{K}&=A_3+(CF/A_1)+(EG/A_2)+(H_3^2/A_3),\\
\mathcal{L}&=J_2-(GJ_1/A_2)-(H_3J_1/A_3),\\
\mathcal{M}&=K_2+(K_1H_1/A_1)-(DL_1/A_2)-(FL_1/A_3),\\
\mathcal{N}&=L_2-(K_1B/A_1)+(L_1H_2/A_2)-(L_1G/A_3),\\
\mathcal{P}&=L_2-(K_1C/A_1)-(L_1E/A_2)+(L_1H_3/A_3),\\
\mathcal{Q}&=1+(J_1L_1)[A_2^{-1}-A_3^{-1}],\\
\mathcal{R}&=(M_1H_1/A_1)-(M_2B/A_2)-(M_3C/A_3),\\
\mathcal{S}&=-(M_1D/A_1)+(M_2H_2/A_2)-(M_3E/A_3),\\
\mathcal{T}&=-(M_1F/A_1)-(M_2G/A_2)+(M_3H_3/A_3),\\
\mathcal{U}&=-N_2+(M_1K_1/A_1)+(M_2L_1/A_2)+(M_3L_1/A_3).
\end{aligned}
\tag{22a–t}
$$

If we solve these equations for the oscillatory magnetization components, and let

$$m_x=m_{1x}+m_{2x}+m_{3x}=\chi_{xx}h_x+\chi_{xy}h_y, \tag{23}$$

$$m_z=\chi_{zx}h_x+\chi_{zy}h_y, \tag{24}$$

we find that

$$
\begin{aligned}
\tilde{D}\chi_{xx}=\;&(\mathcal{K}\mathcal{Q}-\mathcal{L}\mathcal{P})[\mathcal{S}(\mathcal{A}-\mathcal{B})-\mathcal{R}(\mathcal{E}-\mathcal{F})]\\
&+(\mathcal{G}\mathcal{Q}-\mathcal{H}\mathcal{P})[\mathcal{R}(\mathcal{I}-\mathcal{J})-\mathcal{T}(\mathcal{A}-\mathcal{B})]\\
&+(\mathcal{G}\mathcal{L}-\mathcal{H}\mathcal{K})[\mathcal{U}(\mathcal{A}-\mathcal{B})-\mathcal{R}(\mathcal{M}-\mathcal{N})]\\
&+(\mathcal{C}\mathcal{Q}-\mathcal{D}\mathcal{P})[\mathcal{T}(\mathcal{E}-\mathcal{F})-\mathcal{S}(\mathcal{I}-\mathcal{J})]\\
&+(\mathcal{C}\mathcal{L}-\mathcal{D}\mathcal{K})[\mathcal{S}(\mathcal{M}-\mathcal{N})-\mathcal{U}(\mathcal{E}-\mathcal{F})]\\
&+(\mathcal{C}\mathcal{H}-\mathcal{D}\mathcal{G})[\mathcal{U}(\mathcal{I}-\mathcal{J})-\mathcal{T}(\mathcal{M}-\mathcal{N})]\\
&+(\mathcal{J}\mathcal{Q}-\mathcal{L}\mathcal{N})(\mathcal{E}\mathcal{R}-\mathcal{A}\mathcal{S})\\
&+(\mathcal{F}\mathcal{Q}-\mathcal{H}\mathcal{N})(\mathcal{A}\mathcal{T}-\mathcal{I}\mathcal{R})\\
&+(\mathcal{F}\mathcal{L}-\mathcal{H}\mathcal{J})(\mathcal{M}\mathcal{R}-\mathcal{A}\mathcal{U})\\
&+(\mathcal{B}\mathcal{Q}-\mathcal{D}\mathcal{N})(\mathcal{I}\mathcal{S}-\mathcal{E}\mathcal{T})\\
&+(\mathcal{B}\mathcal{L}-\mathcal{D}\mathcal{J})(\mathcal{E}\mathcal{U}-\mathcal{M}\mathcal{S})\\
&+(\mathcal{B}\mathcal{H}-\mathcal{D}\mathcal{F})(\mathcal{M}\mathcal{T}-\mathcal{I}\mathcal{U}),
\end{aligned}
\tag{25}
$$

$$
\begin{aligned}
\tilde{D}\chi_{xy}=\;&(\mathcal{K}\mathcal{Q}-\mathcal{L}\mathcal{P})[M_1(\mathcal{E}-\mathcal{F})-M_2(\mathcal{A}-\mathcal{B})]\\
&+(\mathcal{G}\mathcal{Q}-\mathcal{H}\mathcal{P})[M_3(\mathcal{A}-\mathcal{B})-M_1(\mathcal{I}-\mathcal{J})]\\
&+(\mathcal{G}\mathcal{L}-\mathcal{H}\mathcal{K})[M_1(\mathcal{M}-\mathcal{N})+N_1(\mathcal{A}-\mathcal{B})]\\
&+(\mathcal{C}\mathcal{Q}-\mathcal{D}\mathcal{P})[M_2(\mathcal{I}-\mathcal{J})-M_3(\mathcal{E}-\mathcal{F})]\\
&+(\mathcal{D}\mathcal{K}-\mathcal{C}\mathcal{L})[M_2(\mathcal{M}-\mathcal{N})+N_1(\mathcal{E}-\mathcal{F})]\\
&+(\mathcal{C}\mathcal{H}-\mathcal{D}\mathcal{G})[M_3(\mathcal{M}-\mathcal{N})+N_1(\mathcal{I}-\mathcal{J})]\\
&+(\mathcal{J}\mathcal{Q}-\mathcal{L}\mathcal{N})(M_2\mathcal{A}-M_1\mathcal{E})\\
&+(\mathcal{F}\mathcal{Q}-\mathcal{H}\mathcal{N})(M_1\mathcal{I}-M_3\mathcal{A})\\
&+(\mathcal{H}\mathcal{J}-\mathcal{F}\mathcal{L})(M_1\mathcal{M}+N_1\mathcal{A})\\
&+(\mathcal{B}\mathcal{Q}-\mathcal{D}\mathcal{N})(M_3\mathcal{E}-M_2\mathcal{I})\\
&+(\mathcal{B}\mathcal{L}-\mathcal{D}\mathcal{J})(M_2\mathcal{M}+N_1\mathcal{E})\\
&+(\mathcal{D}\mathcal{F}-\mathcal{B}\mathcal{H})(M_3\mathcal{M}+N_1\mathcal{I}),
\end{aligned}
\tag{26}
$$

$$
\begin{aligned}
\tilde{D}\chi_{zx}=\;&(\mathcal{J}\mathcal{P}-\mathcal{K}\mathcal{N})(\mathcal{A}\mathcal{S}-\mathcal{E}\mathcal{R})+(\mathcal{F}\mathcal{P}-\mathcal{G}\mathcal{N})(\mathcal{I}\mathcal{R}-\mathcal{A}\mathcal{T})\\
&+(\mathcal{F}\mathcal{K}-\mathcal{G}\mathcal{I})(\mathcal{A}\mathcal{U}-\mathcal{M}\mathcal{R})\\
&+(\mathcal{B}\mathcal{P}-\mathcal{C}\mathcal{N})(\mathcal{E}\mathcal{T}-\mathcal{I}\mathcal{S})\\
&+(\mathcal{C}\mathcal{I}-\mathcal{B}\mathcal{K})(\mathcal{E}\mathcal{U}-\mathcal{M}\mathcal{S})\\
&+(\mathcal{B}\mathcal{G}-\mathcal{C}\mathcal{F})(\mathcal{I}\mathcal{U}-\mathcal{M}\mathcal{T}),
\end{aligned}
\tag{27}
$$

$$
\begin{aligned}
\tilde{D}\chi_{zy}=\;&(\mathcal{J}\mathcal{P}-\mathcal{K}\mathcal{N})(M_1\mathcal{E}-M_2\mathcal{A})\\
&+(\mathcal{F}\mathcal{P}-\mathcal{G}\mathcal{N})(M_3\mathcal{A}-M_1\mathcal{I})\\
&+(\mathcal{F}\mathcal{K}-\mathcal{G}\mathcal{J})(M_1\mathcal{M}+N_1\mathcal{A})\\
&+(\mathcal{B}\mathcal{P}-\mathcal{C}\mathcal{N})(M_2\mathcal{I}-M_3\mathcal{E})\\
&+(\mathcal{C}\mathcal{J}-\mathcal{B}\mathcal{K})(M_2\mathcal{M}+N_1\mathcal{E})\\
&+(\mathcal{B}\mathcal{G}-\mathcal{C}\mathcal{F})(M_3\mathcal{M}+N_1\mathcal{I}),
\end{aligned}
\tag{28}
$$

where $\tilde{D}$, the determinant of the coefficients of (21) is given by

$$
\begin{aligned}
\tilde{D}=\;&(\mathcal{A}\mathcal{F}-\mathcal{B}\mathcal{E})(\mathcal{K}\mathcal{Q}-\mathcal{L}\mathcal{P})\\
&+(\mathcal{B}\mathcal{I}-\mathcal{A}\mathcal{J})(\mathcal{G}\mathcal{Q}-\mathcal{H}\mathcal{P})\\
&+(\mathcal{A}\mathcal{N}-\mathcal{B}\mathcal{M})(\mathcal{G}\mathcal{L}-\mathcal{H}\mathcal{K})\\
&+(\mathcal{C}\mathcal{L}-\mathcal{D}\mathcal{K})(\mathcal{F}\mathcal{M}-\mathcal{E}\mathcal{N})\\
&+(\mathcal{C}\mathcal{H}-\mathcal{D}\mathcal{G})(\mathcal{I}\mathcal{N}-\mathcal{J}\mathcal{M})\\
&+(\mathcal{C}\mathcal{Q}-\mathcal{D}\mathcal{P})(\mathcal{E}\mathcal{J}-\mathcal{F}\mathcal{I}).
\end{aligned}
\tag{29}
$$

These general results are much too unwieldy for our present purposes. In what follows, we shall successively consider the antiparallel and triangular cases and evaluate (25)–(29) only to terms of highest order in the λ's which will give us useful approximations.

ANTIPARALLEL CASE

As pointed out before, this can be obtained from our general results by setting $\bar{M}_x=\bar{M}_y=0$. We see from (17)–(20) and (22) that the following quantities are then zero as well: J_1, J_2, K_1, K_2, L_1, L_2, N_1, N_2, $\mathcal{D}$, $\mathcal{H}$, $\mathcal{L}$, $\mathcal{M}$, $\mathcal{N}$, $\mathcal{P}$, and $\mathcal{U}$.

First of all, we see at once from (24), (27), and (28) that $m_z=0$ as we should have for the antiparallel case.

It is now a straightforward but tedious task to evaluate the general solutions explicitly for this case. We shall only quote the results.

The highest order of the molecular field coefficients which occurs in (25), (26), and (29) is the fourth. If we evaluate all of these quantities to the fourth and third order so that we get the leading term plus the first correction term, we find that

$$\Delta\chi_{xx}=M^3H+2M^2H^2(\mathcal{Y}/\mathcal{W})-(\omega^2/\mathcal{W})\{[M_1^2(\lambda_3M_2+\lambda_2M_3)+\lambda_0M_1(M_2+M_3)^2]\gamma_1^{-2}+[M_2^2(\lambda_0M_1+\lambda_2M_3)+\lambda_3M_2(M_1+M_3)^2]\gamma_2^{-2}+[M_3^2(\lambda_0M_1+\lambda_3M_2)+\lambda_2M_3(M_1+M_2)^2]\gamma_3^{-2}+(2M_1M_2/\gamma_1\gamma_2)[M_3(\lambda_2-\lambda_3-\lambda_0)-(\lambda_3M_1+\lambda_0M_2)]+(2M_1M_3/\gamma_1\gamma_3)[M_2(\lambda_3-\lambda_2-\lambda_0)-(\lambda_2M_1+\lambda_0M_3)]+(2M_2M_3/\gamma_2\gamma_3)M_1(\lambda_0-\lambda_3)\}, \quad (30)$$

$$\Delta\chi_{xy}=-i\omega S[M^2+2MH(\mathcal{Y}/\mathcal{W})], \quad (31)$$

$$\Delta=(M^2H^2-\omega^2S^2)+\frac{2H}{\mathcal{W}}\left\{MH^2\mathcal{Y}-\omega^2\left[(\lambda_2+\lambda_3)\left(\frac{M_1}{\gamma_1}\right)^2+(\lambda_2+\lambda_0)\left(\frac{M_2}{\gamma_2}\right)^2+(\lambda_3+\lambda_0)\left(\frac{M_3}{\gamma_3}\right)^2+M_1M_2\left(\frac{\lambda_0}{\gamma_1^2}+\frac{\lambda_3}{\gamma_2^2}+\frac{2\lambda_2}{\gamma_1\gamma_2}\right)+M_1M_3\left(\frac{\lambda_0}{\gamma_1^2}+\frac{\lambda_2}{\gamma_3^2}+\frac{2\lambda_3}{\gamma_1\gamma_3}\right)+M_2M_3\left(\frac{\lambda_3}{\gamma_2^2}+\frac{\lambda_2}{\gamma_3^2}+\frac{2\lambda_0}{\gamma_2\gamma_3}\right)\right]\right\}, \quad (32)$$

$$\mathcal{W}=\lambda_2\lambda_3M_1+\lambda_2\lambda_0M_2+\lambda_3\lambda_0M_3, \quad (33)$$

$$\mathcal{Y}=(\lambda_2+\lambda_3)M_1+(\lambda_2+\lambda_0)M_2+(\lambda_3+\lambda_0)M_3, \quad (34)$$

$$S=(M_1/\gamma_1)+(M_2/\gamma_2)+(M_3/\gamma_3). \quad (35)$$

M is, of course, the total magnetization of the system and S is the total angular momentum.

As a check on our results, we can neglect the terms proportional to $\mathcal{W}^{-1}$, and we find that (30)–(35) yield the well-known first approximation for the case of no damping:

$$\chi_{xx}=\gamma_e^2MH/(\gamma_e^2H^2-\omega^2), \quad \chi_{xy}=-i\omega\gamma_eM/(\gamma_e^2H^2-\omega^2),$$

where the effective gyromagnetic ratio is $\gamma_e=M/S$.

We see at once from (30) and (31) that, to this order, χ_{xy} vanishes at both the magnetization and angular momentum compensation points, while χ_{xx} is proportional to the inverse first power of the molecular field coefficients at the magnetization compensation point and is different from zero to zeroth order at the angular momentum compensation point. In principle, these formulas should be of assistance in accurately analyzing those experimental results which are obtained very close to the compensation points, especially for materials with very small damping.

TRIANGULAR CASE

In this case, of course, we cannot set $\bar{M}_x$ and $\bar{M}_y$ equal to zero. Consequently, none of the symbols (15)–(20) and (22) vanish in general; this makes the task of simplifying the general solutions very much more difficult. The existence of the conditions (2)–(4) does, however, alleviate the situation somewhat. For example, now $E=\beta D$ and $G=\beta F$; in addition, certain frequently occurring combinations have simple values—examples of this are $B+E+G=0$, $BD+E(D+F)=0$, $BF+G(D+F)=0$, $B+D+F=\lambda M$, etc.

In principle, there are two separate cases to be considered here depending upon whether γ_2 and γ_3 are equal or different. If $\gamma_2=\gamma_3$, so that $A_2=A_3$, then N_1, N_2, $\mathfrak{D}$, $\mathfrak{M}$, $\mathfrak{N}$, $\mathcal{P}$, and $\mathcal{U}$ are all zero and, among other things, $m_z=0$. As a check on the work, one can evaluate the results for this case, and one finds that the results are exactly the same as for the two sublattice system as was already known.[4] Consequently, we will not quote the results again here, especially since the two sublattice system including damping has already been completely worked out.[5]

When $\gamma_2\neq\gamma_3$, one finds that the leading terms are proportional to λ^3. If only these are kept, the following results are obtained:

$$\bar{\Delta}\chi_{xx}=\frac{M}{\beta}\left\{(\beta-1)H+\frac{1}{(\bar{M}_x^2+\bar{M}_y^2)}\left[\bar{M}_x^2H-\frac{\bar{M}_y^2\omega^2}{\gamma_2\gamma_3H}-i\omega\bar{M}_x\bar{M}_y\left(\frac{1}{\gamma_2}+\frac{1}{\gamma_3}\right)\right]\right\}, \quad (36)$$

$$\bar{\Delta}\chi_{xy}=\frac{-iM\omega}{\gamma_1}\left\{1-\frac{\bar{M}_x\gamma_1}{\beta(\bar{M}_x^2+\bar{M}_y^2)}\left[\bar{M}_x\left(\frac{1}{\gamma_2}+\frac{1}{\gamma_3}\right)-\frac{i\bar{M}_y}{\omega H}\left(\frac{\omega^2}{\gamma_2\gamma_3}+H^2\right)\right]\right\}, \quad (37)$$

$$\chi_{zx}=\sigma\bar{M}_x-\tau\bar{M}_y, \quad (38a)$$

$$\chi_{zy}=\tau\bar{M}_x+\sigma\bar{M}_y, \quad (38b)$$

[5] R. K. Wangsness, Phys. Rev. **111**, 813 (1958); **113**, 771 (1959).

$$\beta\gamma_1(\gamma_2-\gamma_3)(\bar{M}_x^2+\bar{M}_y^2)\bar{\Delta}\sigma$$
$$=\frac{\omega^2}{\lambda}\left[(\gamma_1-\gamma_2)\frac{M_2}{\gamma_3}+(\gamma_1-\gamma_3)\frac{M_3}{\gamma_2}\right]-\frac{H^2\gamma_1\gamma_2\gamma_3}{\lambda\beta}\left[\frac{M_1}{\gamma_1}+\beta\left(\frac{M_2}{\gamma_2}+\frac{M_3}{\gamma_3}\right)\right], \quad (39)$$

$$\beta^2\gamma_1(\gamma_2-\gamma_3)(\bar{M}_x^2+\bar{M}_y^2)\bar{\Delta}\tau$$
$$=\frac{i\omega H}{\lambda}\left\{\gamma_1 M\left[\frac{\gamma_2\gamma_3H^2}{\omega^2}-\left(\frac{\gamma_2}{\gamma_3}+\frac{\gamma_3}{\gamma_2}\right)\right]+\beta\gamma_3\left(1-\frac{\gamma_1}{\gamma_2}\right)M_2+\beta\gamma_2\left(1-\frac{\gamma_1}{\gamma_3}\right)M_3+\frac{\omega^2}{(\gamma_2\gamma_3H^2)}[(\gamma_1-\beta\gamma_2)M_2+(\gamma_1-\beta\gamma_3)M_3]\right\}, \quad (40)$$

$$\bar{\Delta}=H^2-(\omega^2/\gamma_t^2), \quad (41)$$

$$\frac{1}{\gamma_t^2}=\frac{1}{(\beta-1)}\left[\frac{\beta}{\gamma_1^2}+\frac{1}{\gamma_2\gamma_3}-\frac{1}{\gamma_1}\left(\frac{1}{\gamma_2}+\frac{1}{\gamma_3}\right)\right]. \quad (42)$$

We see that γ_t in (42) is the effective gyromagnetic ratio for the strict triangular case of $\gamma_2\neq\gamma_3$. We also see that γ_t is independent of temperature, or, at most, may have a slight dependence on temperature if the molecular field coefficients and hence the ratio β vary with temperature. This is in contrast to the antiparallel case where the first approximation is M/S and can vary greatly with temperature near compensation points. This sort of behavior does not occur in (42) because it was possible to eliminate the static magnetizations by using the equation of constraint (3).

DISCUSSION

Probably the most interesting result of these calculations is the fact that in the strict triangular case, $m_z\neq 0$, according to (24) and (38)–(40). In other words, the oscillating field can induce an oscillating magnetization which has the same frequency and which is perpendicular to this field and hence along the common direction of the static field and the net static magnetization. This is a new effect in ferrimagnetics, arising as it does from the linearized equations of motion rather than from the more exact nonlinear ones. We can also see at once from the above results that this component also has a resonant behavior with an effective gyromagnetic ratio which is the same as for the transverse components. The existence of this magnetization is due to the triangular structure since it is proportional to $\bar{M}_x$ and $\bar{M}_y$ and vanishes when they do. One may need to use single crystals in order to investigate the existence of this effect since χ_{zx} and χ_{zy} are linear in $\bar{M}_x$ and $\bar{M}_y$ and thus may average out to zero in a polycrystalline sample.

We can understand the origin of this component by referring to Fig. 2. The general effect of the oscillating field is to set the sublattice magnetizations into precession about their static orientations. As shown, the precession of sublattices 2 and 3 will give rise to an oscillating component along the direction of net magnetization in addition to the transverse x and y components. If these z components are produced with appropriate phases, they will combine to form a nonzero resultant. (Sublattice 1 will not contribute to this z component since its own z component is constant according to the linearized equations of motion.) From the figure, we see that we would expect this resultant oscillating magnetization to increase as the angles made by sublattices 2 and 3 with the z direction increase; this is reflected in (38)–(40) by χ_{zx} and χ_{zy} being proportional to $\bar{M}_x$ and $\bar{M}_y$. We also see that σ and τ are both proportional to λ^{-1}. This can again be roughly understood from Fig. 2 since λ^{-1} is approximately the perpendicular susceptibility $\chi_\perp$ of these materials and we would expect the effect produced by h_x and h_y to be proportional to this quantity.

One might wonder about the real existence of these nonzero values of χ_{zx} and χ_{zy} since discussions of the properties of the susceptibility tensor usually conclude that they are zero, so that the susceptibility tensor has only three independent components.[6] These proofs, however, assume that the system possesses a rotational symmetry about the z axis which is not the case for the triangular systems of Figs. 1 and 2.

In general, all of the results obtained here may be useful in interpreting experimental data for three-sublattice systems, particularly in the strict triangular case, for which, for example, the expression for the effective gyromagnetic ratio is quite different from what has heretofore been used.

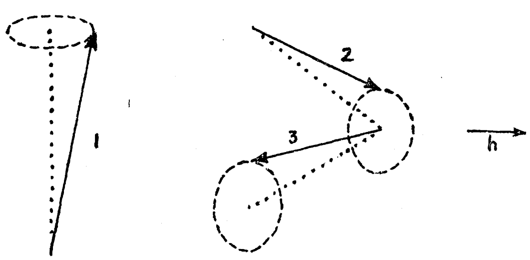

FIG. 2. Origin of the oscillating magnetization component which is transverse to the exciting field.

[6] For example, B. S. Gourary, J. Appl. Phys. 28, 283 (1957).