transition from the lower to the upper "ground-state" level is small. In the case of the magnetoresistivity the Boltzmann factor at 4.2°K is not so small and, for this reason, the calculated magnetoresistive ratio should strictly be compared with values measured at lower temperatures where the population of the triplet level is vanishingly small.

Note added in proof.—One can expect that (with appropriate rephrasement) the above model can be applied to p-type germanium too. This view is supported by measurements[22] which show that the conductivity of a lightly doped and weakly compensated ($K'=N_D/N_A$) p-type germanium crystal increases with a small increase in the compensation. This is expected, since the number of acceptor ions to which holes can jump increases. To arrive at numerical results and compare them with measured data on In-doped[22] and Ga-doped[19] samples, we have assumed a hydrogenlike $1s$ wave function for a hole. This is an admittedly rough choice because it corresponds to only one term of the more exact but complicated hole wave function, derived[23] from the effective mass theory. The effective masses ($m_{\mathrm{In}}{}^*=0.214\,m_0$ and $m_{\mathrm{Ga}}{}^*=0.203\,m_0$) have been determined from the ionization energy of In and Ga acceptors[24] at liquid He temperatures. Calculated and measured data are shown in Table II, where the number of "free" acceptor ions/cc is denoted by $\bar{p}$.

VI. ACKNOWLEDGMENT

The writer wishes to thank Dr. Evan O. Kane for many valuable discussions during the course of this work.

[22] H. Fritzsche and K. Lark-Horowitz, Phys. Rev. **113**, 999 (1959).

[23] W. Kohn and D. Schechter, Phys. Rev. **99**, 1903 (1955).

[24] P. Fisher and H. Y. Fan, Phys. Rev. Letters **2**, 456 (1959).

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Adiabatic Demagnetization and Specific Heat in Ferrimagnets

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The specific heat of yttrium iron garnet (YIG) has been measured at 1.45°K as a function of magnetic field from zero to 18 kilo-oersteds. It is found to drop about 30% over this field range as the field is increased. Adiabatic temperature changes, ΔT, were observed during magnetic cycles over various field intervals in the range from 3 to 18 kilo-oersteds, and at various temperatures between 1 and 4°K. The experimental values of ΔT and the specific heat fit the predictions of spin-wave theory to within experimental error. The data are sufficiently extensive to provide a useful test of spin-wave theory as well as checks on the consistency of the data itself as between the two types of observations. Values of various parameters which characterize the thermal and magnetic properties of YIG are determined from this investigation to have the following values: Landau-Lifshitz exchange constant $A=4.3\times10^{-7}$ erg/cm; Debye temperature $\theta=510$°K. The serious effect of magnetic impurities on investigations of this sort is pointed out.

I. INTRODUCTION

WE have observed the heat capacity as a function of magnetic field and the adiabatic magnetization and demagnetization of yttrium iron garnet (YIG) at temperatures in the liquid helium range. The heat capacity has been observed at only one temperature, namely 1.45°K, and is significantly smaller at zero magnetic field than the earlier results obtained by Edmonds and Peterson[1] and by Meyer and Harris.[2] Our other observations are of the temperature changes in a thermally isolated sample as entropy is transferred back and forth between the magnetic spin system and the crystal lattice by means of adiabatic changes of an applied magnetic field. These experiments require care in isolating the sample thermally and in measuring sample temperature with thermometry which has a heat capacity small compared to that of the sample. Because of the temperature and field dependence of the population of the energy levels of the magnetic lattice, the density of states of the magnetic lattice can be deduced from these results. When combined with specific heat data they provide a new test of the predictions of spin-wave theory.

Adiabatic demagnetization has found wide use in the study of paramagnetics at liquid helium temperatures. In such materials, the difference in entropy between completely disordered states and essentially ordered ones can be achieved experimentally. The maximum entropy change which can be produced in ferrimagnetics like YIG with Curie points above room temperature is much smaller, since the amount of magnetic disorder and thus the amount of magnetic entropy present at liquid helium temperatures is very small compared to

[1] D. T. Edmonds and R. G. Petersen, Phys. Rev. Letters **2**, 499 (1959); **4**, 92 (1960).

[2] H. Meyer and A. B. Harris (private communication); and *Fifth Conference on Magnetism and Magnetic Materials, Detroit, Michigan, November, 1959* [J. Appl. Phys. **31**, 49S (1960)].

that in paramagnetics. As a result, it is not immediately obvious that a significant effect can be achieved in such materials, but closer inspection shows that it is possible. The change in temperature, ΔT, may be shown from thermodynamical reasoning[3] to be:

$$\Delta T = -\int_{H_1}^{H_2} \frac{T(\partial M/\partial T)_H dH}{C_p}, \qquad (1)$$

where M is the magnetization of the material per unit volume, H is the magnetic field, and C_p is the total specific heat at constant pressure. C_p is the sum of contributions from the magnetic lattice, C_M, the crystal lattice, C_L, and the surroundings, C_s. There may also be a contribution to C_p from paramagnetic impurities in an actual sample; we will discuss this possibility later. Both C_M and M are functions of H as well as T. From Eq. (1) it follows that a significant effect will be observed if, (1) the sum of C_s and C_L is smaller than or comparable to C_M, and (2) we make a substantial fractional change in C_M by changing the applied field H. This latter condition requires that $(\partial M/\partial T)_H$ be significantly different from zero. From Kouvel's early data on magnetite,[4] we expect condition (1) to be valid for most ferrimagnetics having negligible electronic specific heat. Spin-wave theory[5] suggests that a field of 10–20 kilo-oersteds is adequate for condition (2) in the range 1° to 2°K. Our result bear out these predictions.

II. EXPERIMENTAL

As mentioned earlier, there are experimental problems in applying adiabatic demagnetization methods to the study of ferrimagnets. These problems relate primarily to the mounting of the sample, which is subjected to various forces, and to the measuring of temperature changes in it. These things must be done without altering the heat capacity of the sample significantly, or destroying the high degree of thermal isolation required, and above all they must be done without introducing even small amounts of a paramagnetic material. Paramagnetics have heat capacities in this range which are quite large, and their presence in significant quantities would have given rise to spurious temperature changes when the applied field was changed. The method adopted was to support the sample, which was spherical, in silicon powder. Silicon is one of the few substances which can be obtained in sufficient purity so that there is no danger of spurious effects in these experiments from traces of paramagnetic impurities in it. At the same time it has a relatively low heat capacity at low temperatures (high Debye temperature). If the particles which make up the powder are made as fine as possible, a sample embedded in it in an evacuated container is so completely isolated thermally that it cannot be cooled effectively. It was found that freshly prepared particles about 50 microns in diameter made the thermal relaxation time between the sample and the helium bath about 7 minutes at these temperatures, and they proved quite satisfactory. The thermal conductivity of silicon powder is very sensitive to its state of oxidation. A discussion of this problem and others associated with the use of powders in this way will be given elsewhere.[6]

A description of the magnet, cryostat, and method of supporting the sample chamber is given elsewhere in connection with other experiments.[7] Figure 1 shows the

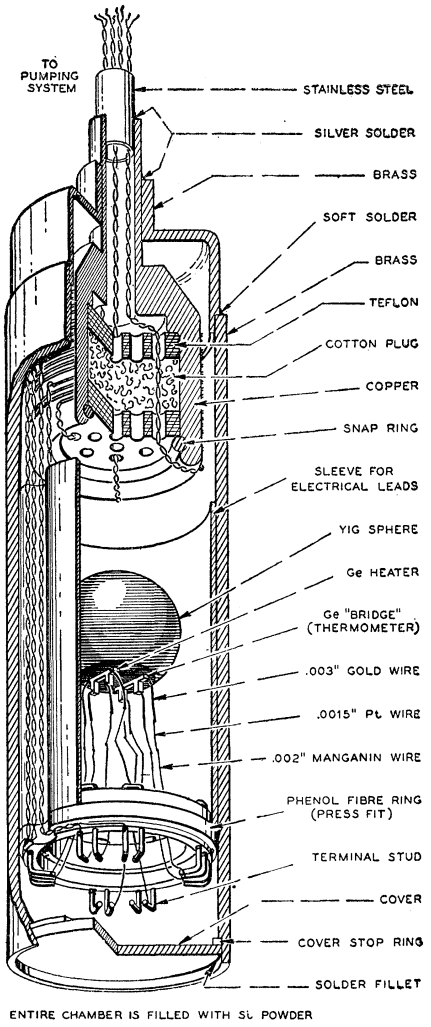

FIG. 1. Sample chamber. This chamber is supported within the cryostat. The sample is embedded in Si powder. The electrical leads are equilibrated with the helium bath by a copper thermal link as shown. Each lead has a section of 0.002-in. diam manganin wire to aid thermal isolation. The platinum and gold wire sections in each lead are for convenience in making electrical contact with the germanium "bridge." The silicon powder is restrained by means of the cotton plug and the chamber is evacuated before use.

[3] H. B. G. Casimir, *Magnetism and Very Low Temperatures* (Cambridge University Press, New York, 1940), Chap. IV.
[4] J. S. Kouvel, Phys. Rev. **102**, 1489 (1956).
[5] T. Holstein and H. Primakoff, Phys. Rev. **58**, 1098 (1940).
[6] J. E. Kunzler (to be published).
[7] C. Herring, T. H. Geballe, and J. E. Kunzler, Phys. Rev. **111**, 36 (1958).

sample chamber with the sample in position. The temperature was measured using a germanium thermal sensing element,[8] one end of which was attached to a "flat" on the sample with G.E. No. 7031 adhesive (approximately 10^{-5} gram). The thermometer was calibrated as a function of magnetic field as well as temperature. In addition, a similar piece of germanium was affixed to the same flat but separated from the thermometer by a distance of 0.7 cm. The latter formed a resistance heater used for adding controlled and measured amounts of heat to the sample in order to measure its specific heat. The total heat capacity of the thermometer, heater resistance leads and all other material in thermal equilibrium with but not a part of the sample, is estimated to be less than 10 ergs/deg at 1.5°K. The thermometer current was 5×10^{-8} amp, and the resistance was less than 10^5 ohms, so that the thermometer heating was less than 0.2 erg/min.

The sample used for most of the experiments was a polycrystalline sphere of YIG ($Y_3Fe_5O_{12}$) approximately 1.6 cm in diameter. A flat approximately 1.0 cm in diameter was ground on it, which was used as mentioned above for mounting the thermometer and heater resistance. The sample weighed 10.31 grams; it was sintered to within 1% of the x-ray density of 5.19 g/cc.

The adiabatic temperature change data were obtained by cycling the sample in the following way. Once the sample achieved equilibrium at the initial temperature and magnetic field for a given cycle, the field was changed and then held fixed at the new value briefly, until the temperature reached a state of "quasiequilibrium" and was observed, whereupon the field was returned to the initial value. At each initial temperature, a series of cycles was performed, each covering a different field range, many of which overlapped. This process was repeated at several temperatures, and the results from the cycles at a given temperature were all quite consistent. The field was changed at a uniform rate in times of approximately 15 seconds. No effect was observed from variations in this rate. The field was held only a fraction of a minute at the noninitial value of the field, as the sample came to "quasiequilibrium" quickly. In each case the temperature change of the sample was recorded continuously on a recorder chart as the magnetic field was varied.

III. RESULTS

Representative values of adiabatic temperature changes ΔT are shown in Fig. 2. The data as observed have been corrected in two ways in order to obtain the points plotted. First, suitable correction (always small) was made for heat exchanged between the sample and the bath by extrapolation of the temperature-time curve back to the time at which equilibrium was first disturbed. Second, a correction was made for irreversible heating effects which occurred during the cycle. It has been shown in separate experiments that the bulk of the irreversible heating is not associated with the thermometer or the leads. These irreversible effects caused the final temperature for all cycles to be higher than the initial temperature even after the first correction mentioned had been made. This temperature difference will be labeled ΔT_{irr}. Such irreversible effects were small but perceptible at the highest fields we have used, and became very much larger at the lowest fields. In fact, at fields below 3000 oersteds, these effects were so large as to make it impossible to observe ΔT with reasonable accuracy, and no data are given in that field range. The maximum ΔT_{irr} observed in the field range for which data are given amounted to 0.05°K and involved a cycle from 3000 to 18 000 oersteds at 1.5°K. The correction of the data for these irreversible changes in temperature was made by assuming that the irreversible heat production occurred in equal amounts in both the magnetization and demagnetization parts of the cycle. It was observed that $C_p\times\Delta T_{\text{irr}}$ (for 3000 to 18 000 oersted cycles) did not vary significantly with temperature.

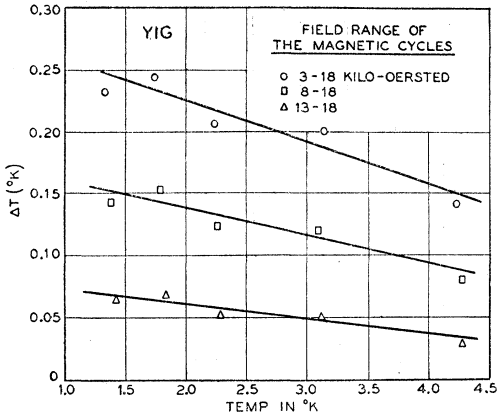

FIG. 2. Adiabatic temperature changes, ΔT, as a function of temperature for typical cycles of the magnetic field. The temperature assigned to each point is the average of the extremes of the cycle. The data have been corrected for irreversible heat production and for heat leaks but include a small effect due to the heat capacity of the thermometer and the heater.

The deviation of the points from the curves in Fig. 2 is systematic at each temperature. We attribute this behavior to the presence of small but significant amounts of magnetic impurities in our sample as discussed later.

The specific heat of the sample was measured as a function of applied field by adding measured amounts of heat through the heater resistance and observing the temperature change of the sample. The measurement has only been made at one temperature, namely 1.45°K; the specific heat at this temperature for $H_{\text{app}}=0$ was observed to be 126 ergs/°K cc. It is estimated that 5 ergs/°K cc of sample is included in this figure for the contribution of the surroundings to the heat capacity observed. The only important sources of this correction

[8] J. E. Kunzler, T. H. Geballe, and G. W. Hull, Rev. Sci. Instr. 28, 96 (1957).

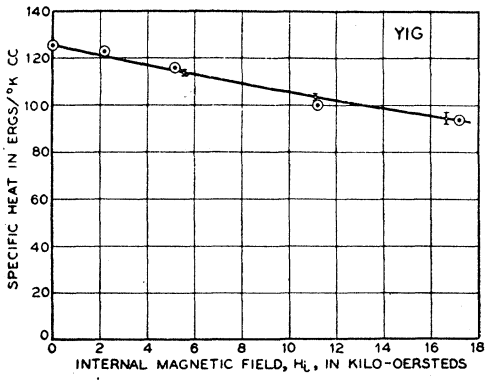

FIG. 3. Specific heat of YIG as a function of the internal magnetic field at 1.45°K. The experimental values are indicated by the points and the circle diameters are estimates of the uncertainty of the points relative to each other. The absolute values of specific heat may be uncertain by 2 or 3 times this amount. The curve is calculated from the constants obtained from spin-wave theory together with adiabatic temperature change data and the uncorrected zero field specific heat, which is 126 ergs/°K cc. The values corrected for the contribution of the thermometer and heater are 5 ergs/°K cc lower. The experimental uncertainty indicated in Eq. (23) leads to uncertainty in the calculated results indicated by the height of the arrows.

are the thermometer and the heater together with their leads. The value of the specific heat given here is considerably lower than those given in references 1 and 2. Meyer and Harris have made extensive preliminary data available to us. Their specific heat at 1.45°K lies approximately midway between the data in reference 1 and ours. Examination of their data up to 20°K suggests to us the presence in their sample of appreciable amounts of magnetic impurities. We are forced to conclude that the differences between the different sets of data arise from the presence of different amounts of impurities in the samples used, and that while our sample may have some such impurities in it, the amount is smaller than that in the others for which data have been reported. These impurities are thought to be either paramagnetic or characterized by a relatively low exchange coupling parameter.

A plot showing the field dependence of the specific heat as observed experimentally is shown in Fig. 3. The value decreases with increasing field as expected; the theoretical curve shown will be discussed later. This is the first experimental observation of this effect known to the authors.

Adiabatic magnetization and demagnetization measurements were also made on a single crystal sphere[9] of YIG with its easy direction oriented parallel to the applied magnetic field. This sphere was about 6 mm in diameter, however, and since it was much smaller than the sample on which the data reported above were taken, the data were less reliable. However, the results on the single crystal do confirm generally the data presented above.

[9] J. W. Nielsen and E. F. Dearborn (to be published).

IV. SPIN-WAVE THEORY

In the temperature range of these experiments the accessible energy levels of the magnetic system will be those of spin-wave excitations long in wavelength compared to the lattice spacing. The relation between he frequency, ω_k, and the wave vector, $\mathbf{k}$ of such spin waves is:

$$\omega_{\mathbf{k}}^2=(\gamma H_i+a\mathbf{k}\cdot\mathbf{k})(\gamma H_i+a\mathbf{k}\cdot\mathbf{k}+4\pi\gamma M\sin^2\theta), \quad (2)$$

where γ is the gyromagnetic ratio; $4\pi M$ is the saturation magnetization; H_i is the demagnetized dc magnetic field within the sample, and θ is the angle between $\mathbf{k}$ and the dc magnetic field. The factor $a=(2g\mu_B/\hbar M)A$, where A is the Landau-Lifshitz exchange constant,[10] g is the spectroscopic splitting factor, and μ_B is the Bohr magneton. At fields as high as those used here the term $4\pi\gamma M\sin^2\theta$ may be ignored to a good approximation and although the most important sample was a polycrystalline sphere, we may take $H_i=H_{\text{applied}}-\frac{4}{3}(\pi M)$, appropriate to a sphere. Then,

$$\begin{aligned}\omega_{\mathbf{k}}&=\gamma H_i+a\mathbf{k}\cdot\mathbf{k},\\ H_i&=H_{\text{app}}-\tfrac{4}{3}(\pi M).\end{aligned} \quad (3)$$

If the spin waves are treated as independent oscillators, capable of unlimited excitations, Z, the partition function, is given by

$$Z=\prod_{\mathbf{k}}[1-\exp(-\hbar\omega_{\mathbf{k}}/k_BT)]^{-1}. \quad (4)$$

When k is treated as a continuous variable the above expression becomes:

$$\ln Z=-\frac{\Omega}{(2\pi)^3}4\pi \times\int_0^{|k|_{\max}}\ln\left\{1-\exp\left[-\frac{\hbar}{k_BT}(\gamma H_i+a\mathbf{k}\cdot\mathbf{k})\right]\right\}\times\mathbf{k}\cdot\mathbf{k}d|k|, \quad (5)$$

where Ω is the volume of the sample and $|k|_{\max}$ the largest value of $|k|$; for the present purposes $|k|_{\max}$ may be replaced by infinity. On expanding the logarithm and integrating one finds:

$$\frac{1}{\Omega}\ln Z=-\frac{1}{8}\left(\frac{k_BT}{\pi\hbar a}\right)^{\frac{3}{2}}f_{\frac{5}{2}}\left(\frac{\bar{H}}{T}\right), \quad (6)$$

where:

$$f_p(x)=\sum_1^\infty\frac{e^{-nx}}{n^p}, \quad (7)$$

and

$$\bar{H}=\frac{\hbar\gamma H_i}{k_B}. \quad (8)$$

[10] C. Kittel and J. K. Galt, in *Solid-State Physics*, edited by F. Seitz and D. Turnbull (Academic Press, New York, 1956), Vol. 3, p. 461.

We use the formulas:

$$S_M=\frac{1}{\Omega}\frac{\partial}{\partial T}[k_BT\ln Z]_H, \tag{9}$$

$$C_M=\frac{k_B}{\Omega}\frac{\partial}{\partial T}\left[T^2\left(\frac{\partial \ln Z}{\partial T}\right)_H\right]_H, \tag{10}$$

$$M_0-M=+k_BT\left(\frac{\partial \ln Z}{\partial H}\right)_T, \tag{11}$$

where S_M, C_M, and M are the entropy, specific heat and magnetization, each per unit volume and M_0 is the value of M at 0°K. From Eqs. (9), (10), and (11), and the above expression for ln Z, one obtains:

$$S_M=\frac{k_B}{8}\left(\frac{k_BT}{\pi\hbar a}\right)^{\frac{3}{2}}\left[\frac{5}{2}f_{\frac{5}{2}}\left(\frac{\bar{H}}{T}\right)+\frac{\bar{H}}{T}f_{\frac{3}{2}}\left(\frac{\bar{H}}{T}\right)\right], \tag{12}$$

$$C_M=\frac{k_B}{8}\left(\frac{k_BT}{\pi\hbar a}\right)^{\frac{3}{2}}\left[\frac{15}{4}f_{\frac{5}{2}}\left(\frac{\bar{H}}{T}\right)+3\frac{\bar{H}}{T}f_{\frac{3}{2}}\left(\frac{\bar{H}}{T}\right)+\left(\frac{\bar{H}}{T}\right)^2f_{\frac{1}{2}}\left(\frac{\bar{H}}{T}\right)\right], \tag{13}$$

$$M_s=M_0-\frac{\hbar\gamma}{8}\left(\frac{k_BT}{\pi\hbar a}\right)^{\frac{3}{2}}\left[f_{\frac{3}{2}}\left(\frac{\bar{H}}{T}\right)\right]. \tag{14}$$

The lattice entropy and specific heat may be written as:

$$S_L=(C/3)T^3, \tag{15}$$

and

$$C_L=CT^3, \tag{16}$$

where the constant C may be obtained from the lattice theory of specific heats.[11]

In comparing the experimental results with the theory we shall use an expression for the entropy, which is conserved in the adiabatic changes and we shall include a term to take account of the entropy of the thermometer and heater. It is instructive, however, to look directly at the formula for the rate of change of temperature with field in such adiabatic processes. The contribution of the thermometer and heater to the entropy and specific heat is small and is assumed to be of the form,

$$S_s=(C'/3)T^3 \text{ ergs/°K cc of sample}, \tag{17}$$

$$C_s=C'T^3 \text{ ergs/°K cc of sample}. \tag{18}$$

Equation (1) may be written in the form

$$\left(\frac{\partial T}{\partial H}\right)_S=\frac{-T(\partial M/\partial T)_H}{C_M+C_L+C_s}, \tag{19}$$

and this may be rewritten as

$$\left(\frac{\partial T}{\partial \bar{H}}\right)_S=\frac{\frac{3}{2}f_{\frac{3}{2}}(\bar{H}/T)+(\bar{H}/T)f_{\frac{1}{2}}(\bar{H}/T)}{15/4f_{\frac{5}{2}}(\bar{H}/T)+3(\bar{H}/T)f_{\frac{3}{2}}(\bar{H}/T)+(\bar{H}/T)^2f_{\frac{1}{2}}(\bar{H}/T)+8/k_B(\pi\hbar a/k_B)^{\frac{3}{2}}(C+C')T^{\frac{3}{2}}}. \tag{20}$$

Two points are worth noting: because M_0-M and C_M contain similar amplitude factors and the same power of T, namely $T^{\frac{3}{2}}$, $(\partial T/\partial H)_S$ is of order unity provided that $\bar{H}/T$ is of order unity and also that the temperature is small enough so that the lattice specific heat is small; secondly, at temperatures and in materials for which the lattice specific heat has become negligible, $(\partial T/\partial \bar{H})_S$ is a function only of $\bar{H}/T$ and does not depend at all upon the specific nature of the magnetic system. For this to occur, it is sufficient that in addition to these conditions the exchange constant be large enough for our assumptions about the spin-wave spectrum to hold and that the applied field be high enough for the demagnetizing term, $4\pi M_0\sin^2\theta$, to be ignored.

V. DISCUSSION

In an adiabatic change the invariance of the entropy implies that the quantity:

$$S_{\text{tot}}=\frac{k_B}{8}\left(\frac{k_BT}{\pi\hbar a}\right)^{\frac{3}{2}}\times\left[\frac{5}{2}f_{\frac{5}{2}}\left(\frac{\bar{H}}{T}\right)+\left(\frac{\bar{H}}{T}\right)f_{\frac{3}{2}}\left(\frac{\bar{H}}{T}\right)\right]+\frac{C+C'}{3}T^3, \tag{21}$$

remains unchanged. This may also be expressed in the form:

$$\frac{\Delta\{T^{\frac{3}{2}}[\frac{5}{2}f_{\frac{5}{2}}(\bar{H}/T)+(\bar{H}/T)f_{\frac{3}{2}}(\bar{H}/T)]\}}{\Delta\{T^3\}}=-\frac{8}{3k_B}\left(\frac{\pi\hbar a}{k_B}\right)^{\frac{3}{2}}(C+C'), \tag{22}$$

where the symbol Δ refers to the difference between the quantity concerned at the initial and final values of temperature and field in an adiabatic change. We have represented a substantial group of experimental results in this manner in Fig. 4. The line represents the best choice of the parameters $(C+C')$. In calculating the internal field from the applied field, the value of M at 0°K must be used; it is 195 cgs units.[12] One finds:

$$\frac{8}{(3k_B)}\left(\frac{\pi\hbar a}{k_B}\right)^{\frac{3}{2}}(C+C')=1.10\pm0.1. \tag{23}$$

It is noticeable that the points in Fig. 4 scatter about the straight line. It is our opinion that this scatter

[11] C. Kittel, *Introduction to Solid-State Physics* (John Wiley & Sons, New York, 1953), 1st ed., p. 75.

[12] M. A. Gilleo and S. Geller, Phys. Rev. **110**, 73 (1958).

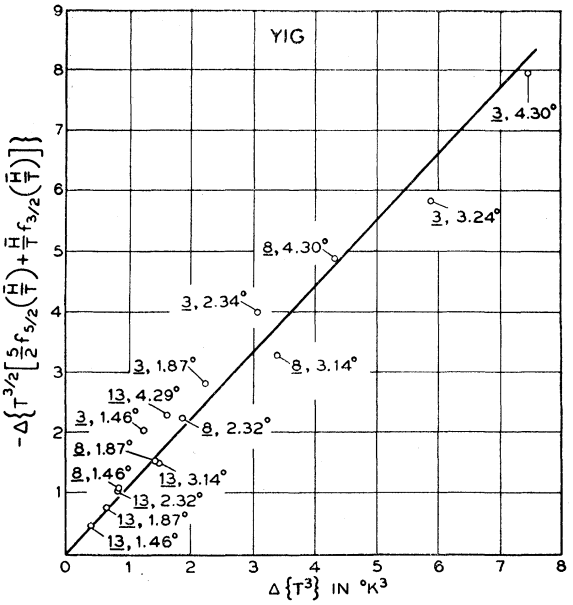

FIG. 4. Correlation of the different observed adiabatic temperature changes with each other by means of spin-wave theory. The different observations involve several temperature and magnetic field intervals and are represented more simply in Fig. 2. The experimental points are identified by the minimum value of the applied magnetic field and the maximum temperature of each interval. The field values are underlined and are in kilo-oersteds. The maximum field in each interval is 18 kilo-oersteds. This plot is useful because the slope of the line is related to the lattice heat capacity and the Landau-Lifshitz exchange constant as given by Eq. (23).

exceeds our experimental errors. A more detailed examination of the points in Fig. 4 shows that points taken at higher temperatures lead to a somewhat higher slope than those taken at lower temperatures. These aspects of the data seem to us to provide added evidence for the presence of paramagnetic or weakly exchange coupled impurities which cause deviations from the behavior described by Eq. (22). Furthermore, it seems likely that the irreversible effects observed in the course of obtaining these data are also connected with the presence of such impurities.

The total specific heat is:

$$C_M+C_L+C_s=\frac{k_B}{8}\left(\frac{k_BT}{\pi\hbar a}\right)^{\frac{3}{2}}\left[\frac{15}{4}f_{\frac{5}{2}}\left(\frac{\bar{H}}{T}\right)+3\left(\frac{\bar{H}}{T}\right)f_{\frac{3}{2}}\left(\frac{\bar{H}}{T}\right)+\left(\frac{\bar{H}}{T}\right)^2f_{\frac{1}{2}}\left(\frac{\bar{H}}{T}\right)\right]+(C+C')T^3. \quad (24)$$

At zero magnetic field this becomes:

$$C_M+C_L+C_s=0.628k_B\left(\frac{k_B}{\pi\hbar a}\right)^{\frac{3}{2}}T^{\frac{3}{2}}+(C+C')T^3. \quad (25)$$

The value of C' for our sample is obtained from Eq. (18) and the estimated value for C_s of 5 ergs/°K cc of sample discussed earlier. It is:

$$C'=1.6. \quad (26)$$

From the value of the specific heat at zero magnetic field and 1.45°K as given in Fig. 3, together with Eqs. (23), (25), and (26), values may be determined for the constants C and a. We obtain:

$$C=20.5,$$
$$a=0.078.$$

The corrected value of the specific heat at zero field and 1.45°K is 121 ergs/°K cc.

The variation of the total specific heat with magnetic field may now be calculated without the use of any new parameters. This has been done, and the results are shown by the solid line in Fig. 3. The fit is quite satisfactory, and represents a check on the internal consistency of our data and on the ability of spin-wave theory to account for it satisfactorily. This calculation was made both with the spin-wave spectrum given by Eq. (2) and with that given by Eq. (3). The difference is small, but the line shown in Fig. 3 reflects the spectrum given by Eq. (2).

Explicitly the formula for the specific heat of YIG at zero magnetic field is:

$$C_P=C_M+C_L=33.6T^{\frac{3}{2}}+20.5T^3 \text{ ergs/°K cc.} \quad (28)$$

Now from the formulas discussed in reference 1, we deduce:

Debye temperature, θ, $=510$°K. (29)

Landau-Lifshitz exchange constant, A, $=4.3\times10^{-7}$ erg/cm. (30)

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