Interactions of 125-Mev K^+ Mesons in Nuclear Emulsion*

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An experimental study of the elastic, inelastic, and charge exchange scattering of K^+ mesons in nuclear emulsion has been made based on 100 meters of track at an average beam energy of 125 Mev. The elastic scattering and total cross sections have been used in a diffuse surface optical model calculation and the inelastic distribution has been analyzed by Monte Carlo techniques. With the assumption of S-wave scattering for the $T=1$ K^+-nucleon state and S- and P-wave scattering for $T=0$, two best fit $T=0$ phase-shift solutions have been found for two different radius parameters. The dependence upon radius is small and the P-wave phase shifts seem to be important at this energy.

INTRODUCTION

ALTHOUGH a number of experiments have been reported[1] on the interaction of the K^+ mesons in nuclear emulsion in the energy region below 150 Mev, few have been accompanied by analyses incorporating exact optical model calculations[2] or detailed studies of the inelastic distributions and the associated problem of pseudoelastic events.[3] There has been no study of the dependence of the analysis on the nuclear radius parameter.

We report here on an experiment based on the examination of 100 meters of K^+-meson track in a nuclear emulsion stack exposed to a separated 150-Mev beam at the Berkeley Bevatron. The experimental results have been analyzed by means of an exact diffuse surface optical model calculation and by means of a Monte Carlo study of the inelastic scattering distribution. These have been used in turn in a K^+-nucleon phase-shift analysis. Two nuclear radius parameters have been used throughout, and, within the framework of the models used, particular care has been taken to include second order effects such as double scattering and the influence of the exclusion principle.

Details of the optical model calculations for this experiment have been given previously[4] and preliminary phase-shift results have been reported earlier[5] for one nuclear radius parameter.

I. EXPERIMENTAL PROCEDURE

A separated beam of 150-Mev K^+ mesons was designed and set up at the Berkeley Bevatron. A schematic diagram showing the details of the experimental arrangement is shown in Fig. 1. Detailed information concerning this beam is given by Stork.[6] An exposure to this beam was made with a stack of 118 Ilford G5 600 micron nuclear-emulsion pellicles of 4 inches $\times 8$ inches area. During the exposure 4×10^{13} protons entered the 1.5-inch long Cu production target. Positive secondary particles of momentum 525 ± 9 Mev/c which left the target at 58° in a solid angle of 1.9 millisteradians were selected by the system. Separation of the momentum-selected pions, protons, and K^+ mesons was achieved with a beryllium degrader from which the K^+ mesons emerged with momentum 420 Mev/c. The exposure yielded approximately one K^+ track per millimeter per

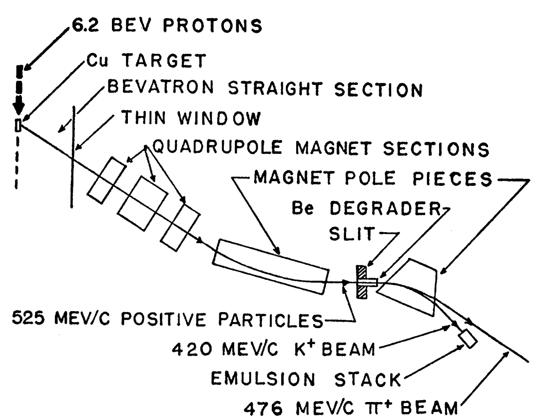

FIG. 1. Experimental arrangement for the separated 150-Mev K^+-meson beam. The target to stack distance was 24 ft.

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[1] J. E. Lannutti, W. W. Chupp, G. Goldhaber, S. Goldhaber, E. Helmy, E. L. Iloff, A. Pevsner, and D. M. Ritson, Phys. Rev. **101**, 1617 (1956); W. W. Chupp, S. Goldhaber, G. Goldhaber, E. L. Iloff, J. E. Lannutti, A. Pevsner, and D. M. Ritson, Suppl. Nuovo cimento 4, 361 (1956); N. N. Biswas, L. Ceccarelli-Fabbrichesi, M. Ceccarelli, K. Gottstein, N. C. Varshneya, and P. Waloschek, Nuovo cimento **5**, 123 (1957); G. Cocconi, G. Puppi, G. Quareni, and A. Stangellini, Nuovo cimento **5**, 172 (1957); M. Baldo-Ceolin, M. Cresti, N. Dallaporta, M. Grilli, L. Guerriero, M. Merlin, G. A. Saladin, and G. Zago, Nuovo cimento **5**, 402 (1957); B. Bhowmik, D. Evans, S. Nilsson, D. J. Prowse, F. Anderson, D. Keefe, A. Kernan, and J. Losty, Nuovo cimento **6**, 440 (1957); T. F. Hoang, M. F. Kaplon, and R. Cester, Phys. Rev. **107**, 1698 (1957); M. Widgoff, A. Pevsner, D. F. Davis, D. M. Ritson, R. Schluter, and V. P. Henri, Phys. Rev. **107**, 1430 (1957); J. E. Lannutti, S. Goldhaber, G. Goldhaber, W. W. Chupp, S. Giambuzzi, C. Marchi, G. Quareni, and A. Wataghin, Phys. Rev. **109**, 2121 (1958).

[2] G. Igo, D. G. Ravenhall, J. J. Tiemann, W. W. Chupp, G. Goldhaber, S. Goldhaber, J. E. Lannutti, and R. M. Thaler, Phys. Rev. **109**, 2133 (1958).

[3] M. Grilli, L. Guerriero, M. Merlin, and G. A. Saladin, Nuovo cimento **10**, 205 (1958).

[4] M. A. Melkanoff, O. R. Price, D. H. Stork, and H. K. Ticho, Phys. Rev. **113**, 1303 (1959).

[5] O. R. Price, D. H. Stork, and H. K. Ticho, Phys. Rev. Letters **1**, 212 (1958).

[6] D. H. Stork, University of California Radiation Laboratory Bevatron Notes 186, 1956 (unpublished).

plate over an area of approximately 3 inches by 2 inches. The minimum-track background was approximately three per K^+ track.

The scanning was carried out by examining a swath in each plate perpendicular to the beam flux and one centimeter from the entering edge. Each track satisfying specified directional criteria and with grain density between 1.5 and 2.2 times minimum ionization was followed until it interacted, scattered, decayed in flight, or had traveled four centimeters. At the end of each such follow a grain count of 625 grains was taken on the primary track. For each interaction or scattering with projected angle greater than 3° the space angle of each secondary was measured. With the exception of a sample of scatters of less than 20 degrees, each secondary was followed to the end of its range or until it interacted or left the stack. In the latter cases the secondary range was determined by ionization measurements. Under the initial assumption that one of the tracks from an interaction was due to a K meson, the end of each stopping track was carefully examined for evidence of decay and if no such evidence was found for a given event, each secondary was identified by ionization vs range measurements. For those events thus determined to have no K-meson secondaries, multiple scattering and ionization measurements were made on the primary tracks and if an incoming particle was indeed a K meson the event was classified as a charge exchange event.

The energy of the primary K meson of each event was determined within a standard deviation of 6% by taking a weighted average of the energy determined from the ionization measurement and that determined from the mean residual range of the beam particles at the event position.

The energy of the K meson after interaction or scattering was determined where possible from the range measurement. The Barkas range energy tables were used.[7] In those cases where the secondary K meson left

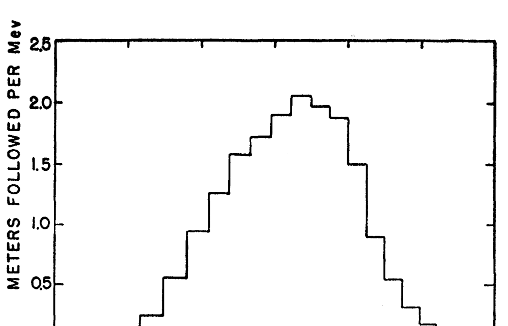

FIG. 2. K^+ meson-track path length followed as a function of energy.

[7] Walter H. Barkas, University of California Radiation Laboratory Report UCRL-3384, 1956 (unpublished).

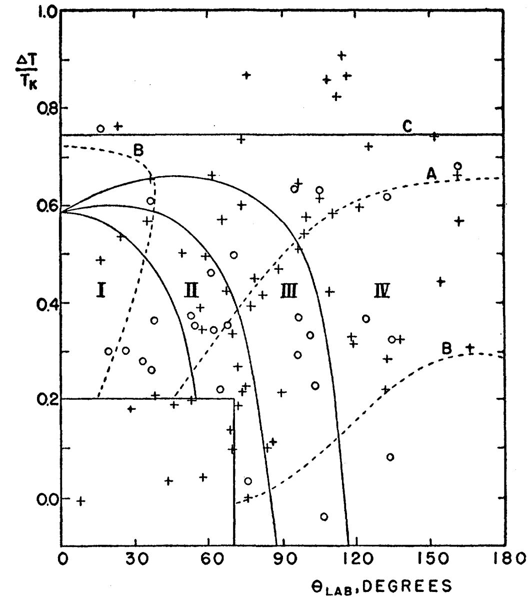

FIG. 3. Scatter diagram of inelastic scattering events. The circles designate events in which no other secondary accompanied the scattered K^+ mesons; the crosses designate events in which there were other secondaries.

the stack or interacted or decayed in flight, the residual range was determined by ionization measurements.

The total path length of K-meson track followed was 93.7 meters after correction for an estimated 0.5% contamination due to noninteracting proton and pion tracks followed without detection. A histogram of the tracklength followed as a function of primary K energy is shown in Fig. 2.

Thirty-one decays in flight were found corresponding to a mean life of $(1.32\pm0.23)\times10^{-8}$ second.

II. EXPERIMENTAL DATA

Eighteen charge exchange events were found, including four disappearances in flight. These disappearances could be decays in flight in which the decay track was not seen. However, they occurred well away from the top or bottom surfaces of the emulsion in regions relatively free from background, and an exhaustive search for decay secondaries was made by several observers. Our efficiency for finding secondaries from stopped K mesons without exhaustive search is 90%, and we conclude that these four disappearances are most probably charge-exchange events.

A total of 457 examples were found of K-meson scatterings with projected angle greater than 3 degrees and of interactions in which the K^+ meson reappeared. We cannot immediately classify each of those in which only the K^+ meson emerges as either elastic or inelastic because of the uncertainty in the measurement of energy lost by the scattered K meson. A scatter diagram of the ratio of K-meson energy lost, ΔT, to the primary energy T_K versus the laboratory angle of scatter $\theta_{\rm lab}$ is shown

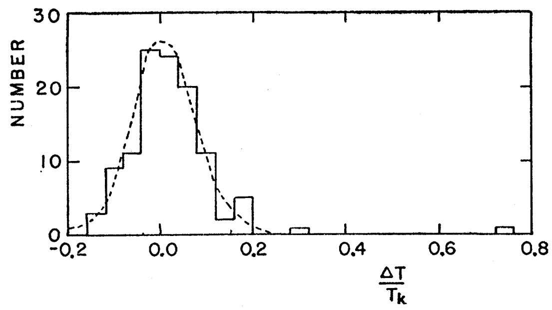

FIG. 4. Distribution of fractional energy loss for a sample of K^+ mesons scattered through less than 20° and having no other secondaries. The dashed curve is a gaussian with standard deviation 0.06.

in Fig. 3. The possible elastic scattering events with $\Delta T/T_K<0.2$ and $\theta_{\rm lab}<70°$ are not plotted. Events which were certainly inelastic because of the appearance of other secondaries are indicated by crosses.[8]

In Fig. 4 is shown a histogram of a sample of the possible elastic scattering events with $3°<\theta_{\rm lab}<20°$ as a function of $\Delta T/T_K$. The majority of these events are elastic and the distribution of the fluctuations of energy loss is well fitted by a Gaussian curve whose width is determined by the standard deviation of the measured energy loss as indicated above. The spread comes primarily from the uncertainty in the primary K energy. On the basis of the Gaussian distributions shown in Fig. 4, less than half an elastic event is expected with measured fractional energy loss greater than 20%. We assume each event with $\Delta T/T_K>0.2$ to be inelastic. Because of the uncertainty in energy loss measurement, however, some inelastic events with small energy loss and no additional secondaries will be masked by the elastic distribution. Although a more accurate estimate of the numbers of inelastic and elastic events is obtained by means of the detailed analysis described below, we have made a preliminary estimate of a total number of 132 inelastic events corresponding to a reaction cross section of 299±26 millibarns per emulsion nucleus (excluding hydrogen). The corresponding preliminary estimate for $d\sigma/d\Omega$ is given in reference 4 and will not be repeated here. These initial results were used to perform a preliminary optical model analysis as described in Part III.

The remaining experimental result is the distribution in energy and angle of charged secondaries other than K mesons from the K-meson interactions. It is instructive to plot in a scatter diagram the ratio of the kinetic energy of the fastest prong (assumed to be a proton) to the incident K energy T_p/T_K versus its space angle relative to the incident K direction. Such a diagram is shown in Fig. 5(a) for charge-exchange events and in Fig. 5(b) for noncharge-exchange events. We have made no quantitative analysis of these data, but remark that the distributions suggest that the charge exchange cross section is peaked forward relative to the K^+-proton cross section. This is derived from the assumption that the majority of the faster protons in the noncharge-exchange distribution come from a primary K^+-proton scattering. The qualitative indication of a forward-peaking-charge-exchange cross section supports the phase-shift results of Part IV below.

III. ANALYSIS OF THE RESULTS

We wish to apply an optical model analysis to the elastic scattering and reaction cross section and, furthermore, to analyze the distribution in energy and angle of inelastically scattered K mesons in terms of the average K-nucleon-scattering angular distribution. However, these two procedures are interdependent, since the optical model calculation requires a clear distinction between inelastic events and the elastic distribution, and this distinction must be based on a study of the inelastic distribution. Conversely, the analysis of the inelastically scattered K mesons requires a knowledge of the real part of the nuclear potential obtained from the optical model. The effects of these interrelations are however sufficiently small such that a method of successive approximations may be easily applied.

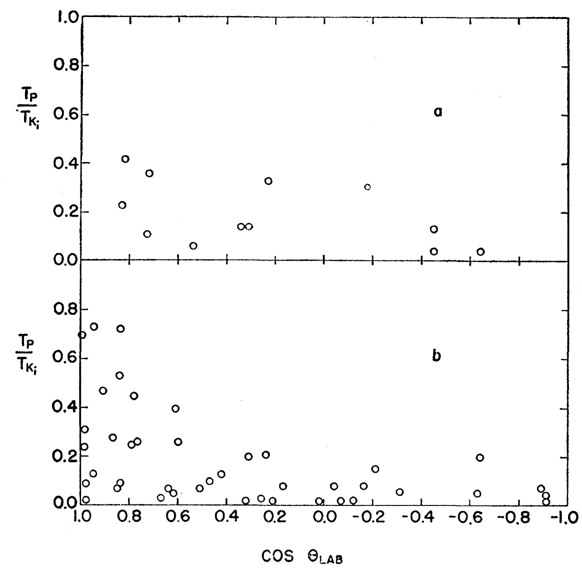

FIG. 5. Scatter diagram of relative energy and laboratory angle of *fastest* secondary other than a K^+ meson for (a) charge exchange events and (b) noncharge exchange inelastic events. T_p/T_{Ki} is the ratio of secondary energy (assuming the secondary to be a proton) to the incident K^+ energy.

1. Preliminary Optical Model Analysis

We start with a preliminary optical model analysis based on the initial estimates of the reaction cross section and of the elastic-scattering cross section given in the previous section. The diffuse-surface optical

[8] Those events having a recoil track which satisfies the kinematics of elastic scattering of the K meson with a light emulsion nucleus are not included in this category.

model with the Saxon potential was used. The details are fully described elsewhere[4] and we give in Table I the results for the central real potential, V, and the central imaginary potential, W, for values of the radius parameter $R_0=1.07$f and 1.20f and for a surface-thickness parameter $a=0.57$f. These values for the real potential were used to determine the kinetic energy of the K meson inside the nucleus for the analysis of the inelastic scattering distribution. In addition, a preliminary phase-shift analysis was carried out analogous to that of Part IV. For this preliminary analysis only the parameters V, W, the K-proton cross section, and the charge-exchange fraction were used. The results are $\delta_{00}=-20°$, $\delta_{10}=-10°$, and $\delta_{11}=\delta_{13}=15°$ (where the notation is that of Part IV). These preliminary phase-shift values were used only to estimate the variation of mean free path with kinetic energy for the K meson inside the nucleus. This was needed for the double scattering calculation given below. The usual linear and cubic momentum dependences of the phase shifts were assumed.

2. Inelastic Scattering Distributions

We proceed next to the analysis of the distribution in energy and angle of the inelastically scattered K mesons.

TABLE I. Results of the preliminary optical model analysis.

R_0	a	V	W
1.07f	0.57f	21 Mev	−11 Mev
1.20f	0.57f	15 Mev	−7 Mev

With the assumption that these inelastically scattered mesons came from the scattering by a single nucleon, attempts have been made to determine the angular distribution for the K-nucleon scattering in the K-nucleon center-of-mass system by (a) neglecting the nucleon motion in the nucleus,[9] (b) calculation of the most-probable center-of-mass angle for each event,[10] and (c) transformation of various center-of-mass angular distributions to the laboratory system by Monte Carlo techniques.[3] While the effects of the target-nucleon momentum distribution and of the exclusion principle on the nucleon final state have been included in (b) and (c) above, according to our study double scattering within the nucleus has been neglected or underestimated.

In order to calculate the expected distribution of inelastically scattered K^+ mesons for chosen K^+ meson-nucleon scattering cross sections we have used a square-well nuclear model with central potentials found in the preliminary optical model analysis. The nucleon momentum distribution was taken to be that of a Fermi degenerate gas with maximum energy $T_F=25$ Mev.

[9] J. E. Lannutti, S. Goldhaber, G. Goldhaber, W. W. Chupp, S. Giambuzzi, C. Marchi, G. Quareni, and A. Wataghin, Phys. Rev. **109**, 2121 (1958).

[10] B. Bhowmik, D. Evans, S. Nilsson, D. J. Prowse, F. Anderson, D. Keefe, A. Kernan, and J. Losty, Nuovo cimento **6**, 440 (1957).

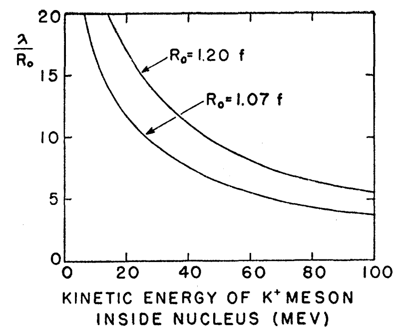

FIG. 6. Mean free paths for two optical model radii as a function of energy—used in the double-scattering Monte Carlo calculations.

A Monte Carlo calculation was carried out in which individual K^+ mesons of initial energy 125 Mev were followed through the nucleus. Refraction at the boundary was neglected as justified by Grilli *et al.*,[3] but the K^+-meson energy was reduced by the repulsive optical model potential plus 11 Mev to take account of the Coulomb barrier. The first scattering was described by an averaged bound-nucleon cross section

$$d\sigma/d\Omega=A+B\cos\theta+C\cos^2\theta,$$

where B/A and C/A are parameters to be fit. Second scatterings, which occurred in 25 to 30% of the cases, were taken to be isotropic. In any case scatterings for which the final nucleon energy was less than 25 Mev were excluded.

To determine the probability of a K-meson interaction in the nucleus an energy-dependent mean free path was used. It was evaluated from the expression $\lambda=(\rho_0\bar{\eta}\bar{\sigma})^{-1}$ where ρ_0 is the central nuclear density and $\bar{\eta}\bar{\sigma}$ is the mean attenuation cross section per nucleon. The cross section $\bar{\sigma}$ was taken to be the free-nucleon cross section determined from the preliminary phase shifts; $\bar{\eta}$ accounts for flux and exclusion principle effects and was taken to be the Sternheimer factor[11]; $\bar{\eta}=1-0.78(T_F/T_K)$ for K^+ energies above 40 Mev. At lower energies the expression fails and we have extrapolated $\bar{\eta}$ smoothly to zero (at the lower energies there is negligible scattering so that the extrapolation is not critical). The resulting mean free path used in the calculation is shown as a

TABLE II. Example of χ^2 fit to the inelastic distribution.

Region	Number of observed events	Predicted number: Single scatter	Double scatter	Total	χ^2
I	10.6±3.8	10.0	1.8	11.8	0.10
II	21.2±4.6	13.6	5.3	18.9	0.12
III	18.8±4.3	15.8	4.0	19.8	0.06
IV	31.0±5.6	17.1	14.0	31.1	0.00
Total	81.6	56.5	25.1	81.6	0.28

[11] R. M. Sternheimer, Phys. Rev. **106**, 1027 (1957).

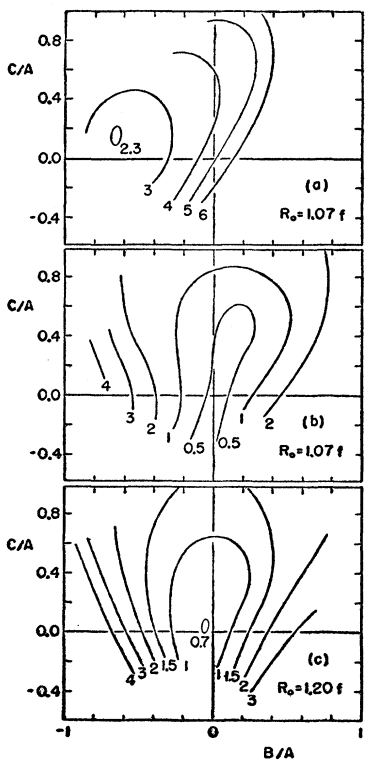

FIG. 7. Contours of constant χ^2 on a C/A versus B/A diagram. In Fig. 7(a) the effect of double scattering has been neglected. In Figs. 7(b) and 7(c) double scattering effects have been taken into account for radius parameters 1.07f and 1.20f. The χ^2 value for each contour is labeled.

function of T_K in Fig. 6 for the two radii tried in these calculations.

In order to facilitate comparison of the calculated inelastic distribution for a given B/A and C/A with the experimental observations, the inelastic scatter diagram of Fig. 3 was divided into the four regions labelled I, II, III, and IV, and a χ^2 test was performed on the relative numbers of events in these intervals.[12] Several events of small final energy ($\Delta T/T$ near 1) give evidence of scattering in the diffuse nuclear surface. These have been redistributed to give the total numbers of events shown in Table II for comparison with the square well calculations. An example of χ^2 comparison is also shown in Table II and illustrates the effect of double scattering at these energies. These results are for $B/A=C/A=0$ and for radius parameter $R_0=1.07$f.

Contours of constant χ^2 in the B/A, C/A space are shown in Fig. 7. In Fig. 7(a) double scattering was neglected. Good fits are found near isotropy although a fairly broad range of parameters is acceptable for the one degree of freedom. Although the internal momentum distribution contributes to this broadening, the double scattering also has an important smearing effect. The appreciable backward peaking found by Grilli *et al.*[3] gives a very poor fit when double scattering is taken into account.

[12] It should be noted that the region $\Delta T/T<0.2$ and $\theta<70°$ is excluded in order to preclude uncertainty due to pseudoelastic events. The boundaries of the four regions were chosen in order to achieve a maximum sensitivity of the test to the angular distribution: the boundaries *approximately* follow loci of constant most-probable center-of-mass scattering angle.

3. Adjusted Value of the Optical Model Potential

The above analysis also permits one to calculate the number inelastic scatterings with $\Delta T/T<0.2$ and with $\theta_{\text{lab}}<70°$ for the best fit parameters B/A and C/A. This calculation showed that the number of inelastic events in this region had been overestimated in the preliminary analysis. Thus a final number of 118 inelastic events including the 18 charge exchanges was obtained corresponding to a reaction cross section of 275±25 millibarns. Minor corrections to the preliminary elastic scattering distribution were also made on the basis of these results yielding the final values shown in Table III.

TABLE III. Corrected elastic scattering events and cross sections.[a]

θ_{lab} interval	(1)	(2)	(3)	(4)	(5)	$d\sigma/d\Omega$, mb/sr
3°– 4°	18	62	0	0	62	24 400± 5800
4 – 6	75	134	0	1	133	15 800± 1810
6 – 8	70	98	1	1	98	8300± 1000
8 –10	50	65	1	1	65	4200± 590
10 –12	27	33	0	0	33	1770± 340
12 –14	22	26	0	1	25	1120± 220
14 –16	14	16	0	0	16	640± 170
16 –18	8	9	0	1	8	280± 100
18 –20	10	11	0	0	11	350± 110
20 –30	20	22	1	2	21	100± 23
30 –40	17	18	0	2	16	56± 14
40 –50	8	8	2	1.5	8	25± 9
50 –60	6	6	2	1.5	6	16± 7
60 –70	1	1	1.5	1.5	1	2± 2

[a] (1) Number of possibly elastic events with $\Delta T/T<0.2$; (2) Number of possibly elastic events corrected for projected angle cut off of 3°; (3) Number of certainly inelastic with $\Delta T/T<0.2$; (4) Number of inelastic events predicted for $\Delta T/T<0.2$; (5) Number of elastic events: (2)+(3)−(4).

The optical model analysis was repeated as described in reference 4. New values of V and W were obtained as shown in Table IV. The uncertainties listed give deviations such that the χ^2 probability, $P(\chi^2)$ is reduced by a factor of three from that of the minimum χ^2 value. These new values of V and W differ little from the old ones (Table I); as a result it did not appear necessary to repeat the Monte Carlo calculations of the inelastic distribution. These results are in fair agreement with those of Igo *et al.*[2] For a radius parameter $R_0=1.07$f they obtained $V=27\pm3$ Mev and $W=10.3\pm1.6$ Mev. Although the discrepency in V could be statistical, it should be noted that Igo fitted the total elastic scattering cross section for angles greater than 10°, while we fitted the elastic scattering distribution for angles greater than 6° by means of a χ^2 test.[4]

4. Final Charge Exchange Fraction

From the revised number of 118 inelastic scatters, the ratio $f=0.152\pm0.033$ was obtained for the observed

fraction of charge exchange scatters among all inelastic events. Because of the poor statistics of this result, Lannutti's result[9] of 23 charge exchange events (including four disappearances in flight) in 96.6 meters of K^+-meson track followed at energies from 100 to 140 Mev was combined with our data. The resulting fraction $f=0.168\pm0.026$ was used in the following phase-shift analysis.

IV. K^+-NUCLEON PHASE SHIFTS

Since the energy of the K^+ meson is below pion production threshold there are but three K^+-nucleon scattering modes:

$$(1)\quad K^+ + p \to K^+ + p,$$

$$(2)\quad K^+ + n \to K^+ + n,$$

$$(3)\quad K^+ + n \to K^0 + p.$$

There is now considerable data on the first reaction from nuclear emulsion,[13] bubble chamber,[13] and counter experiments[14] with free protons. On the other hand, the scattering from neutrons must be obtained with bound neutrons since the more conveniently studied reactions that are equivalent through change independence (as found, for example, with pion scattering) are not available. It is, therefore, instructive to inquire what information may be obtained from the nuclear emulsion results derived in the previous section.

TABLE IV. Optical model potential based on the corrected cross sections.

R_0	a	V	W	$P(\chi^2)$
1.07f	0.57f	23±4 Mev	−9.7±1.3 Mev	0.10
1.20f	0.57f	14±3 Mev	−6.4±0.9 Mev	0.09

The complex central potential $V+iW$ may be related to the scattering phase shifts by means of multiple scattering theory. While more powerful and detailed techniques are available[15] for such an analysis, the simplest form of multiple scattering theory[16] will be used here. Correlation effects are estimated to have an effect on $V+iW$ of less than 10% and will be neglected. Exclusion principle suppression of W will, however, be included following the method of Sternheimer.[11] The kinematical effects resulting from the energy dependence of the potential[17] are negligible because of the very slow variation of the K^+ potential with energy.[2,4,18] Analogous studies of pions have had some qualitative success[17] in spite of the additional problems due to the presence of a pure absorption mechanism, a relatively short mean free path in nuclear matter, a strongly energy dependent potential and a lack of detailed diffuse-surface optical model studies. Similarly, analogous analyses for nucleons show striking semiquantitative success in spite of a much enhanced exclusion principle effect and more serious correlation problems.

The inelastic distribution and charge exchange results of this experiment can be related to the free nucleon scattering through the direct interaction model which has already been implied in the analysis of the previous section. While strict conditions for the impulse approximation[19] are not rigorously satisfied, it is worth noting that considerable success has been obtained with such a model in the case of nucleons[15] under conditions where the impulse approximation seems less likely to be of value.

1. The $T=1$ Phase Shifts

The scattering on protons occurs in a pure $T=1$ state. The evidence[13,14] for an isotropic scattering that is energy independent supports the assumption that the $T=1$ scattering is pure S wave. A survey of the experimental data available indicates a total cross section of 14.4 ± 1.7 mb which gives for the phase shift

$$\delta_{10}=\pm20.2\pm1.2°$$

at the average energy studied here (93 Mev in the laboratory system inside the nucleus). This value will be used throughout the following analysis of the $T=0$ phase shifts.

2. Phase-Shift Functions

With the assumed $T=1$ phase shift above, there remains the determination of the $T=0$ phase shifts. Since $p/\mu c$ is 0.4 in the center-of-mass K^+-nucleon system, only S and P waves will be considered. The scattering is then described by the four phase shifts δ_{10}, δ_{00}, and δ_{01}, and δ_{03} where the first subscript denotes the isotopic spin and the second is 0 for S states and $2J$ for P states.

More convenient algebraic quantities are

$$ka_T=e^{i\delta_{T0}}\sin\delta_{T0};\quad T=0,1,$$

$$kb_0=2e^{i\delta_{03}}\sin\delta_{03}+e^{i\delta_{01}}\sin\delta_{01},$$

$$kc_0=e^{i\delta_{03}}\sin\delta_{03}-e^{i\delta_{01}}\sin\delta_{01},$$

where k is the center-of-mass wave number of the K^+-nucleon system (1.00f^{-1} at the mean energy in this experiment).

From multiple scattering theory[16]

$$V+iW=-2\pi\rho_0\frac{\hbar^2c^2}{E_{K\text{ lab}}}\frac{p_{\text{lab}}}{p_{\text{c.m.}}}\langle f_{\text{c.m.}}(0°)\rangle,$$

[13] Proceedings of the International Conference on Elementary Particles, Padova, Italy, 1957 (unpublished).

[14] L. T. Kerth, T. F. Kycia, and L. van Rossum, Phys. Rev. **109**, 1784 (1958).

[15] K. M. Watson, Revs. Modern Phys. **30**, 565 (1958); and references cited there.

[16] R. M. Frank, J. L. Gammel, and K. M. Watson, Phys. Rev. **101**, 891 (1956).

[17] K. M. Watson and C. Zemach, Nuovo cimento **10**, 452 (1958).

[18] D. H. Stork, *Proceedings of the International Conference on the Nuclear Optical Model, Florida State University Studies*, edited by A. E. S. Green, C. E. Porter, and D. S. Saxon (The Florida State University, Tallahassee, Florida, 1959), p. 216.

[19] G. F. Chew and M. L. Goldberger, Phys. Rev. **87**, 778 (1952).

where p_{lab} and $E_{K\text{ lab}}$ are the momentum and total energy of the K meson in the nucleus, and $\langle f_{\text{c.m.}}(0°)\rangle$ is the forward scattering amplitude in the K meson-nucleon system averaged over reaction (1) and (2).

$$\text{Re}\langle f_{\text{c.m.}}(0°)\rangle = 0.45[\text{Re}(a_1)] + 0.55[\tfrac{1}{2}\,\text{Re}(a_1+a_0+b_0)],$$

where the numerical factors 0.45 and 0.55 are the proportion of protons and neutrons averaged over nuclear emulsion species. In the expression for W it is convenient to use the cross-section theorem

$$\text{Im}\langle f_{\text{c.m.}}(0°)\rangle = (k/4\pi)\bar{\sigma}_b,$$

where $\bar{\sigma}_b$ is the average K meson-bound-nucleon cross section

$$\bar{\sigma}_b = 0.45\eta_1\sigma_1 + 0.55(\eta_2\sigma_2 + \eta_3\sigma_3).$$

Here σ_1, σ_2, and σ_3 are the total cross sections for reactions 1, 2, and 3, respectively;

$$\sigma_1 = 4\pi|a_1|^2,$$
$$\sigma_2 = \pi\{|a_1+a_0|^2 + \tfrac{1}{3}|b_0|^2 + \tfrac{2}{3}|c_0|^2\},$$
$$\sigma_3 = \pi\{|a_1-a_0|^2 + \tfrac{1}{3}|b_0|^2 + \tfrac{2}{3}|c_0|^2\};$$

and the η_i's are the Sternheimer factors[11] appropriate to the σ_i angular distributions.

The parameters B/A and C/A of the inelastic distribution are also directly related to the phase shifts:

$$A = 0.45|a_1|^2 + 0.55\tfrac{1}{4}\{|a_1+a_0|^2 + |c_0|^2\},$$
$$B = 0.55\{\tfrac{1}{2}\,\text{Re}[b_0^*(a_1+a_0)]\},$$
$$C = 0.55\tfrac{1}{4}\{|b_0|^2 - |c_0|^2\}.$$

The fraction F of charge exchange for *single* scattering is

$$F = 0.55\eta_3\sigma_3/\bar{\sigma}_b.$$

The measured fraction, f, depends however on *double scattering* effects as well. We write

$$f = \xi F,$$

where ξ is a correction factor for double scattering.

This correction factor was obtained from the relation

$$f = F(1-\alpha^0) + \alpha^+(1-F),$$

where α^+ is the fraction of K^+ mesons which scatter a second time by charge exchange and α^0 is the fraction of double charge exchanges. The dependence of the fractions α^0 and α^+ on the phase shifts was obtained by Monte Carlo techniques analogous to those described in Sec. III-2. In the phase-shift region of the χ^2 tests discussed below, the correction factor, ξ, varied from 1.00 to 1.27.

3. χ^2 Fitting of the Phase Shifts

For chosen values of the three $T=0$ phase shifts δ_{00}, δ_{01}, δ_{03}, the five quantities f, V, W, B/A, and C/A were computed as described above. These were compared by means of a χ^2 test to the values derived in Sec. III from the experimental data. This procedure led to the χ^2 contours shown in Fig. 8. The contours are orthogonal projections of the surface of constant χ^2 on the appropriate planes; the χ^2 value is labeled for each contour.

The ratio $(V+iW)/\rho_0$ which is used in the determination of the phase shifts in this approach is much less sensitive to the assumed radius parameter R_0 than the potentials themselves (see Table IV). Also as shown in Fig. 7, the ratios B/A and C/A show little R_0 dependence. As a result the phase shift fit does not depend sensitively on R_0.

There are two equally good best-fit solutions. These are shown in Table V; solution A is characterized by a predominant δ_{01} and solution B by a predominant δ_{03}. P-wave phase shifts with $\delta_{01}=\delta_{03}=0°$ lead to an exceeding poor fit. It is of interest to note that the presence of P waves is required by the fitting of f and W alone.

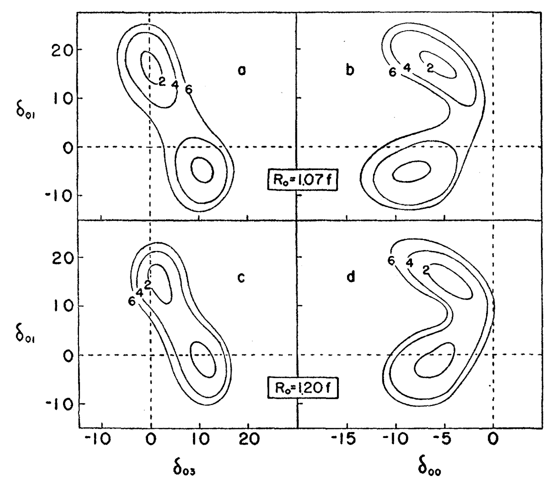

FIG. 8. Contours of constant χ^2 on phase-shift diagrams.

Inclusion of the real potential V in the fitting localizes the good fit regions to those called solutions A and B above. The contribution to χ^2 from the fitting of parameters B/A and C/A is not sensitive to the phase shifts in the good fit regions. A fair way to put this is that the solutions above are determined by f, V, and W and that the inelastic scattering distribution described by parameters B/A and C/A is consistent with these solutions. The total K^++n cross section is about 12.5 mb with about $\frac{1}{3}$ charge exchange for both solutions. Forward-peaking charge exchange is predicted with a front-to-back ratio of about three. The noncharge-exchange K^++n scattering is backward peaking for these phase-shift solutions with a front to back ratio of about $\frac{1}{3}$. These results are also insensitive to R_0.

V. DISCUSSION

We have attempted in this analysis to consider with some care various details that have not previously been considered or have been neglected. Igo *et al.*[2] have

already pointed out the importance of an exact diffuse-surface optical-model calculation. Much of the weight of our results rests upon such a calculation. Grilli *et al.*[3] have demonstrated the need for a careful treatment of pseudoelastic events. Although they also used a Monte Carlo calculation for their analysis of the inelastic distribution, they did not extend it to include double scatterings which we find here to be important. To be explicit, Fig. 7(a), which neglects double scattering, is in essential agreement with the conclusions of Grilli *et al.*, but Figs. 7(a) and 7(b) which include double scattering are not. Finally, the arbitrary, or in some cases unstated, choice of the radius parameter R_0 has seemed to us to raise some question since some methods of analysis give results that are strongly radius dependent. Melkanoff *et al.*,[4] find good optical model fits for a broad range of R_0 and we therefore have no *a priori* radius value. Our phase-shift results, however, are seen from Fig. 8 to be insensitive to radius.

In the separation of elastic and inelastic events we have made no attempt to take into account the excitation of specific low-lying nuclear levels. If the typical effect in the nuclear species of nuclear emulsion is as strong as that found for the 4-Mev level of C^{12} and if the relative probability of excitation is determined essentially by the momentum transfer as found in the case of electrons,[20] pions,[21] and protons,[22] then the total reaction cross sections would be increased by less than 10%. This would require some change of W, but little change in V since the major modification in the elastic scattering would be at large angles which receive small weight in the optical model analysis. In fact, a reduction of the large angle elastic cross section would lead to an improved optical model fit.

TABLE V. Phase-shift solutions.

	R_0	δ_{00}	δ_{01}	δ_{03}
Solution A	1.07f	$-6°$	16°	0°
	1.20f	$-4°$	15°	1°
Solution B	1.07f	$-8°$	$-5°$	11°
	1.20f	$-6°$	$-1°$	11°

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Since the inelastic scattering distribution did not contribute important restrictions on the phase shift solutions, a discussion of the details of the model used becomes unnecessary. The Sternheimer correction factor appears explicitly, however, in the expressions for f and for W. In each case the effect of this factor is to reduce the calculated values by a factor of about 0.8 in the good-fit phase shift regions. This effect is small enough then so that a change of as much as 30% in the Fermi energy value would have little effect on the final results. It should also be noted that the approximations used by Sternheimer[13] in his application of the Goldberger model[23] have been shown to lead to negligible error at the energies considered here.[24]

On the other hand the question of the basic validity of the multiple scattering theory and, for example, the Goldberger model is more difficult. Although correlation effects are estimated to be small, some of the severe and formal conditions required in the impulse approximations are not well met. We remark, however, that qualitative and, where pursued, even quantitative agreement with experiments is obtained[17,25] under much more trying conditions for pions and nucleons where the elementary interactions are known directly from experiment.

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