

Damping of a Torsionally Oscillating Cylinder in Liquid Helium at Various Temperatures and Densities

BENJAMIN WELBER

Lewis Research Center, National Aeronautics and Space Administration, Cleveland, Ohio

(Received September 25, 1959; revised manuscript received April 21, 1960)

A method is described for measuring the product of the viscosity and the density of liquid helium by determining the energy dissipation of a piezoelectric cylinder of quartz oscillating in a torsional mode. Data are reported for liquid helium under its own vapor pressure as well as at higher densities.

INTRODUCTION

INTEREST in the hydrodynamic properties of liquid helium dates back more than twenty years, when it was found that liquid helium possessed an abnormally low viscosity. It was also discovered at that time that the viscosity as measured with an oscillating disk^{1,2} exhibited an anomalous temperature dependence below the λ point. On the other hand, investigations with the capillary flow method^{3,4} yielded vanishingly small values for the viscosity (below the λ point); indeed it was the apparent contradiction between these and the oscillating disk values that first suggested the inadequacy of a description of the fluid in terms of classical hydrodynamics, thus leading directly to the two-fluid model.

Following the earlier measurements, numerous determinations of the viscosity of liquid helium were made by other workers.⁵⁻¹⁰ These have largely been variations of the original method in which the decay of torsional oscillations of a plane circular disk is measured at each temperature. Such a technique possesses the virtue of simplicity; unfortunately, it also has its special difficulties. One of these, rather obviously, is the necessity of maintaining a constant temperature for long periods of time, a restriction which proves particularly troublesome at temperatures below the λ point. A second is the fact that the "nuisance decrement" is not always small compared to the decrements of the system when immersed in liquid helium and must be determined with considerable accuracy. Finally, there are the natural limitations imposed by the disk geometry to experimentation below 1°K. It is the purpose of this paper to report measurements obtained with a different method which has none of these disadvantages, and may readily

be employed down to the lowest attainable temperatures. We include also a more complete description of the experimental technique than appeared when we reported our preliminary investigations.^{11,12}

Consider a suitably prepared right circular cylinder of quartz which is excited piezoelectrically in its fundamental mode of torsional vibration at a constant amplitude. If the crystal is immersed in liquid helium, the observed dissipation of electrical energy required to maintain oscillations of a constant amplitude will originate from (a) the internal friction of the quartz, (b) the mounting losses, and (c) losses due to the interaction between the liquid and the crystal surface through purely viscous forces. We have been able to prepare and mount a crystal of this kind so that the combined losses (a) and (b), which together constitute the "nuisance decrement," were small compared to (c) down to the lowest temperatures investigated.¹³

To relate the electrical characteristics of such a crystal oscillator to the viscous losses, it is useful to recall that the behavior of the system in the neighborhood of the resonant frequency f_0 may be described by an equivalent electrical circuit consisting of a series resonant branch L, C, R in parallel with a capacity C' . The resistance R is proportional to the logarithmic decrement Δ of the system, defined by

$$\Delta = W^d/2W^v, \quad (1)$$

where W^d = energy dissipated per cycle and W^v = vibrational energy of the system. The quantity Δ is related to R by the relation

$$R = KMf_0\Delta, \quad (2)$$

¹¹ B. Welber and S. L. Quimby, *Phys. Rev.* **107**, 645 (1957).

¹² B. Welber and S. L. Quimby, *Proceedings of the Fifth International Conference on Low-Temperature Physics and Chemistry, Madison, Wisconsin, August 30, 1957*, edited by J. R. Dillinger (University of Wisconsin Press, Madison, 1958), p. 6.

¹³ The actual value of the "nuisance decrement" was found to be independent of temperature in the helium range although its value was not exactly reproducible if the crystal were dismounted and handled between "runs." While its magnitude was never more than 1% of the measured decrement in liquid helium at 4.2°K, this value was in all cases carefully determined and subtracted from the measured data. It should be emphasized also that the "nuisance decrements" in this experiment are very much smaller than in the oscillating disk measurements of reference 9 where they were as high as 10%. It is only fair to point out, however, that this was with a small apparatus designed especially to study liquid He³. When a larger apparatus was employed, as in reference 8, the magnitude of the "nuisance decrement" was only about 4%.

¹ J. O. Wilhelm, A. D. Misener, and A. R. Clark, *Proc. Roy. Soc. (London)* **A151**, 342 (1935).

² W. H. Keesom and G. E. MacWood, *Physica* **5**, 737 (1938).

³ J. F. Allen and A. D. Misener, *Nature* **141**, 75 (1938).

⁴ P. Kapitza, *Nature* **141**, 74 (1938).

⁵ A. de Troyer, A. van Itterbeek, and G. J. van den Berg, *Physica* **17**, 50 (1951).

⁶ A. C. Hollis-Hallett, *Proc. Roy. Soc. (London)* **A210**, 404 (1952).

⁷ E. L. Andronikashvili, *J. Exptl. Theoret. Phys. (U.S.S.R.)* **18**, 429 (1948).

⁸ J. G. Dash and R. Dean Taylor, *Phys. Rev.* **105**, 7 (1957).

⁹ R. Dean Taylor and J. G. Dash, *Phys. Rev.* **106**, 398 (1957).

¹⁰ K. M. Eisele and A. C. Hollis-Hallett, *Can. J. Phys.* **36**, 25 (1958).

where M is the mass of the crystal and K is a constant dependent on the electrode geometry and is obtained experimentally.

If we designate by η the viscosity and by ρ the density of the fluid it can be easily shown that, as in the case of the oscillating disk experiments, the quantity $\eta\rho$ is proportional to $(\Delta - \Delta_0)^2$. (Δ_0 denotes the value of Δ in vacuum.) If we assume a constant pressure and periodic conditions the Stokes-Navier equation of motion for a fluid becomes¹⁴

$$\eta \nabla^2 \mathbf{V} + i\omega \rho \mathbf{V} = 0, \quad (3)$$

where \mathbf{V} is the vector velocity, $\omega = 2\pi f$, and the coefficients define a characteristic length $\lambda = (2\eta/\omega\rho)^{1/2}$, which is the thickness of the "boundary layer" of fluid dragged along by the moving surface. Under the conditions of measurement ($f \approx 32.4$ kc/sec) λ is only $10^{-4} - 10^{-5}$ cm. Accordingly, to calculate the work done by the crystal on the fluid it is sufficient to utilize the plane wave solution of Eq. (3) in order to obtain the velocity gradient at the surface of the cylinder.¹⁵ Elementary calculation, taking into account the sinusoidal variation of the velocity vector, then yields

$$W_{\text{liquid}} = -\frac{\eta}{f} \int_S \mathbf{V} \cdot \frac{\partial \mathbf{V}}{\partial n} dS = \frac{S\eta}{4f\lambda} V_m^2, \quad (4)$$

where S = surface area (including the ends) of the crystal and V_m is the *maximum* velocity of a point on the rim of the cylinder. Furthermore, by a simple integration,

$$W^v = \frac{1}{8} M V_m^2. \quad (5)$$

Using Eqs. (4) and (5) in Eq. (1) we obtain as a close approximation

$$\eta\rho_n = \left(\frac{M(\Delta - \Delta_0)}{S} \right)^2 \frac{f}{\pi}, \quad (6)$$

where ρ_n designates the "normal" component of the fluid density when we are dealing with a superfluid. We record here also for future reference the expression for V_m as a function of the driving voltage ϵ impressed on the crystal,

$$V_m = (2\epsilon/R) K^{1/2} 10^{7/2}, \quad (7)$$

where ϵ is in rms volts, R in ohms, K is in ohm-sec/cm and V_m in cm/sec. Equation (7) is obtained very readily with the aid of (1), (2), and (5) by equating the electrical energy supplied to the crystal during a cycle of vibration with the energy dissipated during the same time interval.

¹⁴ The complete Stokes-Navier equation is

$$\rho \partial \mathbf{V} / \partial t = -\text{grad} P + \eta \nabla^2 \mathbf{V}.$$

Assuming periodic conditions it can be shown that at a sufficiently small velocity and with a high frequency the term involving the pressure is negligible.

¹⁵ See, for example, J. W. S. Rayleigh, *Theory of Sound* (Dover Publications, New York, 1945), Vol. II, p. 318.

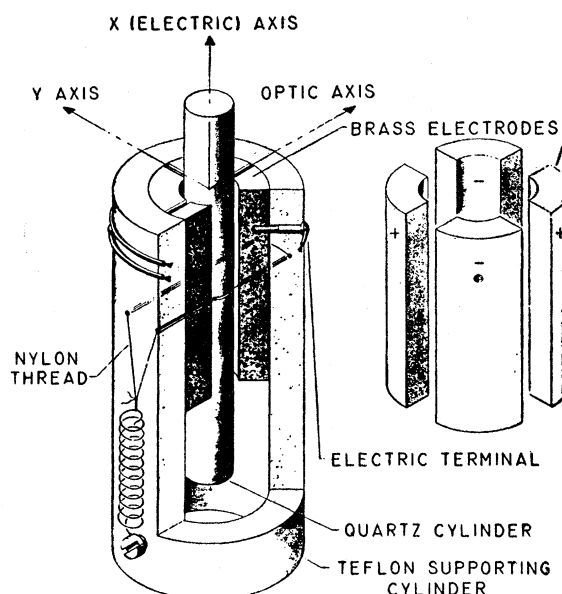


FIG. 1. Sketch of quartz crystal oscillator viewed cut away to show electrode system and method of mounting. The crystal is supported by the nylon threads at the nodal plane.

EXPERIMENTAL PROCEDURE

The quartz crystal with which the measurements reported here were made was a 30° cut cylinder 6.00 cm long in the direction of the x (electric) axis by 0.60 cm in diameter. Its fundamental frequency of torsional vibration was 32 429 cps at 4.2°K. A pair of taut nylon threads (Fig. 1) supported the crystal near its midpoint in a vertical position in the liquid. Two special grooves for this purpose had been sawed into the opposite sides of the cylindrical surface in a direction parallel to the optic or z axis. To excite the lowest torsional mode, four electrodes were employed. These were fabricated from a single brass cylinder, somewhat shorter than the crystal, which was cut lengthwise into four equal segments. The electrodes were positioned around the crystal in a special Teflon holder in such a way that each segment was entirely within a quadrant of the y, z axes of the quartz. Then the electrodes were connected so that adjacent pairs possessed opposing polarities. If a sinusoidally varying potential is applied to this system, opposing shears are produced on opposite sides of the crystal, due to the oppositely directed electric fields, yielding a periodically varying torsion of the cylinders. With such an arrangement, when the crystal oscillates at its resonant frequency, there is a sinusoidal distribution of velocity along the length of the cylinders and the displacement node occurs at the center where the shear is a maximum. This has the effect of minimizing any contribution to the "nuisance decrement" Δ_0 from the supports. The "nuisance decrement" turned out to be quite modest ($\Delta_0 \approx 10^{-7}$) though by no means negligible. Indeed, although at 4.2°K it amounted to only about one percent of the value of Δ in the fluid, the fraction

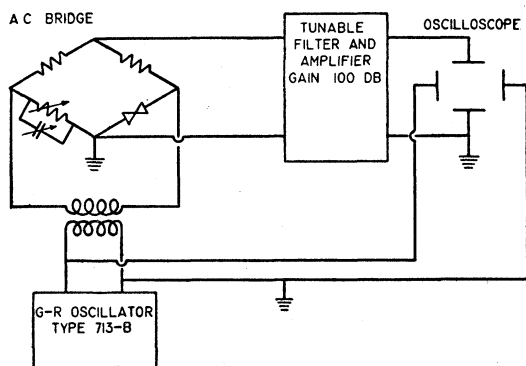


FIG. 2. Schematic diagram of the measuring circuit for determining the resistance of the crystal oscillator at resonance. The frequency meter has been omitted.

risers to as much as five percent at 1.3°K, where $\eta\rho_n$ is so much smaller. Further details of the crystal mount and geometry are contained in Fig. 1. It need be remarked that the electrodes were separated from the crystal surface by about one mm. This was still very much larger than λ and was not expected to affect the validity of Eq. (6). Considering the small value of λ , special precautions were taken to assure an optically smooth finish on the surface of the quartz cylinder.

The impedance of the torsional oscillator was measured with a conventional ac bridge (Fig. 2), one arm of which contained the quartz crystal in parallel with a condenser sufficiently large to ensure that the reactance of the arm remained capacitive over the resonance range of frequencies. The adjacent arm consisted of a variable capacitor and a 0–100 kilohm resistor variable in steps of 0.1 ohm, while the ratio arms were pure resistances. The bridge was fed from a General Radio type 713-B beat frequency oscillator suitably modified with a vernier system to obtain fine frequency control. To observe the bridge balance, the output was passed through suitably tuned amplifiers and impressed on the vertical deflection plates of an oscilloscope while the oscillator signal was simultaneously applied to the horizontal deflection plates. Now, if one represents the impedance of the series resonant branch of the crystal by a (frequency-dependent) resistance \bar{R} in parallel with a reactance \bar{X} then clearly

$$\begin{aligned}\bar{R} &= (R^2 + X^2)/R, \\ \bar{X} &= (R^2 + X^2)/X,\end{aligned}\quad (8)$$

where $X = I\omega - 1/C\omega$. It is obvious that the quantity we seek to measure, R , is the minimum value of \bar{R} at the resonant frequency. In practice R was not found laboriously from the "resonance curve" but rather directly from a single bridge balance. To see this it is only necessary to note that far from the resonant frequency the impedance of the crystal is a pure capacity C' (which includes the effect of the electrodes, leads, and the externally connected capacity) and that at

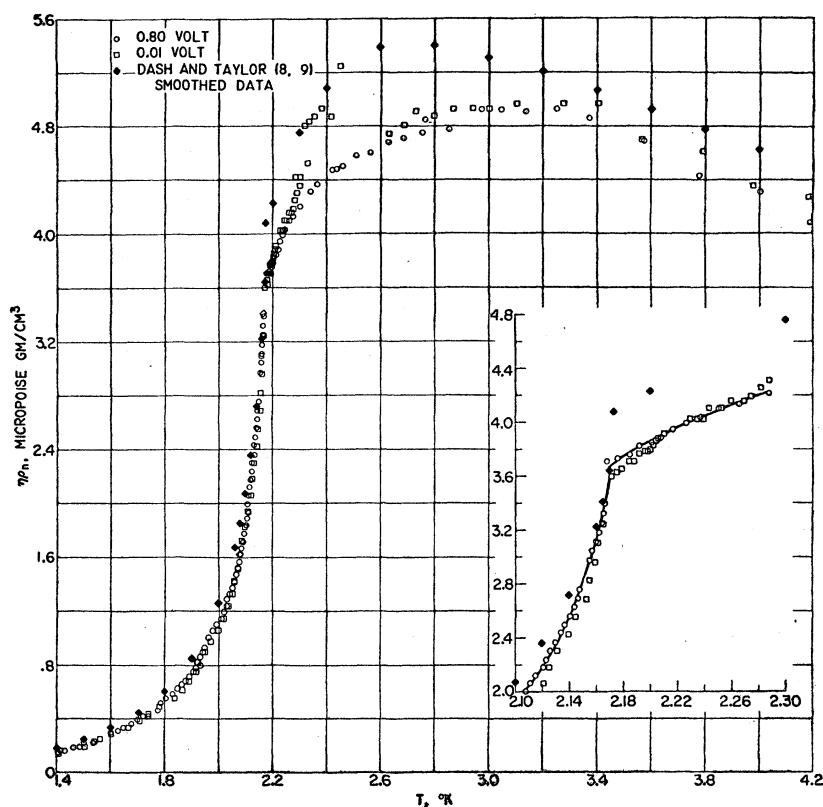
resonance the impedance is represented again by this same capacity in parallel with R . Hence, after having balanced the bridge with the aid of the variable capacitor at a frequency remote from f_0 , it was required merely to leave this setting unaltered, and obtain R (and f_0) from a single bridge balance in the resonance region. This allowed very rapid measurements to be made. To relate R to Δ via Eq. (2) it was of course necessary finally to have recourse to the "resonance curve," but one set of such measurements sufficed since K in Eq. (2) is sensibly independent of temperature. However, owing to the sharpness of the resonance, the frequency determinations required in finding K had to be made with some care, viz., with a precision of ± 0.01 cps.

INVESTIGATION OF LIQUID HELIUM UNDER ITS OWN VAPOR PRESSURE

A conventional Dewar arrangement was employed for these measurements, with the crystal located near the bottom of the helium Dewar. To ensure a frost-free surface on the crystal, the following procedure was adopted in preparing for the measurements. Starting at room temperature the inner Dewar was thoroughly pumped and then filled to a positive pressure with clean helium gas. The system was precooled by pouring liquid nitrogen into the outer Dewar, all the time maintaining a positive pressure in the inner Dewar. Finally, liquid helium was introduced with the aid of the transfer tube which entered the inner Dewar through a tight-fitting rubber stopper, while the entire system was vented through a single small hole.

Our temperatures are based on the L55 scale. The vapor pressure of the bath was measured with a mercury manometer corrected to 0°C when that pressure was above 50 mm. For lower pressures it proved convenient to make our readings on one or the other of a pair of Wallace and Tiernan absolute manometers of ranges 0–20 mm Hg and 0–50 mm Hg, respectively. These instruments were recalibrated on several occasions with a U-tube manometer filled with Octoil-S, and proved to be very reliable. Our procedure for gathering the data varied somewhat for different temperature ranges. Well above the λ point the temperature of the bath was maintained constant by pumping through a Cartesian manostat, until temperature equilibrium was achieved. A heater was employed to assure adequate stirring, a precaution which was particularly necessary to avoid severe temperature gradients in the bath when it was warmed from a lower temperature. In determining the temperature from the vapor pressure a correction was made for the height of the bath above the crystal. This correction was negligible at 4°K but became increasingly more important as one approached the neighborhood of the λ point. To obtain data in this region, we adopted the following procedure. The λ point was approached very gradually from above by setting the pump to operate at a suitably slow speed making sure always that the liquid was boiling throughout its

FIG. 3. Values of $\eta\rho_n$ vs temperature in the normal and superfluid regions, as measured with the torsional crystal. Two different voltages were employed. At any given temperature these are proportional to the average velocity of the cylinder. The inset shows the data near the λ point.



volume. Under these conditions the initiation of the λ transition at the surface of the bath was clearly marked by the fact that the bath pressure became stationary,

TABLE I. Smoothed values of the product $\eta\rho_n$ vs temperature for liquid helium under its own vapor pressure.

T , °K	$\eta\rho_n$, micropoise g/cm ³
1.4	0.16
1.5	0.20
1.6	0.28
1.7	0.40
1.8	0.54
1.9	0.76
2.00	1.11
2.06	1.47
2.08	1.65
2.10	1.89
2.12	2.18
2.14	2.56
2.160	3.14
2.165	3.35
T_λ	3.67
2.2	3.86
2.3	4.21
2.4	4.42
2.6	4.66
2.8	4.83
3.0	4.93
3.2	4.93
3.4	4.84
3.6	4.69
3.8	4.49
4.0	4.28

say at time T_1 . From this point on the bridge was continuously balanced and readings recorded as frequently as possible as a function of time. The transition of the fluid in the neighborhood of the crystal, located about ten inches below the surface, was again very clearly marked by the sudden fall in the bridge readings, i.e., the value of $\eta\rho_n$, occurring say at time T_2 . To determine the temperatures during this interval of a few minutes, duration, the hydrostatic head corrections were diminished linearly with time starting with the full value of T_1 and ending with zero at T_2 . Below the λ point the hydrostatic head correction was omitted entirely. Furthermore, with no fear of possible temperature gradients we did not trouble to regulate the pressure but instead took numerous data while gradually pumping down, and while warming up.

Figure 3 shows the results of two typical sets of measurements from 4.2°K down to 1.3°K, which was the lowest temperature then attainable with our equipment. Table I provides a set of smoothed values derived from one of the sets. The data shown in Fig. 3 were obtained on two different occasions in a matter of a few hours. It is evident that the general character of the data with respect both to the magnitude and temperature dependence of $\eta\rho_n$ is similar to what has already been obtained with the oscillating disk method, but that the scatter of the data is in general considerably smaller.

Considering now these data in more detail, we observe that apart from the anticipated behavior at the

λ point, which will be discussed presently, there is apparently a maximum in η in the helium I region. Taking account of the temperature variation in ρ ,¹⁶ we find that this maximum occurs at 3.3°K thus confirming a similar observation by de Troyer *et al.*⁵ It is very probable that this maximum is intimately connected with the strong variation of density with temperature of the fluid, and that a measurement of $\eta\rho_n$ at constant density would reveal the viscosity to have a steady rise with temperature above the λ point. A similar maximum is found in the data of Dash and Taylor^{8,9} which we have included in Fig. 3 for comparison, but the peak in this case is at about 2.7°K.

We have been able to make a large number of measurements in the immediate vicinity of the λ point, but because of a lack of elaborate temperature regulation we could not exploit to the full the possibilities of an investigation of this region. Thus, the gap in the data just below T_λ is simply due to the sharpness of the drop and the fact that we could not "sit" on the temperature long enough to make our readings. Nevertheless, we have obtained enough data in these and other measurements to be quite certain that $\partial(\eta\rho_n)/\partial T$ is discontinuous at T_λ . Approaching from above, $\partial(\eta\rho_n)/\partial T \approx 7 \times 10^{-5}$ cgs units in the "run" at higher velocities and somewhat larger for the other run; from below $\partial(\eta\rho_n)/\partial T \gtrsim 13 \times 10^{-5}$ cgs units. The uncertainty in the latter figure could presumably be removed by further measurements employing more refined techniques for regulating the temperature near the λ point. It should be borne in mind, however, that this would not be sufficient to determine the precise behavior near the λ point of η , the "normal" fluid viscosity, because very large uncertainties still exist regarding the rate of change of ρ_n , the "normal" fluid density, in this region. It is obvious from Fig. 3 that the values of $\eta\rho_n$ in the region above T_λ obtained with the quartz cylinder are somewhat smaller than those obtained with the oscillating disk. More significantly, perhaps, the quantities $\partial(\eta\rho_n)/\partial T$ agree surprisingly well in the two cases, especially if the intercomparison is made on the basis of the "run" at lower velocity. This may not be apparent from Fig. 3 where the data of Taylor and Dash are not shown in any detail near the λ point, but is nevertheless found to be true upon closer scrutiny of their data.⁹ Considering the widely different frequencies employed in the two cases, this agreement suggests that at least in the range 0–32 kc/sec the quantity $\partial(\eta\rho_n)/\partial T$ near T_λ is actually independent of the frequency.

One of the interesting features of these measurements is the fact that away from the λ point they appear to be independent of the velocity of the crystal. In making our measurements it was convenient to operate with a constant voltage ϵ impressed upon the crystal as we ran through the temperature range. This meant that the velocity V_m increased sharply in accordance with Eq. (7) as the temperature fell below the λ point. The data

shown in Fig. 3, however, seem to indicate that except for some erratic behavior in one of the "runs" near 2.4°K,¹⁷ a change in velocity V_m at any given temperature by a factor of eighty produced no remarkable change in the resultant $\eta\rho_n$.

It should be noted that the two "runs" shown in the figure were obtained on two separate occasions between which the crystal was brought to room temperature and handled. They thus provide a fairly stringent test of the reproducibility of the measurements. A direct comparison was also obtained between "runs" made on the same day, differing by as much as a factor of ten in velocity. These were confined to the region below the λ point, and they reproduced each other so well that the runs are virtually indistinguishable. They have not been included in Fig. 3 in order to avoid unnecessary crowding. Finally, to satisfy ourselves still further on the question of velocity independence we made special measurements of $\eta\rho_n$ against voltage ϵ at several fixed temperatures, 1.34°, 1.40°, 1.94°, and 2.14°K. In each case we varied the voltage from 0.002 volt to 0.5 volt. This voltage range corresponded to a range in V_m from about 0.1 cm/sec to 25 cm/sec at 1.34°K and from about 0.02 cm/sec to 5 cm/sec at 2.14°K. The sensitivity of the bridge was such that even at 0.1 cm/sec $\eta\rho_n$ was still measureable to $\pm 5\%$, so that within an error at least as good as this we found no evidence of velocity dependence over the stated ranges. We note also that for even the biggest velocity the Reynolds number, Re , calculated from the expression $Re = V_m \rho \lambda / \eta$, is very small.

MEASUREMENTS AT HIGHER DENSITIES

With only slight modifications it has proved convenient to employ the apparatus already described to investigate the viscosity of liquid helium at higher densities. These modifications consisted mainly of enclosing the torsional oscillator in a thick-walled stainless steel container which was immersed in a bath of liquid helium and was connected by means of a capillary to a source of helium gas at high pressure. The capillary served also as the coaxial shield for the wire leading to the crystal. The flange and mating cover of the container were machined with sharp serrations which, together with the soft copper gasket, assured a very satisfactory low-temperature seal even at 1000 psi.

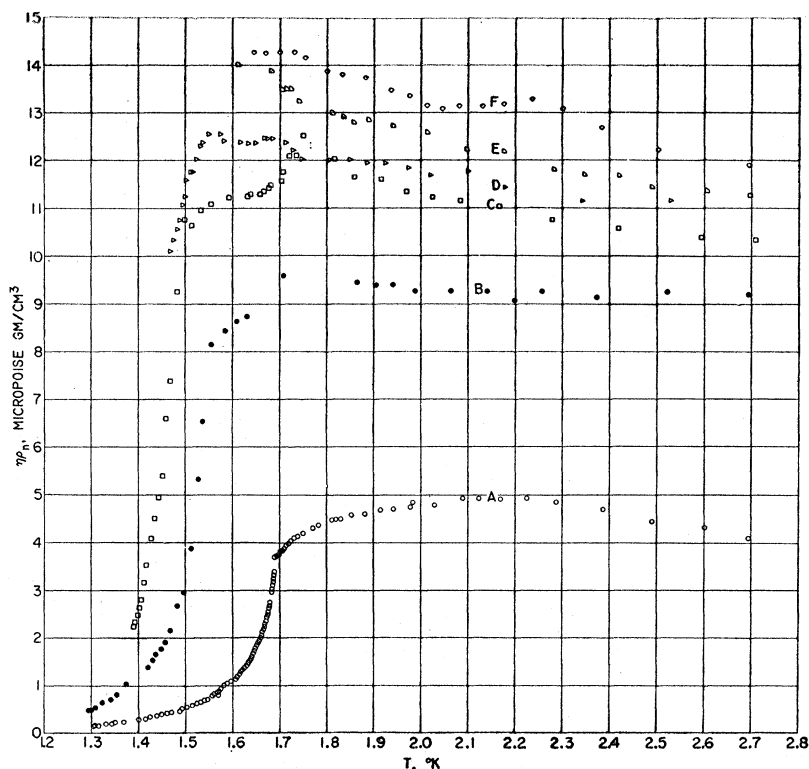
Liquid helium at 4.2°K was condensed into the container by admitting gas through the capillary until the desired pressure was attained and then closing the inlet valve. A set of viscosity measurements was made at various stages while pumping the surrounding bath

¹⁶ E. C. Kerr, *J. Chem. Phys.* **26**, 511 (1957).

¹⁷ A probable explanation of the peak which occurs in one of the two "runs," near 2.4°K, is that we have here an effect produced by a small amount of compressional wave excitation via the eccentricity of the crystal. This will be discussed more fully in a later section, but it should be remarked here that the occurrence of the effect depends in some complex way on the electrode geometry and probably not at all on the voltage. The crystal in fact was dismounted and reassembled between the two "runs" and this resulted, unavoidably, in a slight alteration of the geometry.

FIG. 4. Viscosity of liquid helium vs temperature for several densities. The following table gives the various densities together with the expected values of T_λ , the lambda temperature, and T_s , the solidification temperature.

	ρ , g/cm ³	T_λ , °K	T_s , °K
A	Vapor pressure data	2.17	...
B	0.175	1.90	...
C	0.181	1.79	1.64
D	0.187	...	1.875
E	0.190	...	2.01
F	0.194	...	2.23



either to the lowest attainable temperature or until the liquid around the crystal solidified, the latter event being sharp and unmistakable.

Our results at several liquid densities are shown in Fig. 4. For each isochore are indicated the densities deduced from the pressure at 4.2°K¹⁸; we have neglected the slight decrease in density with temperature and bath level which must occur because of the nonvanishing volume of the capillary and connecting fittings. For the sake of completeness we have also included in the figure some of our previous measurements on helium under its own vapor pressure even though these do not refer to a strictly constant density. We indicate also in an accompanying table, the temperatures T_s and T_λ at which solidification and superfluidity are expected to occur¹⁹ on the basis of the known density. In point of fact we found that solidification around the crystal occurred at a somewhat lower temperature and indeed there was evidence (from the observed pressure) that the capillary became plugged at T_s . Therefore, the data in Fig. 4 at temperatures below T_s should be ignored as they refer necessarily to a changing density of the liquid component in equilibrium with the solid as one proceeds part way along the liquid-solid phase boundary.

¹⁸ F. J. Edeskuty and R. H. Sherman, *Proceedings of the Fifth International Conference on Low-Temperature Physics and Chemistry, Madison, Wisconsin, August 30, 1957*, edited by J. R. Dillinger (University of Wisconsin Press, Madison, 1958), p. 105.

¹⁹ C. A. Swenson, *Phys. Rev.* **89**, 538 (1953).

A notable feature of these results is the progressive increase in the (negative) slope of the isochores in the helium I region as the density is increased. This is not unexpected, since with a higher density and a smaller atomic spacing liquid helium I may be plausibly expected to behave more like a classical liquid.²⁰ The increase of the viscosity with density may be similarly interpreted. Tjerkstra²¹ has made an investigation of the influence of pressure on the viscosity of liquid helium flowing in a capillary, and from the smoothed data has constructed a family of curves at constant density. On comparing we find that his results agree qualitatively with ours but not quantitatively. We have measured considerably smaller viscosities at corresponding densities, and also we found the temperature coefficient to be somewhat larger. We do not know the reason for these discrepancies.

A closer examination of the curve in Fig. 4 for density $\rho=0.181$ g/cm³ (pressure ~ 900 psi at 4.2°K) reveals the presence of an apparent peak in the viscosity near the λ point. A similar, but not so pronounced peak, is present in the curve $\rho=0.190$ g/cm³. This we feel is not a real effect; rather we suspect that it may be caused by the strong-focusing action of the electrode geometry which produces an undesirable resonance. In this connection we recall that longitudinal sound waves in liquid

²⁰ F. London, *Superfluids* (John Wiley & Sons, Inc., New York, 1957), Vol. II, p. 37.

²¹ H. H. Tjerkstra, *Physica* **18**, 853 (1952).

helium, unlike shear waves, are not damped out readily. Therefore, at pressures where the velocity of sound will satisfy the boundary conditions imposed by the concentric electrodes, conditions will be particularly favorable for coupling to a longitudinal mode. Our measurements indicate that for the geometry employed this situation prevailed at several different pressures but the peak which occurred at the lowest pressure, around 200 psi, proved to be the most pronounced and hence also the most troublesome. It is for this reason also that measurements in this region have been avoided in Fig. 4. To ascertain that the above is the correct explanation we made detailed measurements on the "lowest excited mode" around 200 psi as shown in Figs. 5 and 6. The first figure gives $\eta\rho_n$ vs pressure at 4.2°K and two lower temperatures, with the very pronounced spurious maximum at 200 psi. The change in position of this maximum with temperature is given by Fig. 6. It is clear from this figure, together with the well-known data on the velocity of first sound at high pressures,²² that the peak occurs consistently at those pressures where the velocity of sound is 325 m/sec, thus lending support to our hypothesis about the geometry. The existence of these spurious maxima is troublesome, but we suppose that, except in the neighborhood of these resonances, the values obtained for $\eta\rho_n$ are unaffected by them. It may very well be that ultimately the origin of the spurious maxima lies in eccentricity in the quartz cylinder. Indeed, at room temperature the difference between maximum and minimum diameters was as high as 2.0×10^{-3} cm. But if the absence of eccentricity is really so important, then the difficulty is scarcely avoidable even with a perfect cylinder in view of the pronounced crystal anisotropy. (At room temperature, for instance, the coefficient of thermal expansion of quartz perpendicular to the z axis is nearly twice that parallel to z .)

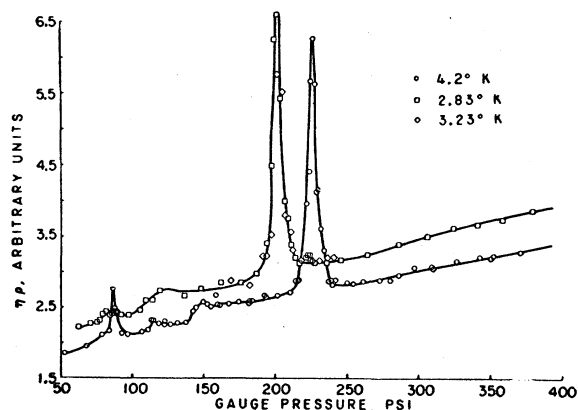


FIG. 5. Diagram showing spurious peaks and the shifting of the main peak with temperature. The points corresponding to data at 3.23°K have not been connected by a solid line.

²² K. R. Atkins and R. A. Stasior, *Can. J. Phys.* **31**, 1156 (1953).

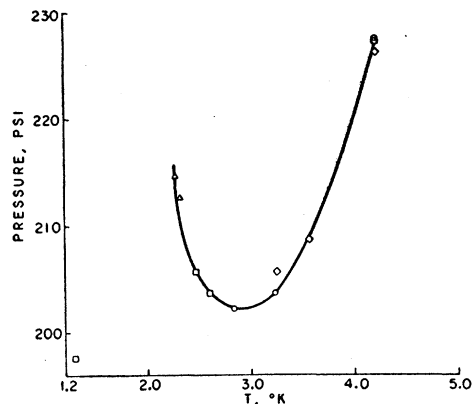


FIG. 6. Location of the main peak as a function of pressure and temperature. The solitary point below the λ point should not be overlooked. The different symbols represent data taken at different times.

However, it should be possible to avoid these maxima in any given region of interest by appropriate modification of the electrode geometry. One attractive possibility is to eliminate the girdling brass electrodes by evaporating a suitable set directly onto the surface of the crystal.

MEASUREMENTS AT LOWER TEMPERATURES

With the aid of a high-speed pumping system which we have recently installed in our laboratory we are seeking to extend the range of our measurements on liquid helium under its own vapor pressure to yet lower temperatures, where the phonon contribution to the viscosity must make itself felt. This work has not progressed sufficiently to allow a detailed comparison with the theoretical predictions of Khalatnikov²³ for this region. Our preliminary results, however, suggest that the rate of rise of viscosity of the normal component with falling temperature in the neighborhood of 1°K is considerably steeper than observed by Zinov'eva.²⁴ We hope to carry these measurements to yet lower temperatures by means of a helium-3 cryostat, after which a full account will be published.

ACKNOWLEDGMENTS

The author wishes to thank Dr. John Wilks and Dr. Darrell Osborne for their sound advice and encouragement. He is also indebted to R. W. Webeler for assistance in the measurements and to Harold Linkert for the construction of the apparatus. Finally, he is very grateful to the Clarendon Laboratory, Oxford, where this paper was written, for the hospitality extended to him during the past year.

²³ I. M. Khalatnikov, *J. Exptl. Theoret. Phys. (U.S.S.R.)* **23**, 8, 21 (1952).

²⁴ K. N. Zinov'eva, *J. Exptl. Theoret. Phys. (U.S.S.R.)* **31**, 31 (1956) [translation: *Soviet Phys.-JETP* **4**, 36 (1957)].