Dielectric Constant of a Dense Electron Gas

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The problem of absorption and dispersion of electromagnetic waves in a dense electron gas is treated semiclassically through the use of Boltzmann equation and Fermi-Dirac statistics. The singularity in the dispersion formula is treated by the method of Van Kampen. Expressions for the dielectric constant and conductivity as functions of frequency have been obtained for temperatures at and near absolute zero.

1. INTRODUCTION

DISPERSION formulas for the plane monochromatic oscillations of a gas of charged particles generally contain integrals over velocity space and invariably these integrals are singular. Integration across the pole of such integrands can be done in a variety of ways and is hence arbitrary. It could be made unique by putting boundary conditions in space and/or in time, but such boundary conditions do not exist for plane monochromatic oscillations. Consequently, there do not exist unique dispersion formulas for such oscillations. This was first pointed out by Van Kampen.[1] He showed that it is possible to get unique dispersion formulas for oscillations, which are certain linear superpositions of monochromatic oscillations. His method has been applied by Pradhan[2] to the problem of circularly polarized electromagnetic oscillations of a Maxwellian plasma, propagating in the direction of a steady magnetic field impressed on it. The results are in agreement with those of Bernstein[3] obtained by the Laplace transform method of Landau.[4] It is the purpose of the present investigation to extend the method of Van Kampen to the problem of dispersion and absorption of electromagnetic waves in a dense electron gas. Lindhard[5] has obtained expressions for the dielectric constant of such a gas for monochromatic waves, and has recognized the fact that its imaginary part is not unique. He has not shown how to get a unique value for it. We shall, in the present work, obtain unique value for this constant by constructing wave packets. We shall give explicit formulas for it that are valid for temperatures at and in the neighborhood of absolute zero.

2. DISPERSION FORMULA FOR MONOCHROMATIC WAVES

The dispersion formulas are obtained by simultaneously solving the Boltzmann and Maxwell equations[6]:

$$\frac{\partial f}{\partial t}+\mathbf{u}\cdot\nabla_r f-\frac{e}{m}\mathbf{E}\cdot\nabla_u f=\left(\frac{\partial f}{\partial t}\right)_{\text{coll}}, \tag{1}$$

$$\nabla^2\mathbf{E}-\frac{1}{c^2}\frac{\partial^2\mathbf{E}}{\partial t^2}=-\frac{4\pi e}{c^2}\int d\mathbf{u}\,\mathbf{u}f-4\pi e\int d\mathbf{u}\,\nabla_r f, \tag{2}$$

with the restriction $\nabla_r\cdot\int d\mathbf{u}\,\mathbf{u}f+(\partial/\partial t)\int d\mathbf{u}\,f=0$, imposed by the equation of continuity. We shall solve these equations in the linear approximation:

$$f(\mathbf{r},\mathbf{u},t)=n_0f_0(\mathbf{u})+f_1(\mathbf{r},\mathbf{u},t),\quad f_1\ll f_0, \tag{3}$$

so that terms of order f_1^2 can be neglected. We shall also make the approximation:

$$\left(\frac{\partial f}{\partial t}\right)_{\text{coll}}=-\frac{f-n_0f_0}{\tau}=-\omega_cf_1=-ku_cf_1, \tag{4}$$

where τ = collision time and ω_c = collision frequency. For monochromatic waves propagating along the z axis,

$$f_1^{k,u_s}(\mathbf{r},\mathbf{u},t)=g^{k,u_p}(\mathbf{u})e^{ik(z-u_st)}, \tag{5a}$$

$$\mathbf{E}^{k,u_s}(\mathbf{r},t)=\mathbf{A}(k,u_p)e^{ik(z-u_st)}, \tag{5b}$$

where $u_s=u_p-iu_c$, u_p = phase velocity of the waves and is real. Substituting Eqs. (3), (4), and (5) in Eqs. (1) and (2) and neglecting terms of order f_1^2, we obtain,

$$g^{k,u_p}(\mathbf{u})=\frac{n_0e}{imk}\frac{1}{u-u_p}\left[\frac{df_0}{dv}\{A_1(k,u_p)\cos\theta+A_2(k,u_p)\sin\theta\}+\frac{df_0}{du}A_3(k,u_p)\right], \tag{6a}$$

$$A_{\text{tr}}(k,u_p)=A_1(k,u_p)=\frac{4\pi ieu_s}{k(u_s^2-c^2)}\int_{-\infty}^{+\infty}du\times\int_0^{+\infty}v^2dv\int_0^{2\pi}d\theta\cos\theta g^{k,u_p}(u,v,\theta),$$

$$A_{\text{tr}}(k,u_p)=A_2(k,u_p)=\frac{4\pi ieu_s}{k(u_s^2-c^2)}\int_{-\infty}^{+\infty}du\times\int_0^{+\infty}v^2dv\int_0^{2\pi}d\theta\sin\theta g^{k,u_p}(u,v,\theta). \tag{6b}$$

We have used cylindrical polar coordinates for velocity space integrations in these equations, i.e., $u_1=v\cos\theta$,

[1] N. G. Van Kampen, Physica **21**, 949 (1955).
[2] T. Pradhan, Phys. Rev. **107**, 1222 (1957).
[3] I. B. Bernstein, Phys. Rev. **109**, 10 (1958).
[4] L. Landau, J. Phys. (U.S.S.R.) **10**, 25 (1946).
[5] J. Lindhard, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **28**, No. 8 (1954).
[6] Notations of the paper in reference 2, with their meanings unchanged, will be used throughout the present work.

$u_2=v\sin\theta$, $u_3=u$. Eliminating $A_{\rm tr}(k,u_p)$ and $g^{k,u_p}(\mathbf{u})$ from Eqs. (6), we get the dispersion formula for the electromagnetic or transverse modes:

$$\frac{c^2-u_s^2}{2\pi u_0^2 u_s}=\int_{-\infty}^{+\infty} du\frac{F(u)}{u-u_p}, \tag{7}$$

where $F(u)=\int_0^\infty vdvf_0(u,v)$, and $u_0^2=4\pi n_0 e^2/mk^2$. In obtaining this formula we have made an integration by parts and have used the fact that $\lim_{v\to\infty} v^2 f_0(u,v)=0$. Since the complex dielectric constant is defined as $\epsilon=c^2/u_s^2$, Eq. (7) gives us ϵ as a function of frequency provided the prescription for integration across the singular point $u=u_p$ is known from boundary conditions. Since there do not exist boundary conditions for plane monochromatic waves, the integration across this point is quite arbitrary. According to Van Kampen[1]

$$\frac{c^2-u_s^2}{2\pi u_0^2 u_s}=\mathrm{P}\int_{-\infty}^{+\infty} du\frac{F(u)}{u-u_p}+\lambda(u_p)F(u_p), \tag{8}$$

where λ is an arbitrary function of u_p. In the work of Lindhard,[5] this arbitrariness has been pointed out; he mentions that the imaginary part of the dielectric constant lies between $-\pi$ and $+\pi$ times a certain expression. We shall, in the following section, show that for wave packets, the dispersion formula does not have this kind of arbitrariness and hence the imaginary part of the dielectric constant, obtained from it, is unique.

3. DISPERSION FORMULA FOR WAVE PACKETS

We construct wave packets by linear superposition of monochromatic waves as follows:

$$E_{\rm tr}^k(z,t)=e^{ikz-ku_ct}\times\int_{-\infty}^{+\infty} du_p A_{\rm tr}(k,u_p)e^{-iku_pt}C(k,u_p), \tag{9}$$

$$f_1^k(z,\mathbf{u},t)=e^{ikz-ku_ct}\times\int_{-\infty}^{+\infty} du_p g^{k,u_p}(\mathbf{u})e^{-iku_pt}C(k,u_p), \tag{10}$$

where $C(k,u_p)$ is an arbitrary function. Let us now define a function

$$\mathcal{G}_{\rm tr}^k=\frac{2ie}{k}\int_{-\infty}^{+\infty}\int du_1du_2u_{1,2}f_1(z,\mathbf{u},0)e^{-ikz},$$

to describe the initial condition. This function, integrated over u, is proportional to the amplitude of the initial transverse current density that generates the electromagnetic waves. In other words, this function tells us how much is contributed by electrons with a given velocity component u to the initial transverse current density. The electric vector of the electromagnetic wave is determined by this function as well as the impedances $1/Z^*$ and $1/Z$ as can be seen from Eq. (13).

It then follows from Eq. (10) and (6a) that

$$\mathcal{G}_{\rm tr}(u)=u_0^2F(u)\left[\mathrm{P}\int_{-\infty}^{+\infty} du_p\frac{D_{\rm tr}(u_p)}{u_p-u}+\lambda(u)D_{\rm tr}(u)\right], \tag{11}$$

where $D_{\rm tr}(u_p)=A_{\rm tr}(u_p)C(u_p)$. We have suppressed the suffix k in Eq. (11). If we now decompose $\mathcal{G}$, D, and F into their positive and negative frequency parts and eliminate λ from Eqs. (8) and (11), we obtain,

$$D_{\rm tr}^{(+)}(u_p)=\mathcal{G}_{\rm tr}^{(+)}(u_p)Z^{-1}(u_p),\quad D_{\rm tr}^{(-)}(u_p)=\mathcal{G}_{\rm tr}^{(-)}(u_p)Z^{*-1}(u_p), \tag{12a}$$

where

$$Z(u_p)=\frac{(u_p-iu_c)^2-c^2}{2\pi(u_p-iu_c)}+2\pi iu_0^2F^{(+)}(u_p), \tag{12b}$$

$$Z^*(u_p)=\frac{(u_p-iu_c)^2-c^2}{2\pi(u_p-iu_c)}-2\pi iu_0^2F^{(-)}(u_p). \tag{12c}$$

Using Eq. (12a) in Eq. (9) we get for the electric field of the packet,

$$E_{\rm tr}(z,t)=e^{ikz-ku_ct}\int_{-\infty}^{+\infty} du_pe^{-iku_pt}\times[\mathcal{G}_{\rm tr}^{(+)}(u_p)Z^{-1}(u_p)+\mathcal{G}_{\rm tr}^{(-)}(u_p)Z^{*-1}(u_p)]. \tag{13}$$

For $k>0$ and $t>0$, the integral appearing on the right-hand side is most easily evaluated by considering it as a part of a contour integral in the complex u_p-plane, the contour running from $-\infty$ to $+\infty$ on the real axis and then returning to the starting point via the semicircle of infinite radius in the lower half u_p-plane. Since the zero of $Z^*(u_p)$ occur in the upper half of the complex u_p-plane, the second term in the integrand gives no contribution to the integral. We have, of course, to demand that $\mathcal{G}_{\rm tr}^{(-)}(u_p)$ are well-behaved functions of u_p without any singularity in the lower half u_p-plane. This we can do, since these functions describe our initial condition. The entire contribution to the integral then comes from the 1st term, because $Z(u_p)$ has its zero in the lower half plane. Since the integrand vanishes on the infinite semicircle as a consequence of the factor $e^{-iku_pt}(k>0, t>0)$, we have,

$$E_{\rm tr}(z,t)\sim\mathcal{G}_{\rm tr}^{(+)}(\bar{u}_p)\exp[ik(z-\bar{u}_pt-iu_ct)], \tag{14}$$

where $\bar{u}_p$ is a point in the lower half-plane such that

$$Z(\bar{u}_p)=\frac{(\bar{u}_p-iu_c)^2-c^2}{2\pi(\bar{u}_p-iu_c)}+2\pi iu_0^2F^{(+)}(\bar{u}_p)=0. \tag{15}$$

Equation (15) is thus the dispersion formula for the wave packet represented by Eq. (14). Using the definition $\epsilon=c^2(\bar{u}_p-iu_c)^{-2}$ for the complex dielectric constant, and $K=\mathrm{Re}(\epsilon)=c^2k^2\omega^{-2}$, we get, from Eq. (15), the

following equation for ϵ:

$$(\epsilon-1)\epsilon^{-\frac{1}{2}}=\frac{2\pi\omega_0^2c}{\omega^2K}\left[i\pi F^*\left(\frac{c}{\epsilon^{\frac{1}{2}}}+i\frac{c}{\omega\tau K^{\frac{1}{2}}}\right)+i\pi F\left(\frac{c}{\epsilon^{\frac{1}{2}}}+i\frac{c}{\omega\tau K^{\frac{1}{2}}}\right)\right], \quad (16)$$

where $\omega_0^2=k^2u_0^2$. For $\omega\tau\gg1$, we get from (16), by Taylor expansion,

$$\begin{aligned}(\epsilon-1)\epsilon^{-\frac{1}{2}}=&(2\pi\omega_0^2c/\omega^2K)[i\pi F^*(c\epsilon^{-\frac{1}{2}})+i\pi F(c\epsilon^{-\frac{1}{2}})]\\&+(2\pi i\omega_0^2c^2/\omega^3\tau K^{\frac{3}{2}})\\&\times[i\pi F^{*\prime}(c\epsilon^{-\frac{1}{2}})+i\pi F'(c\epsilon^{-\frac{1}{2}})], \quad (17)\end{aligned}$$

where prime denotes differentiation. This equation gives us the following approximate expressions for K and $\sigma=(\omega/4\pi)\,\mathrm{Im}(\epsilon)$:

$$K(\omega)=1+\frac{2\pi\omega_0^2c}{\omega^2K^{\frac{1}{2}}}i\pi F^*(cK^{-\frac{1}{2}})-\frac{2\pi^2\omega_0^2c^2}{\omega^3\tau K}F'(cK^{-\frac{1}{2}}), \quad (18a)$$

$$\sigma(\omega)=\frac{\pi\omega_0^2c}{2\omega K^{\frac{1}{2}}}F(cK^{-\frac{1}{2}})+\frac{\omega_0^2c^2}{2\omega^2\tau K}i\pi F^{*\prime}(cK^{-\frac{1}{2}}). \quad (18b)$$

These formulas are valid for any of the three statistics. However we are interested in explicit forms of these equations for Fermi-Dirac statistics which we shall do in the following section.

4. CALCULATION OF $K(\omega)$ AND $\sigma(\omega)$ FOR FERMI DISTRIBUTION

In this section we shall calculate $F(u)$ and $F^*(u)$ for temperatures at and near absolute zero, so that we can obtain explicit formulas for $K(\omega)$ and $\sigma(\omega)$ given in Eqs. (18). According to our definition,

$$\begin{aligned}F(u)&=\int_0^{+\infty}vdvf_0(u,v),\\ i\pi F^*(u)&=\mathrm{P}\int_{-\infty}^{+\infty}du'\frac{F(u')}{u'-u}=2u\mathrm{P}\int_0^{+\infty}du'\frac{F(u')}{u'^2-u^2},\end{aligned} \quad (19)$$

since $F(u)=F(-u)$. For Fermi statistics, which our electron gas obeys at the temperatures we are interested in,

$$n_0f_0(u,v)=2\left(\frac{m}{h}\right)^3\{\exp[-\nu+\tfrac{1}{2}m\beta(u^2+v^2)]+1\}^{-1}, \quad (20)$$

where h = Planck's constant, ν is a constant, $\beta=1/\kappa T$ with κ = Boltzmann's constant, and T = absolute temperature of the electron gas. Simple calculation gives

$$\begin{aligned}F(u)&=\int_0^{+\infty}vdv\ f_0(u,v)\\&=\frac{2}{mn_0\beta}\left(\frac{m}{h}\right)^3\ln\{1+\exp[\beta(g-\tfrac{1}{2}mu^2)]\}. \quad (21)\end{aligned}$$

Here we have put $\nu=\beta g$, g being a parameter with the dimensions of energy and is a function of temperature. For $T=0$, $\beta=\infty$ and $g=g_0=\frac{1}{2}mv_0^2$ (say), so that

$$F_0(u)=(m^3/n_0h^3)(v_0^2-u^2)\theta(v_0^2-u^2). \quad (22)$$

In this formula θ is the usual step function, i.e., $\theta(x)=0$, $x<0$, $\theta(x)=1$, $x>0$. Substituting $F_0(u)$ from Eq. (22) in Eq. (19), we get

$$\begin{aligned}i\pi F_0^*(u)&=\frac{2um^3}{n_0h^3}\int_0^{v_0}du'\frac{v_0^2-u'^2}{u'^2-u^2}\\&=-\frac{m^3}{n_0h^3}\left[2uv_0+(v_0^2-u^2)\ln\left|\frac{u+v_0}{u-v_0}\right|\right]. \quad (23)\end{aligned}$$

The parameter v_0 is the velocity at the top of the Fermi distribution and is found from the normalization condition of $F_0(u)$, i.e.,

$$\int_0^{v_0}du\ F_0(u)=\frac{1}{4\pi}$$

which gives $v_0=(h/m)(3n_0/8\pi)^{\frac{1}{3}}$. Using the results of Eqs. (22) and (23) in Eqs. (18) we get, for $T=0$, $c\neq v_0K^{\frac{1}{2}}$,

$$\begin{aligned}K_0(\omega)=1&-\frac{3\omega_0^2c}{4\omega^2K^{\frac{3}{2}}v_0^3}\\&\times\left[2v_0cK^{\frac{1}{2}}+(Kv_0^2-c^2)\ln\left|\frac{c+v_0K^{\frac{1}{2}}}{c-v_0K^{\frac{1}{2}}}\right|\right]\\&+\frac{3\omega_0^2c^3\pi}{2\omega^3\tau K^{\frac{3}{2}}v_0^3}\theta(Kv_0^2-c^2), \quad (24a)\end{aligned}$$

$$\begin{aligned}\sigma_0(\omega)=\frac{3\omega_0^2c}{16\omega K^{\frac{3}{2}}v_0^3}\bigg[&(Kv_0^2-c^2)\theta(Kv_0^2-c^2)\\&-\frac{2cv_0K^{\frac{1}{2}}}{\pi\omega\tau}+\frac{c^2}{\pi\omega\tau}\ln\left|\frac{c+v_0K^{\frac{1}{2}}}{c-v_0K^{\frac{1}{2}}}\right|\bigg]. \quad (24b)\end{aligned}$$

These formulas differ from those of Lindhard in that the imaginary part of the logarithmic term has been uniquely determined in the present ones. For frequencies for which $K(\omega)\cong1$, formulas (24) take the simple form:

$$K(\omega)=1+\omega_0^2/2\omega^2,\quad \sigma(\omega)=\omega_0^2/8\pi\tau\omega^2, \quad (25)$$

since $v_0\simeq10^8$ cm/sec$\ll c\cong10^{10}$ cm/sec. On the other hand, for frequencies for which $K(\omega)$ is large, i.e., $K(\omega)\simeq10^2$ so that $v_0^2\simeq c^2/K$, $(v_0^2\neq c^2/K)$ the logarithmic term in Eq. (24b) gives a very large contribution and $\sigma_0(\omega)$ becomes very large.

For temperatures in the neighborhood of absolute zero, we shall employ Sommerfeld's method[7] of integration for evaluating $i\pi F^*(u)$. In order to do this we first bring the desired integral into a form best suited to

[7] A. Sommerfeld, Z. Physik 47, 1 (1928).

apply Sommerfeld's results. By integration by parts, and subsequent change of variable of integration, we get

$$i\pi F^*(u)=2u\mathrm{P}\int_0^\infty du'\frac{F(u')}{u'^2-u^2}$$
$$=-\int_0^\infty du'\frac{\partial F(u')}{\partial u'}\ln\left|\frac{u+u'}{u-u'}\right|$$
$$=\frac{2m^3}{n_0h^3m\beta}I(u), \quad (26)$$

where

$$I(u)=\int_0^\infty dx\frac{d\varphi/dx}{e^{-\nu+x}+1},\quad \frac{d\varphi}{dx}=\ln\left|\frac{u+(2x/m\beta)^{\frac{1}{2}}}{u-(2x/m\beta)^{\frac{1}{2}}}\right|. \quad (27)$$

According to Sommerfeld,

$$I(u)=\varphi(x=\nu)+2\sum_{n=1}^{\infty}c_{2n}\left(\frac{d^{2n}\varphi}{dx^{2n}}\right)_{x=\nu}, \quad (28)$$

where $c_n=\sum_{s=1}^\infty(-1)^{s+1}(s)^{-n}$. Explicit calculation gives $c_2=\pi^2/12$ and $c_4=7\pi^4/720$. Retaining terms up to $n=1$ in Eq. (28) and substituting it in Eq. (26) we get

$$i\pi F^*(u)=\frac{2m^3}{n_0h^3m\beta}\left[\varphi(x=\nu)+\frac{\pi^2}{6}\left(\frac{d^2\varphi}{dx^2}\right)_{x=\nu}\right]. \quad (29)$$

Making a Taylor expansion of $\varphi(\nu)$ and $\varphi''(\nu)$ about $\nu=\nu_0$, and neglecting terms of second and higher order in $(\nu-\nu_0)$, Eq. (29) reduces to

$$i\pi F^*(u)=i\pi F_0^*(u)+\frac{2m^3}{n_0h^3m\beta}\times\left[\frac{\pi^2}{6}\left(\frac{d^2\varphi}{dx^2}\right)_{x=\nu_0}+(\nu-\nu_0)\left(\frac{\partial\varphi}{\partial\nu}\right)_{\nu=\nu_0}+(\nu-\nu_0)\frac{\pi^2}{6}\left\{\frac{\partial}{\partial\nu}\left(\frac{d^2\varphi}{dx^2}\right)_{x=\nu}\right\}_{\nu=\nu_0}\right]. \quad (30)$$

Similarly we can get $F(u)$, to this order of approximation,

$$F(u)=F_0(u)+(\nu-\nu_0)\left(\frac{\partial F(u)}{\partial\nu}\right)_{\nu=\nu_0}.$$

The difference $(\nu-\nu_0)$ that appears in Eqs. (30) and (31) can be determined from the requirement that the total number of electrons is same at all temperatures, which is equivalent to the condition $\int_0^\infty du\,F_0(u)=\int_0^\infty du\,F(u)$, or in other words

$$\int_0^\infty du(v_0^2-u^2)\theta(v_0^2-u^2)=\int_0^\infty du\ln\{1+\exp[\tfrac{1}{2}m\beta(v^2-u^2)]\}. \quad (32)$$

Putting $v=v_0+v'$ and differentiating both sides of this equation with respect to v_0, we get,

$$\frac{v_0^2}{v}=\int_0^\infty du\frac{1}{1+\exp[-\frac{1}{2}m\beta(v^2-u^2)]}=\frac{1}{(2m\beta)^{\frac{1}{2}}}\int_0^\infty dx\frac{x^{-\frac{1}{2}}}{e^{-\nu+x}+1}. \quad (33)$$

Integration of the right-hand side of Eq. (33) by Sommerfeld's method, and subsequent rearrangement gives

$$v^2-v_0^2=\pi^2/6m^2\beta^2v_0^2 \quad\text{or}\quad \nu-\nu_0=\pi^2/12m\beta v_0^2. \quad (34)$$

Now we can explicitly write down the expressions for $i\pi F^*(u)$ and $F(u)$ with the help of Eqs. (27), (30), (31), and (34) as follows:

$$i\pi F^*(u)=i\pi F_0^*(u)+\frac{\pi u}{4m^2\beta^2v_0^4(u^2-v_0^2)}-\frac{\pi}{16m^2\beta^2v_0^5}\ln\left|\frac{u+v_0}{u-v_0}\right|+O\left(\frac{1}{\beta^3}\right), \quad (35)$$

$$F(u)=F_0(u)+(\pi/16m^2\beta^2v_0^5)\theta(v_0^2-u^2). \quad (36)$$

Substituting Eqs. (35) and (36) in Eqs. (18) and retaining terms up to order $1/\beta^2$, we get, for $c\neq v_0K^{\frac{1}{2}}$,

$$K(\omega)=K_0(\omega)+\frac{\pi^2\omega_0^2c}{8m^2\beta^2\omega^2K^{\frac{1}{2}}v_0^4}\times\left[\frac{4cK^{\frac{1}{2}}}{c^2-v_0^2K}-\frac{1}{v_0}\ln\left|\frac{c+v_0K^{\frac{1}{2}}}{c-v_0K^{\frac{1}{2}}}\right|\right], \quad (37a)$$

$$\sigma(\omega)=\sigma_0(\omega)+\frac{\pi^2\omega_0^2c}{32m^2\beta^2\omega K^{\frac{1}{2}}v_0^4}\times\left[\frac{1}{v_0}\theta(v_0^2K-c^2)-\frac{2c(c^2-3v_0^2K)K^{\frac{1}{2}}}{\pi\tau\omega(c^2-v_0^2K)^2}\right]. \quad (37b)$$

For frequencies for which $K\cong1$, these formulas reduce to

$$K(\omega)=1+\frac{\omega_0^2}{2\omega^2}\left(1+\frac{\pi^2}{2m^2\beta^2v_0^4}\right),$$
$$\sigma(\omega)=\frac{\omega_0^2}{8\pi\tau\omega^2}\left(1-\frac{\pi^2}{2m^2\beta^2v_0^4}\right). \quad (38)$$

It is to be noted further that both $K(\omega)$ and $\sigma(\omega)$ become very large when $c^2\cong v_0^2K$.

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