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Classical Analysis of the Reaction $Be^9(Li^7,Li^8)Be^8$†

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The mechanism of the reaction $Be^9(Li^7,Li^8)Be^8$, $Q=0.36$ Mev, from which the angular distribution of Li^8 has been measured by Norbeck, Blair, Pinsonneault, and Gerbracht (NBPG), is analyzed from a classical viewpoint. The reaction is one of neutron pickup by the Li^7, and it is assumed that due to the low reaction energy and relatively low mass of the neutron the Li^7 and Be^9 are not unrecognizably perturbed from the classical hyperbolic orbits they would follow under Coulomb repulsion. For the NBPG data taken at 2-Mev Li^7 bombarding energy it is noted that the blurring of the classical trajectories due to diffraction is about 37% in the angle of deviation. Comparison of NPGB's cross sections per steradian for Li^8 formation with those calculated from Rutherford's formula show that 1 in 10^6 passing Li^7 nuclei can capture a neutron from Be^9 in encounters in which the perinuclear distance is as great as $(3.0\pm0.55)\times10^{-12}$ cm.

The rate of radial attenuation of the probability of finding the Be^9 neutron in unit volume is determined from the angular distribution of Li^8, using the classical equation for the perinuclear separation as a function of angular deviation. In the region $(1.5<r<3)\times10^{-12}$ cm it is consistent with the slope of the function $nr^{-2}\exp(-2\beta r)$ with $\beta=(2.1\pm0.5)\times10^{12}$ cm^{-1}. The theoretical value for a neutron bound to a Be^8 structure by $W_n=1.63$ Mev is $(2\mu W_n)^{\frac{1}{2}}/\hbar$ or 2.6×10^{12} cm^{-1}.

It is shown that the extraordinarily low (n,γ) cross section for Li^7 determined by Imhoff, Vaughn, Johnson, and Walt cannot be used here to describe the capture. Using a plausible cross section of 5×10^{-25} cm^2, the value of n in the specified region is 1.3×10^{13} cm^{-1}, giving, for instance, the absolute value of neutron probability per cm^3 at 2.0×10^{-12} cm radius as 6.2×10^{32} cm^{-3}.

The function $nr^{-2}\exp(-2\beta r)$ must join on to one less steep at a radius $\geqq 8\times10^{-13}$ cm, or the neutron probability integrated over all space becomes greater than unity.

I. INTRODUCTION

BERYLLIUM-9 is the most easily photodissociated of any of the stable nuclei:

$$Be^9 \rightarrow Be^8+n^1-1.63 \text{ Mev}. \tag{1}$$

If one explores a three-body model of this nucleus, i.e., two alpha particles and one neutron, a striking feature is the very low "packing energy," since

$$Be^9-(2\alpha+n)=-0.73 \text{ Mev}. \tag{2}$$

In this paper we will use a crude three-body model of the Be^9 nucleus, in which the two α particles are assumed to be confined to a volume not widely different from that consistent with the formula

$$r=1.5\times10^{-13}\sqrt[3]{A}, \tag{3}$$

where r is the radius and A the nearest integer to the atomic weight. In contrast, the neutron, like the neutron in the deuteron, will be assumed occasionally to wander far outside the range of nuclear forces, the structure possessing an obviously imperfect analogy to a dense but small planet with a rarified but extended atmosphere. If we disregard the evidence from the shell model, place the neutron in an $l=0$ orbit, and use a "square" well potential, the radial part of its wave function should have the form

$$\psi_0(r)\propto(1/\beta r)e^{-\beta r}, \tag{4}$$

with

$$\beta=(2\mu W)^{\frac{1}{2}}/\hbar, \tag{5}$$

for values of r beyond the well radius. W is the ionization energy of the neutron from Eq. (1). In the three-body problem the interpretation of μ is not unique; it would certainly seem to have a value less than the reduced mass of a two-body system composed of a Be^8 core and an external neutron.

According to the shell model the neutron should have a wave function characterized by $l=1$, in which case the corresponding external radial part is

$$\psi_1(r)\propto(1/\beta r+1/\beta^2r^2)e^{-\beta r}. \tag{6}$$

However, we shall be discussing the situation far from the center of the Be^9 nucleus, and under this condition the differences between Eq. (4) and Eq. (6) are comparable to other uncertainties in the analysis.

II. NUCLEAR REACTIONS OF Be^9 AND THE THREE-PARTICLE MODEL

The Be^9 nucleus is involved in several nuclear reactions having extraordinarily large cross sections at low bombarding energy. In these reactions, there is always a possible mechanism by which the Be^9 nucleus can react without decomposing into particles other than the three postulated. For instance, consider $Be^9(p,d)Be^8$ and $Be^9(p,\alpha)Li^6$; in the first of these the Be^9 can be considered to split in the fashion $\alpha\alpha|n$, and the neutron picked up. In the second, the split could be $\alpha|\alpha n$.

Consider the first of these reactions. Deuterons from it are easily observable under laboratory conditions of bombardment with proton energies as low as 0.100 Mev. The formula for the Coulomb potential barrier height B in Mev for protons recommended by Blatt and Weisskopf[1] is

$$B=(Z_PZ_Te^2)(r_0A^{\frac{1}{3}}+\rho), \tag{7}$$

with Z_P and Z_T the atomic numbers of projectile and target, respectively, and $\rho=0$ for protons. Using $r_0=1.5$

† This research supported in part by the U. S. Atomic Energy Commission.

[1] J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley & Sons, New York, 1952).

TABLE I. Some exothermic reactions possible from Be^9 nuclei bombarded with Li^7 projectiles.

Reaction	Q (Mev)	Yield at 2 Mev from thick targets per microcoulomb
$Be^9(Li^7,Li^8)Be^8$	0.37	3.8×10^4 [a]
$Be^9(Li^7,\alpha)B^{12}$	10.47	2.3×10^3
$Be^9(Li^7,n)N^{15}$	18.10	$(1.5–2.2)\times10^3$
$Be^9(Li^7,p)C^{15}$	9.10	$\leqq1.9\times10^2$

[a] Thin-target yield corresponds to 3.0×10^{-27} cm² cross section.

$\times10^{-13}$ cm we obtain $B=1.85$ Mev. At 0.180 Mev, or $0.1B$, the total cross section for this reaction is 5×10^{-26} cm² according to Hatch.[2] If it were necessary that a compound nucleus be formed before this reaction and its competitor $Be^9(p,\alpha)Li^6$, which has approximately equal cross section, could proceed, the maximum cross section would be about half that estimated from Coulomb barrier penetration calculations. According to Blatt and Weisskopf[1] this maximum should be about 3×10^{-27} cm². Thus at this energy the cross section of $Be^9(p,d)Be^8$ is some 16 times the maximum value consistent with penetration through the barrier to a distance of 3.1×10^{-13} cm [Eq. (3)] from the electrical center of the Be^9 nucleus. The classical minimum distance of separation, given in general by

$$r_{\min}=(2Z_PZ_Te^2/\mu V_P^2) \tag{8}$$

is in this case 3.1×10^{-12} cm. V_P is the speed of the projectile in the laboratory system. The inference is clear that in Be^9 the neutron may be picked up, with detectable probability, at radial distances large compared with those of Eq. (3).

In search of further information about the Be^9 nucleus, we may turn to the reactions of Be^9, as a target, caused by Li^7 nuclei as projectiles. It is characteristic of nuclear reactions initiated by heavy ions that many competitive exit channels are open under the same conditions of bombardment. Murphy,[3] who investigated proton groups from the reaction $Be^9(Li^7,p)C^{15}$, using 2-Mev Li^7 ions, has listed twelve competitive reactions, all exothermic. In Table I we list only those on which some cross-section data are available.

The first three reactions are from Norbeck and Littlejohn, the result on the total cross section of $Be^9(Li^7,Li^8)Be^8$ is from Norbeck *et al.*[4] (hereafter referred to as NBPG). Murphy quotes the cross section at 90° in the laboratory system for $Be^9(Li^7,p)C^{15}$ as 1.3×10^{-30} cm² per steradian. The figure given in Table I as a target yield is an upper limit, assuming isotropic distribution and Murphy's quoted result. Inspection of Table I shows again that the high cross sections correspond to reactions in which the Be^9 can be considered an aggregate of α particles and a neutron; the more profound rearrangement required for $Be^9(Li^7,p)C^{15}$ occurs with noticeably less probability.

The reaction of outstanding probability in Table I is $Be^9(Li^7,Li^8)Be^8$; it has a certain analogy with $Be^9(p,d)Be^8$ in that in the former case a Li^7, in the latter a proton, could cause the reaction by picking up a neutron from the outer regions of the Be^9 nucleus, leaving the residual Be^8 relatively undisturbed. The probability of compound nucleus formation for this reaction has been published by Murphy[3] based on the computations of Feshbach, Shapiro, and Weisskopf.[5] They are given again here, in Table II, for the first four l values. We see that this one of at least 12 possible competitive reactions, all of much higher Q value, is 17 times more probable than indicated by the barrier penetration of the $l=0$ partial wave, and five times as probable as the maximum indicated by the sum of the partial waves from $l=0$ to $l=3$.

By investigating the angular distribution of the Li^8 as a function of bombarding energy as it increases from 2 to 4 Mev in the laboratory system, NPBG have shown that the Li^8 comes from a projectile Li^7 which has been tagged as Li^8 by neutron capture. *A priori*, there was the improbable possibility that this particular reaction could have proceeded by proton transfer from the Be^9 target to the projectile Li^7, in which case the Li^8 would have appeared as a modified target. This paper will attempt to show that additional information concerning the Be^9 nucleus may be extracted from the angular distributions observed by NPBG.

III. ASSUMPTIONS OF THE PRESENT CALCULATION

In the original Butler[6] treatment of stripping and pickup reactions, it was assumed that Coulomb effects could be neglected. Later, there were attempts to include Coulomb effects into the calculations. In the meantime experimental physicists had found that broadened Butler-type distributions persisted, even in cases where it was clear that the Coulomb effects were not to be considered as absent.

In this paper the $Be^9(Li^7,Li^8)Be^8$ reaction will be con-

TABLE II. Probability of compound nucleus formation for 2-Mev Li^7 projectiles on Be^9 targets. Observed σ for $Be^9(Li^7,Li^8)Be^8$: 3.0×10^{-27} cm².

l	$\sigma=(2l+1)\pi\lambda^2T_l$ σ_l (cm²)	$\sum_0^l\sigma_l$
0	$1.7\ \times10^{-28}$	1.7×10^{-28}
1	$2.9\ \times10^{-28}$	4.6×10^{-28}
2	$1.2\ \times10^{-28}$	5.8×10^{-28}
3	0.25×10^{-28}	6.0×10^{-28}

[2] G. T. Hatch, Phys. Rev. **54**, 165 (1938).
[3] P. G. Murphy, Phys. Rev. **108**, 421 (1957).
[4] E. Norbeck, Jr., and C. S. Littlejohn, Phys. Rev. **108**, 754 (1957); E. Norbeck, J. M. Blair, L. Pinsonneault, and R. J. Gerbracht, Phys. Rev. **116**, 1560 (1959).
[5] Feshbach, Shapiro, and Weisskopf, Atomic Energy Commission Report, NYO-3077 (unpublished).
[6] S. T. Butler, Proc. Roy. Soc. (London) **A208**, 551 (1951).

TABLE III. Values of η, etc., for various nuclear collisions.

Experimental situation	η [Eq. (9)]	Fraction of barrier height	$10^{12}r_{\min}$ [Eq. (8)]	Remarks
8.6-Mev α's on Au target	35	0.34	2.6	Pure Rutherford scattering.
2-Mev Li^7 on F target	16	0.22	1.4	Coulomb excitation; no nuclear reactions.[a]
2-Mev Li^7 on Be target	7	0.35	1.6	Neutron pickup observed; reactions with Be^9 "core" 100 times less probable.
4-Mev Li^7 on Be target	5	0.70	0.8	Nuclear reactions of all types
9-Mev d's on Al	0.19	0.46	0.21	Butler stripping, no Coulomb effects influencing reactions.[b]

[a] C. S. Littlejohn and G. C. Morrison, Phys. Rev. 116, 1526 (1959).
[b] See Butler, reference 6.

sidered in quite the opposite fashion. We will assume Rutherford scattering as the principal factor in the observed angular distributions, and treat the nuclear reaction as causing a mild perturbation of the classical hyperbolic orbits. The perturbation will be estimated in a very elementary fashion. Such a treatment must be justified in two steps, the first of which is to inquire whether, for the elastically scattered Li^7 nuclei, there is sufficient validity to the orbital picture. This question has been considered by Bohr[7] and depends on the value of η, where

$$\eta = 2Z_PZ_Te^2/hV_P. \tag{9}$$

η is thus the ratio of $r_{\min}$, [Eq. (8)] to $\lambda\!\!\!^{-}$, the de Broglie wavelength divided by 2π. If $\eta \gg 1$, diffraction effects are negligible and an elastically scattered projectile, detected at angle θ_c in the center-of-mass coordinate system, may with confidence be said to have traversed a hyperbolic orbit given in that system by

$$r(\alpha) = \tfrac{1}{2}r_{\min}(\csc\tfrac{1}{2}\theta_c \cos\alpha - 1)^{-1}\cot^2(\tfrac{1}{2}\theta_c). \tag{10}$$

Figure 1 illustrates the coordinate system used. Table III lists values of η and other significant quantities in certain experimental situations. In this table the barrier height was calculated in a conventional manner, i.e.,

$$B = Z_PZ_Te^2/r_0(P^{\frac{1}{3}}+T^{\frac{1}{3}}), \tag{11}$$

where $r_0 = 1.5\times10^{-13}$ cm and P and T stand for the masses of projectile and target, respectively. It is seen that the case of Be^9 under bombardment with 2-Mev Li^7 ions, with $\eta = 7$, lies close to the region in which we have become accustomed to the use of orbital pictures, for instance, consider the classical calculations of Coulomb excitation.

The uncertainty measured by η can be expressed in another way; i.e., Bohr[7] shows that the uncertainty expressed in terms of the variation of the angle of deviation is approximately $\eta^{-\frac{1}{2}}$, or 37% for $\eta = 7$. This can be interpreted as an uncertainty in the perinuclear distance for an observed angle of deviation, through Eq. (10) with $\alpha = 0$. For instance, with $\theta_c = 45°$ (Li^8 observed at 45° in the barycentric system) the perinuclear distance is 2.91×10^{-12} cm at 2 Mev, and the region of uncertainty due to diffraction lies between 3.5 and 2.6×10^{-12} cm. At $\theta_c = 135°$, the uncertainty is less, the perinuclear distance lying in the range $(1.55 < r_{\text{per}} < 1.85)\times10^{-12}$ cm.

The second step which needs to be justified concerns the fact that even if an elastically scattered charged particle may be said with confidence to have traversed a classical orbit, one which has been involved in a nuclear reaction will in general have its orbit distorted beyond all recognition. There are, however, two circumstances, special to the $Be^9(Li^7,Li^8)Be^8$ reaction, which greatly reduce the orbital distortion. These are that the reaction has the low Q of 0.36 Mev, and the mass of the neutron picked up is 1/7 of that of the Li^7 which caught it. For a numerical example, consider the situation in the barycentric system, for the case of Li^7 ions of 2-Mev kinetic energy as measured in the laboratory system. In the barycentric system the Li^7 initially is moving toward the Be^9 with 0.633-Mev kinetic energy, and the Be^9 is approaching it with 0.492 Mev. If we consider that the Li^7 takes up the momentum of a 0.36-Mev neutron, a simple vector diagram shows that the maximum deviation of a Li^7 nucleus which would otherwise pass essentially undeviated, is about 15°. The corresponding value for a 4.0-Mev Li^7 is $9\frac{1}{2}°$. For Li^7

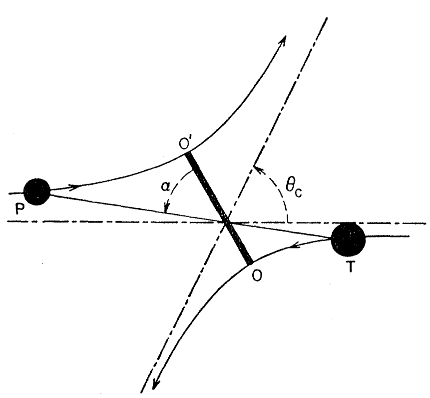

FIG. 1. The coordinate system used to describe the projectile-target collision in the barycentric system. α is the angular deviation of the line connecting the objects from the perinuclear position OO'. θ_c is the deflection of the projectile.

[7] N. Bohr, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. 18, 8 (1948).

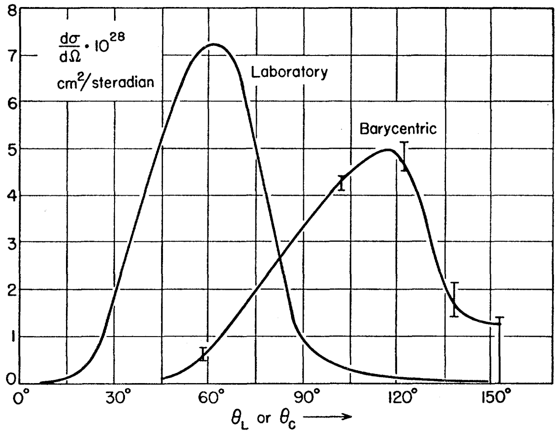

FIG. 2. The angular distribution of Li^8 from the reaction $Be^9(Li^7,Li^8)Be^8$ at 2-Mev Li^7 energy in the laboratory and barycentric coordinate systems.

ions recoiling backward the angular perturbation due to neutron capture should be less.

IV. TREATMENT OF THE DATA

In describing the treatment of the NBPG data we shall speak specifically of the treatment of the 2-Mev angular distribution they observed in the laboratory system. The treatment of their 4-Mev data was entirely analogous. Figure 2 of NBPG shows the angular distribution in the laboratory of the Li^8 produced at 2 Mev. Their ordinates are arbitrary units; let us represent the numerical value of any such ordinate by f. Thus

$$d\sigma(\theta_L)/d\Omega = kf, \tag{12}$$

where θ_L is the laboratory angle, and k is the numerical factor required to express the result in cm² per steradian. The total cross section is then

$$\sigma = 2\pi k \int_0^\pi f \sin\theta_L d\theta_L, \tag{13}$$

and NBPG determined σ by sinking a Be^9 target into a well in a NaI(Tl) crystal scintillator and counting the total activity. They found $\sigma = 3.0\times10^{-27}$ cm². In order to use this value a new distribution curve was constructed by multiplying each ordinate of their Fig. 2 by the appropriate value of $\sin\theta_L$. This curve was then integrated as indicated in Eq. (13), and k could be found and absolute values assigned to the NBPG ordinates. The result is shown in our Fig. 2.

The curve was then transformed to the barycentric system by following standard procedures.

The results of the transformation to the barycentric system are shown in Fig. 2. In the transformation, the absolute values of the experimental errors in points taken at large angles are greatly increased. If these points in the original data were low and close to the background, the magnification of the error may make them unusable. Thus at a point at $\theta_c = 167°$ in the barycentric curve, obtained by the transformation from 135° in the original laboratory data, the NPBG experimental error of $\pm 0.20\times10^{-28}$ cm² becomes $\pm 1.9\times10^{-28}$ cm² in a total value of 4.7×10^{-28} cm². A slight mistake in estimating the background would totally invalidate such points, and they were discarded in the analysis. The angular range, in θ_L, of the data used extended from 20° to 105° at 2 Mev, and from 10° to 100° at 4 Mev.

V. RATE OF ATTENUATION OF NEUTRON DENSITY WITH INCREASING RADIUS

Pursuing the idea of using orbital pictures, we can calculate from the Rutherford formula the cross section for elastic scattering of the Li^7 projectiles, and compare this with the cross section observed by NBPG for those which have become radioactive by neutron pickup. In the barycentric system, the Rutherford formula is

$$\{d\sigma(\theta_c)/dr\}_R = \tfrac{1}{16} r_{\min}^2 \csc^4(\tfrac{1}{2}\theta_c). \tag{14}$$

A crude attempt to allow for the perturbation of the orbits of those Li^7's which capture a neutron was made as follows. It was assumed that the increment of momentum increased the angle of deviation, which was primarily determined by the impact parameter and the Coulomb repulsion. Thus a Li^8 product nucleus appearing at θ_c is assumed to be a modified scattered Li^7 which would otherwise have appeared at an angle $(\theta_c - \delta)$. $\delta = 15°$ was used for 2 Mev, and $\delta = 9\frac{1}{2}°$ for 4 Mev, neglecting its variation with θ_c. Thus the probability that a Li^7 nucleus catches a neutron along its orbit will be

$$P(\theta_c) = \left\{\frac{d\sigma(\theta_c)}{dr}\right\}_N \sin\theta_c \Big/ \left\{\frac{d\sigma(\theta_c-\delta)}{dr}\right\}_R \sin(\theta_c-\delta), \tag{15}$$

where the subscript N refers to the experimental data of NBPG expressed barycentrically. The effect of the δ correction is small compared to uncertainties introduced by the lack of definiteness of the orbital picture.

Let us now suppose that the capture of the neutron occurs essentially at the perinuclear distance, and that the region of capture extends over a length of arc of the orbit equal to the perinuclear separation,[8] which from Eq. (10) is

$$r_{\text{per}}(\theta_c) = \tfrac{1}{2} r_{\min}[\csc\tfrac{1}{2}(\theta_c-\delta) - 1]^{-1} \cot^2[\tfrac{1}{2}(\theta_c-\delta)]. \tag{16}$$

Equation (19) is to be applied to a Li^8 nucleus which appears at θ_c in the barycentric system.

Let $|\psi(r)|^2$ be the probability of finding the neutron in unit volume at distance r from the electrical center

[8] The assignment of effective length of arc equal to perinuclear separation is obviously rough. However, the perinuclear distances with which we are concerned are so great, and the decay of the neutron probability function so rapid, that use of various low powers of r_{per} in the effective length of arc has very little effect in the determination of β.

TABLE IV. Summary of computational steps.

Description of operation	Numerical values
1. Selection of original NPBG datum at 2 Mev (see Fig. 1)	$\{d\sigma(\theta_c)/d\Omega\}_N = 5.15\times10^{-28}$ cm^2/steradian at $\theta_L = 45°$
2. Transformation to barycentric system.	$\{d\sigma(\theta_c)/d\Omega\}_N = 2.4\times10^{-28}$ cm^2/steradian at $\theta_c = 77°54'$
3. Adjustment for perturbation by neutron capture, Eq. (15)	$\{d\sigma(\theta_c-\delta)/d\Omega\}_N = 2.6\times10^{-28}$ cm^2/steradian at $\theta_c-\delta = 63°$
4. Computation of Rutherford cross section, Eq. (14)	$\{d\sigma(\theta_c)/d\Omega\}_R = 2.3\times10^{-24}$ cm^2/steradian at $\theta_c = 63°$
5. Calculation of r_{per}, Eq. (16)	2.35×10^{-12} cm
6. $10^{-12}P(\theta_c)/r_{\text{per}}$	4.8×10^{-5} cm^{-1}

of the Be^9 nucleus. Then the simple assumption we have just made is

$$P(\theta_c) = |\psi(r_{\text{per}})|^2\sigma_c r_{\text{per}}(\theta_c), \tag{17}$$

where σ_c is a kind of capture cross section of the Li^7 nucleus, suitable to the conditions of the encounter. It is expected that the attenuation of $|\psi(r)|^2$ as r increases will be extremely rapid at the large values of r which concern us; 0.8×10^{-12} cm$<r<4\times10^{-12}$ cm. We shall show later that σ_c cannot be the radiative capture cross section of Li^7 for a free neutron; in the absence of any direct knowledge concerning it we shall assume that its variation throughout the barycentric velocities with which we are concerned is small compared to the rate of variation of $|\psi(r)|^2$. If this is correct, each value of $P(\theta_c)/r_{\text{per}}$ deduced from the data of NBPG will be proportional to the neutron probability density function per unit volume at distance r_{per} from the Be^9 electrical center. To assist in following the calculation, Table IV traces the interpretation of one particular NPBG datum through the computations.

In Fig. 3 the computed values of $10^{-12}P(\theta_c)/r_{\text{per}}$ are plotted as ordinates against radial distance in Be^9. Points marked with a cross are from NPBG's 2-Mev data; circled points are from their 4-Mev results.

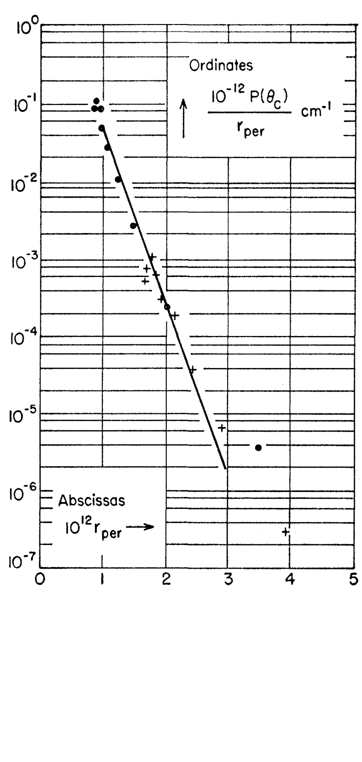

FIG. 3. The rate of attenuation of the neutron probability density in the Be^9 nucleus as the radius increases. + indicates a point derived from 2-Mev data, ● one derived from 4-Mev data. The solid line is a plot of the function $r^{-2}\times\exp(-4.2\times10^{12}r)$ adjusted to the points at $r=2\times10^{-12}$ cm.

The data seem most reliable which lead to radial distances between 1.5 and 3.0×10^{-12} cm. At larger radial distances the data arise from experimental observations on Li^8 in the forward direction and at low intensities with respect to the background. At smaller radial distances the unreliability of the 4-Mev data due to the effect of diffraction and barrier pentration make our simple interpretation unreliable.

One of the firm predictions of the quantum mechanical model of a nucleus is that the radial part of an acceptable characteristic function for a particle outside the range of nuclear forces where the nuclear potential energy is zero, is given by Eqs. (4) and (5). Thus we have

$$|\psi(r)|^2 \propto r^{-2}\exp(-2\beta r), \tag{18}$$

and

$$\{d|\psi(r)|^2\}/|\psi(r)|^2 = -2r^{-1}(1+\beta r)dr. \tag{19}$$

We assume that Eq. (18) is the form of the function of Fig. 3, and determine β through the slope at $r=2\times10^{-12}$ cm, using Eq. (19). This gives

$$\beta_{\text{exp}} = (2.1\pm0.5)\times10^{12}\text{ cm}^{-1}. \tag{20}$$

If we approximate the Be^9 nucleus by a two-body model in which the neutron moves with respect to a Be^8 core, the theoretical value of β may be deduced from Eq. (5), using $W=1.63$ Mev. It is

$$\beta_{\text{th}} = 2.64\times10^{12}\text{ cm}^{-1}.$$

Considering the uncertainties involved there is reasonable agreement. The curve drawn in Fig. 3 is a plot of the function $r^{-2}\exp(-2\beta r)$ with $\beta=2.1\times10^{12}$, the ordinate being adjusted so that at $r=2\times10^{-12}$ cm the value is 2.5×10^{-4}. It is seen that in the range 1.5×10^{-12} cm to 3×10^{-12} cm such a function expresses the experimental attenuation quite well; at larger r, however, the attenuation of the probability seems less rapid. The explanation of such an effect is not at hand; as we have noted, the points concerned represent measurements of Li^8 in the forward direction in the laboratory at low intensity, very near the background, and may be in error. Furthermore, for the small angles of deviation from which these points at large r arise,

the uncertainty in $r_{\rm per}$ due to diffraction effects is much increased.

VI. ABSOLUTE VALUE OF THE NEUTRON PROBABILITY

Although no rigorous deductions were obtained concerning the absolute values of $|\psi(r)|^2$ in the region $1.5\times10^{-12}<r<3\times10^{-12}$, an attempt to calculate it uncovered some points of interest. The procedure was based on the following idea. If the absolute values of $|\psi(r)|^2$ were known along the orbit of the Li^7 as it passes through the neutron "atmosphere" in the outer reaches of the Be^9 nucleus, the probability of capturing a neutron could be expressed as

$$P(\theta_c)=\int_0^\infty |\psi(r)|^2\sigma_c dl, \tag{21}$$

which is an obvious refinement of Eq. (17). dl is an infinitesimal length of arc along the hyperbolic orbit and is connected with an increment $d\alpha$ in α (Fig. 1) by

$$dl=[(dr)^2+r^2(d\alpha)^2]^{\frac{1}{2}}, \tag{22}$$

or

$$dl=\{(dr)^2\sin^2\alpha+r^2[d(\cos\alpha)]^2\}^{\frac{1}{2}}\csc\alpha. \tag{23}$$

The integral of Eq. (21) was numerically evaluated, using various assumptions about $|\psi(r)|^2$ and σ_c, as outlined below. The integral was expressed as a sum of contributions from increments of path Δl associated by Eq. (23) with increments $\Delta(\cos\alpha)$, near $\cos\alpha=1$, which corresponds to the perinuclear separation.

Let us assume that the absolute value of the neutron density is given by

$$|\psi(r)|^2=nr^{-2}\exp(-2\beta r), \tag{24}$$

with $\beta=(2.1\pm0.5)\times10^{12}$ cm^{-1}. The most obvious attempt to evaluate n is to do it independently of Eq. (21), namely by space normalization of Eq. (24) to unity, setting

$$4\pi n\int_0^\infty \exp(-2\beta r)dr=1. \tag{25}$$

This is done in the elementary theory of the deuteron, and neglects the fact that inside the region of the potential well the function is no longer the one postulated in Eq. (24). Carrying out this step Eq. (25) produces $n=\beta/2\pi=3.34\times10^{11}$ which is unacceptably small. To show this, consider the observations on Li^8 produced at 90° in the barycentric system from a 2-Mev Li^7 beam. The perinuclear distance in this case is $(2.03\pm0.25)\times10^{-12}$ cm. Evaluation of the integral of Eq. (21) along a length of arc 3×10^{-12} cm shows that in order to account for the observed $P(\theta_c)$ of 6.5×10^{-4} the value of σ_c would have to be 2.9×10^{-23} cm^2, too large by a factor of ~100.

It is interesting to note, however, that the cross section for the capture of a free neutron by Li^7, followed by γ radiation to the ground state of Li^8, is completely unacceptable here The velocity of the Li^7 nucleus through the neutron "atmosphere" of Be^9 corresponds to a neutron kinetic energy of a few hundred kiloelectron volts. The capture cross section of Li^7 for free neutrons in this energy region has recently been measured[9] and has the extraordinarily low values of $(5–20)\times10^{-30}$ cm^2. The factor of discrepancy is more than 10^6. It is clear that there must be sufficient coupling with other parts of the nuclear constituents concerned to remove the excitation energy in some other form than electromagnetic radiation.[10]

Finally, we may obtain a "plausible" value of n and the absolute probability density by assigning a "plausible" cross section to the Li^7 nucleus for neutron pickup under these conditions. Such a plausible cross section is 5×10^{-25} cm^2, corresponding to a geometrical cross section whose radius is the sum of 2.8×10^{-13} cm for Li^7 and 1.2×10^{-13} cm for the neutron.

At this point another consideration enters, namely that the diameter of the Li^7 cross section is not negligibly small compared to the perinuclear separation (8×10^{-13} cm vs 2.03×10^{-12} cm). Thus the change in the neutron probability function across this diameter will be appreciable, and the average across the Li^7 disk will not equal the value at its center. A purely geometrical calculation of this effect gave the result that the probability of the Li^7 capturing a neutron is 35% higher than would be indicated by the value of $|\psi(r)|^2$ taken at the electrical center of the Li^7 nucleus. Thus the correct value of $P(\theta_c)$ for an orbit with perinuclear distance 2.03×10^{-12} cm is $0.65\times6.5\times10^{-4}$ or 4.2×10^{-4}. Using this value of $P(\theta_c)$ in Eq. (21), with $|\psi(r)|^2$ expressed as in Eq. (24), and carrying out the integration numerically gives $n=1.3\times10^{13}$ cm^{-1} giving, for instance, $|\psi(r)|^2=6.2\times10^{32}$ cm^{-3} at 2.03×10^{-12} radial distance.

This value of n is surprisingly large, in fact, it can be shown that this value of n cannot be correct and at the same time have Eq. (24) represent the neutron function all the way out from the Be^9 center to the radii under consideration. Applying the normalization integral Eq. (25) with the lower limit set at $r=a$ instead of zero, with $n=1.3\times10^{13}$ cm^{-1}, $\beta=2.1\times10^{12}$ cm^{-1}, we find $a=8.8\times10^{-13}$ cm. This may mean that the neutron probability function, rising rapidly in value as r decreases, must flatten out somewhat before $r=a$.

The points plotted in Fig. 3 in the region $0.8\times10^{-12}<r<1.5\times10^{-12}$ cm seem to invalidate this conclusion. There is no obvious flattening as r decreases. But these points are all from the 4.0-Mev data where we are at 70% of the barrier height (Table III) and the tunnelling effect into the classically forbidden radii can no longer be ignored. It is probable that due to this tunnelling, near 1×10^{-12} cm radial distance, the values of $P(\theta_c)/r$

[9] W. Imhoff, F. J. Vaughn, R. G. Johnson, and M. Walt (1958). 1959 Nuclear Data Tables U. S. Atomic Energy Commission.

[10] This point became clear in a discussion with G. C. Morrison.

are actually characteristic of shorter radii than those at which they are plotted in Fig. 3, and the predicted flattening is thus concealed.

VII. CONCLUSION

Although the methods of calculation used in this work could be considerably refined, for instance, in allowing for the effects of diffraction and orbit perturbation by neutron capture, it seems better to await further data which confirm the rather surprisingly large size of the Be^9 nucleus indicated here. As the study of heavy ion interactions proceeds, neutron pickup may be found from beryllium using even heavier projectiles and under circumstances where the orbital picture is less blurred than in the present case.

The idea of using such a classical approach for the interpretation of heavy ion nuclear collisions is not new. It has been developed in a series of papers by Breit and his collaborators,[11] and has been applied to the reaction $Au^{197}(N^{14},N^{13})Au^{198}$ by McIntyre, Watts, and Jobes.[12]

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Elastic Scattering of Fast Neutrons by Tritium and He^3†

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Differential cross sections have been obtained for the elastic scattering of neutrons by tritium and by He^3 at $E_n=1.0$, 2.0, 3.5, and 6.0 Mev over the angular range 27° to 161° in the c.m. system. The Los Alamos large Van de Graaff accelerator and pulsed-beam time-of-flight facility were employed, and the scattering samples were contained in small thin-walled stainless steel spheres. One-third mole of He^3 was contained at 5000 psi and one-half mole of tritium was prepared in the form of CaT_2. Absolute cross sections were determined from a comparison with the scattering from a thin shell of CH_2 at each energy, together with calibration of the relative sensitivity of the detector as a function of energy by the known forward yield of the $T(p,n)He^3$ reaction. The angular distributions for n-T scattering are in excellent agreement with the calculations of Bransden, Robertson, and Swan based on a Serber exchange force. The n-He^3 measurements favor qualitatively the Serber rather than the symmetrical exchange force used in the calculations, but the agreement is poorer than that obtained for n-T scattering. The polarization of elastically scattered 1-Mev neutrons was found to be less than 5% for both samples, unlike the strong polarization observed in n-He^4 scattering.

INTRODUCTION

THE measurements reported in this paper are concerned with the elastic scattering of fast neutrons by nuclei of mass three. They are part of an extensive program of investigation at this Laboratory of the interactions of the very light nuclei. The long-range objective of this program is to reconcile the scattering, reaction, and ground-state properties of the very light nuclei with the information about nuclear forces derived from analysis of n-p and p-p scattering. Supplementary knowledge of nuclear forces may be sought in the study of excited states and interactions in systems of a few nucleons. From the latter point of view, neutron scattering by the very light nuclei is of special interest since direct observation of the scattering of neutrons is not feasible at present, and the character of the n-n force must be inferred from a study of the interactions of neutrons with bound systems of nucleons. The possibility of many-body nuclear forces can also be investigated through the study of systems of a few nucleons.

Much theoretical and experimental effort has been devoted to the study of p-d and n-d interactions. Reviews of the subject have been given by Massey[1] and by Mather and Swan.[2] A bibliography of more recent experimental work will be found in recent papers on n-d elastic and p-d inelastic scattering.[3,4] The calculation of nucleon-deuteron interactions with tensor forces has been formulated,[5] but the present state of nucleon-deuteron scattering theory is inconclusive.

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