Interaction Between a Pair of K^+ Mesons

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An analysis is performed for the reaction $\bar{\Xi}^+ + p \to K^+ + K^+ + \gamma$, with subsequent materialization of the photon into an electron-positron pair. In the circumstance that (a) the annihilation of low-energy $\bar{\Xi}^+$ proceeds largely via the reaction $\bar{\Xi}^+ + p \to K^+ + K^+$ and that (b) the elastic scattering of the $\bar{\Xi}^+$ on hydrogen is largely a diffraction effect, a study of the radiative reaction provides, in principle, a means of determining a function related to the phase shifts for the scattering of a pair of K^+ mesons. Under assumption (b) alone, an absence of all meson-meson interactions among pions and K mesons could be inferred from the vanishing of a certain angular correlation term in the radiative process.

I. INTRODUCTION

THE interaction between a pair of mesons, be they pions, K mesons, or a pion and a K meson, has been the subject of many recent theoretical and phenomenological studies.[1,2] One of the problems involved in this subject is the difficulty in devising analyses of experimental results which would yield direct information on the presence of interactions between mesons. A number of analyses have been suggested which might, in a favorable set of circumstances, yield semiquantitative information on a pion-pion interaction.[3–5] These attempts have appealed to the possibility of isolating the physical effects of that part of the matrix element for the process $\pi + N \to \pi + \pi + N$ which is due to the interaction of the incident pion with a pion in the cloud of the target nucleon, under the assumption that this is an important contribution and a strongly varying contribution for small values of the laboratory three-momentum transfer to the target nucleon. Characteristic effects of the absolute square of this part of the matrix element,[3,4] or of its interference terms with the residual part of the matrix element,[5] may be compared with experimental results. Some information on the interaction between a pion and a K meson might be obtained from similar analyses applied to the reaction $K + N \to K + \pi + N$. A study of the interaction between a pair of K mesons may be considerably less susceptible to this type of analysis, owing to the greater mass of the K meson and the consequent larger range of momentum transfer involved.

In this note we consider an analysis, which, given a favorable set of circumstances, could, in principle, yield information on the interaction between a pair of K^+ mesons. We are aware of the considerable experimental difficulties involved in the analysis,[6] as well as of the likelihood that the favorable conditions necessary may not be met with in reality. We present the discussion in the spirit of an example as to how, in principle, one could obtain information on the interaction in a system which, in most circumstances, appears in a hopelessly complicated system of strongly interacting particles.

II. ANALYSIS

Consider that we have produced $\bar{\Xi}^+$ via the reaction

$$\bar{p} + p \to \Xi^- + \bar{\Xi}^+. \tag{1}$$

The $\bar{\Xi}^+$ will, in general, have some polarization along the normal to the production plane, which we denote by the unit vector $\mathbf{n}$. The existence of some polarization induced by the production reaction (1) would be confirmed by the observation of an up-down asymmetry with respect to the production plane in the subsequent decay process

$$\bar{\Xi}^+ \to \bar{\Lambda} + \pi^+. \tag{2}$$

We consider now the following rare reaction induced by a very low-energy (essentially S-wave), polarized $\bar{\Xi}^+$ interacting with a proton:

$$\bar{\Xi}^+ + p \to K^+ + K^+ + \gamma. \tag{3}$$

We denote the momentum and polarization of the photon by $\mathbf{k}$ and $\mathbf{e}$, respectively; we denote the relative momentum of the K^+ pair by $\mathbf{q}$. The analysis involves the observation of the photon via its materialization into an electron-positron pair. We may then observe a distribution in the angle φ between the plane of the electron-positron pair and the plane defined by its normal, $\mathbf{q} \times \mathbf{k}$. *Any* term in this distribution proportional to $\sin 2\varphi$ would indicate a nonvanishing interaction between the pair of K^+ mesons, provided that the following circumstances were observed to hold: (a) the annihilation of low-energy $\bar{\Xi}^+$ on hydrogen proceeds largely via the reaction $\bar{\Xi}^+ + p \to K^+ + K^+$, and (b) the elastic scattering of low-energy $\bar{\Xi}^+$ on hydrogen is largely a diffraction effect. In this case, for sufficiently large values of $\mathbf{k}$ such that S and D waves are the dominant partial waves in the two-K^+ system (Bose statistics rules out odd orbital states), the coefficient of the $\sin 2\varphi$ term in the distribution would be propor-

[1] G. F. Chew, University of California Radiation Laboratory Report, UCRL-9028 (unpublished).
[2] S. Barshay, Phys. Rev. **109**, 2160 (1958); **110**, 743 (1958).
[3] C. Goebel, Phys. Rev. Letters **1**, 337 (1958).
[4] G. F. Chew and F. E. Low, Phys. Rev. **113**, 1640 (1959).
[5] S. Barshay, Phys. Rev. **111**, 1651 (1958).
[6] F. E. Low and D. H. Frisch (private communications). Professor Low has emphasized that there is a considerable difficulty in reconciling the conversion of the photons into pairs in a high-Z material and the subsequent distortion of the plane of the pair by multiple scattering effects.

tional to $\mathrm{Im}\, e^{i(\delta_D-\delta_S)}$, where δ_S and δ_D are the S- and D-wave phase shifts, respectively, for the elastic scattering of a pair of K^+ mesons at a relative momentum $|\mathbf{q}|$. For values of $|\mathbf{q}|$ below the threshold for the production of pion in a K^+-K^+ collision, this coefficient will be proportional to $\sin(\delta_D-\delta_S)$, with δ_S and δ_D real. Conversely, under assumption (b) alone, the absence of all meson-meson interactions among pions and K mesons could be inferred from the vanishing of a correlation term proportional to the $\sin 2\varphi$.

We now give the derivation of these results. Consider the case of even relative cascade-nucleon parity. The derivation for the odd case is identical, with a small modification in the final formula that will be noted in conclusion. The transition operator for reaction (3) may now be written as follows:

$$t=\alpha\mathbf{q}\times\mathbf{e}\cdot\mathbf{k}+\beta\boldsymbol{\sigma}\cdot\mathbf{e}, \tag{4}$$

where α and β are complex functions of $|\mathbf{q}|^2$, $|\mathbf{k}|^2$, and $\mathbf{q}\cdot\mathbf{k}$, and $\boldsymbol{\sigma}$ is the spin operator. For simplicity and explicitness, let us consider a range of values of $|\mathbf{k}|$ from the maximum value, $|\mathbf{k}_{\max}|$, to a lower limit, $|\mathbf{k}_{\min}|$, corresponding to a value of $|\mathbf{q}|$ just below the threshold for pion production in a K^+-K^+ collision; we then approximate the two-K^+ interaction by real S- and D-wave phase shifts. Redefining α and β, we rewrite the transition operator

$$t=\alpha\mathbf{q}\cdot\mathbf{k}\mathbf{q}\times\mathbf{e}\cdot\mathbf{k}+\beta\boldsymbol{\sigma}\cdot\mathbf{e}. \tag{5}$$

The first and second terms correspond to D- and S-wave production, respectively, of the two-K^+ system. We may now expand the state vector for the photon in terms of a complete set of photon polarization states, with coefficients given by Eq. (5):

$$|\gamma\rangle=(i\alpha\mathbf{q}\cdot\mathbf{k}q_+-\beta\sigma_-)|e_+\rangle+(i\alpha\mathbf{q}\cdot\mathbf{k}q_-+\beta\sigma_+)|e_-\rangle. \tag{6}$$

The $|e_+\rangle$ and $|e_-\rangle$ denote states of right and left circular polarization, respectively; $q_\pm=(q_1\pm iq_2)/\sqrt{2}$; $\sigma_\pm=(\sigma_1\pm i\sigma_2)/\sqrt{2}$. The incident Ξ^+ is described by a density matrix given by

$$\rho=(1+P\boldsymbol{\sigma}\cdot\mathbf{n}), \tag{7}$$

where P is the polarization of the particle along $\mathbf{n}$. We assume that the cascade particle has spin $\frac{1}{2}$. The density matrix for the final state is given by

$$\rho_f=|\gamma\rangle(1+P\boldsymbol{\sigma}\cdot\mathbf{n})\langle\gamma|=\tfrac{1}{2}(1+\boldsymbol{\tau}\cdot\boldsymbol{\lambda}). \tag{8}$$

In this equation, the components of $\boldsymbol{\tau}$ are the Pauli matrices, and $\boldsymbol{\lambda}$ is the Stokes vector for the photon whose components are given by the following equations:

$$\lambda_+=\tfrac{1}{2}(\lambda_1+i\lambda_2)=(i\alpha\mathbf{q}\cdot\mathbf{k}q_-+\beta\sigma_+)\times(1+P\boldsymbol{\sigma}\cdot\mathbf{n})(-i\alpha^*\mathbf{q}\cdot\mathbf{k}q_--\beta^*\sigma_+),$$

$$\lambda_-=\tfrac{1}{2}(\lambda_1-i\lambda_2)=(i\alpha\mathbf{q}\cdot\mathbf{k}q_+-\beta\sigma_-)\times(1+P\boldsymbol{\sigma}\cdot\mathbf{n})(-i\alpha^*\mathbf{q}\cdot\mathbf{k}q_++\beta^*\sigma_-),$$

$$\lambda_3=(i\alpha\mathbf{q}\cdot\mathbf{k}q_+-\beta\sigma_-)(1+P\boldsymbol{\sigma}\cdot\mathbf{n})(-i\alpha^*\mathbf{q}\cdot\mathbf{k}q_--\beta^*\sigma_+)-(i\alpha\mathbf{q}\cdot\mathbf{k}q_-+\beta\sigma_+)(1+P\boldsymbol{\sigma}\cdot\mathbf{n})(-i\alpha^*\mathbf{q}\cdot\mathbf{k}q_++\beta^*\sigma_-), \tag{9}$$

where the * denotes complex conjugation.

The result of an experiment which determines $\mathbf{q}$ and also $\mathbf{k}$, via the materialization of the photon into an electron-positron pair, will be given by

$$R=\mathrm{tr}\rho_f A, \tag{10}$$

where $A=r(1+\boldsymbol{\tau}\cdot\mathbf{u})$ is the density matrix which describes the pair formation; r is therefore related to the rate for this process, and $\mathbf{u}$ is a vector with components

$$u_1=|\mathbf{u}|\cos 2\varphi,$$
$$u_2=|\mathbf{u}|\sin 2\varphi,$$
$$u_3=0. \tag{11}$$

Here, φ is the angle between the plane of the pair and the plane defined by its normal $\mathbf{q}\times\mathbf{k}$; $|\mathbf{u}|\leq 1$ is again a quantity determined by the mechanism of pair formation. We may define our coordinate system with $\mathbf{k}$ as the 3-axis, $\mathbf{q}\times\mathbf{k}$ as the 1-axis, and $\mathbf{k}\times(\mathbf{q}\times\mathbf{k})$ as the 2-axis. A straightforward evaluation of Eq. (10), with the trace being taken over both $\boldsymbol{\sigma}$ and $\boldsymbol{\tau}$, then gives

$$R=2r[1+|\mathbf{u}|\{-|\alpha|^2|\mathbf{q}|^4|\mathbf{k}|^2\tfrac{1}{4}\sin^2 2\theta-P|\mathbf{q}|^2|\mathbf{k}|n_1\sin 2\theta\,\mathrm{Re}\alpha^*\beta\}\cos 2\varphi+|\mathbf{u}|\{-P|\mathbf{q}|^2|\mathbf{k}|n_2\sin 2\theta\}\,\mathrm{Re}\alpha^*\beta\sin 2\varphi], \tag{12}$$

where n_1 and n_2 are the 1 and 2 components, respectively, of the vector $\mathbf{n}$, and θ is the polar angle between $\mathbf{q}$ and $\mathbf{k}$. This is the essential result of the calculation. We now supplement the result with the observation that under the conditions (a) and (b) enumerated previously, for values of $\mathbf{k}$ with $|\mathbf{k}_{\min}|<|\mathbf{k}|\leq|\mathbf{k}_{\max}|$, time-reversal invariance requires that

$$\mathrm{Re}\alpha^*\beta=|\alpha||\beta|\sin(\delta_D-\delta_S), \tag{13}$$

where $|\alpha|$ and $|\beta|$ are the absolute values of the production amplitudes into D and S states, respectively, for the two-K^+ system, and δ_D and δ_S are real phase shifts for elastic K^+-K^+ scattering at a relative momentum $|\mathbf{q}|$.

In the case of even relative cascade-nucleon parity, the transition operator corresponding to Eq. (5) is given by

$$t=\bar{\alpha}\mathbf{q}\cdot\mathbf{k}\mathbf{q}\cdot\mathbf{e}+\bar{\beta}\boldsymbol{\sigma}\times\mathbf{e}\cdot\mathbf{k}. \tag{14}$$

The formula corresponding to equation (12) is simply obtained by letting $\alpha\rightarrow\bar{\alpha}$, $\beta\rightarrow\bar{\beta}$, and by multiplying the coefficient of the $\cos 2\varphi$ term by -1. We have here a means of determining the relative cascade-nucleon parity, provided that the interference term between α ($\bar{\alpha}$) and β ($\bar{\beta}$) is negligible and provided that the D wave production amplitude, α ($\bar{\alpha}$), is nonzero. For then the coefficient of $\cos 2\varphi$ is negative definite in the case of odd relative parity and positive definite in the case of even relative parity. This is reminiscent of the situa-

tion in the analysis of the correlation between the planes of the Dalitz pairs in π^0 decay.[7]

III. CONCLUSIONS

Under the conditions (a) and (b) enumerated above, the observation of a nonvanishing term proportional to $\sin 2\varphi$ in the distribution in the angle between the plane of the pair and the plane defined by its normal, $\mathbf{q}\times\mathbf{k}$, would indicate a nonvanishing interaction between a pair of K^+ mesons. Under condition (b) alone (which is perhaps the more likely of the two conditions to be fulfilled in reality), the absence of any correlation proportional to $\sin 2\varphi$ would imply that all meson-meson interactions were of negligible influence.[7a] This follows from the fact that time-reversal invariance requires $\mathrm{Re}\alpha^*\beta=0$ in the event that the low-energy Ξ^+-p elastic scattering is largely a diffraction effect (i.e., vanishing of the real part of the phase shifts) and that there are negligible meson-meson interactions in a system of pions and K mesons. This conclusion would not hold in the unlikely circumstance that only a single partial wave were produced in the two-K^+ system, for all values of $|\mathbf{q}|$, or in the circumstance that many partial waves conspired to give an accidental cancellation in the interference between the amplitudes α and β in Eq. (4), again for all values of $|\mathbf{q}|$.

Finally, we note that reaction (3) involves four positively charged particles, two with magnetic moments, and the possibility of the emission of a very energetic photon. All of these may contribute to an increased probability for this radiative process.

IV. ACKNOWLEDGMENT

The analysis in this note was stimulated by a remark made to the author by Professor Francis Low. Professor Low has applied a similar analysis to the process

$$K^+\to\pi^++\pi^0+\gamma, \tag{15}$$

with subsequent materialization of the photon via creation of a pair. For this process, the presence or absence of a correlation proportional to $\sin 2\varphi$ could, in principle, clearly establish the existence or nonexistence, respectively, of a real pion-pion phase shift in the state of orbital angular momentum unity (but not in the S wave, owing to the absence of a zero-zero transition). I would like to thank Professor Low for his most helpful remarks.

[7] R. Plano *et al*, Phys. Rev. Letters **3**, 525 (1959).

[7a] Under a similar condition, this statement could also be made with reference to a study of the reactions $\bar{p}+p\to 2\pi+\gamma$ and $\bar{p}+p\to K+\bar{K}+\gamma$.

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High-Energy Electron-Electron Scattering*

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The radiative corrections to the electron-electron scattering to order α^3 are calculated for (1) the colliding beam experiment and (2) the experiment in which the target electron is at rest initially. The contributions from high-energy real photons are included. The two-photon exchange diagrams are found to give only negligible contributions to the cross sections after infrared cancellation. The effect due to the possible breakdown of quantum electrodynamics is discussed. A preliminary study on the electron-positron colliding beam experiment involving various interactions is made. The vacuum polarizations involving heavier particles than an electron pair in the closed loop are investigated.

I. INTRODUCTION

MANY experiments[1] have been suggested to test whether the electron has any finite size or if quantum electrodynamics (QED) is valid at small distances, say at 10^{-14} cm. Among them the interactions $e^-+e^-\to e^-+e^-$, $e^++e^-\to e^++e^-$, and $\gamma+e^-\to\gamma+e^-$ are pure quantum electrodynamical[2] or, in other words, they do not involve the structures of other particles whose effects are often difficult to distinguish from the effect due to the breakdown of QED at small distances. In this paper we are primarily concerned with the evaluation of cross sections for two specific experiments on electron-electron scattering by using the standard technique of QED to order α^3. Any significant deviation of the observed cross sections from the present calculation must be attributed to the breakdown of QED at small distances. The effect due to possible breakdown of QED at small distances is discussed in Sec. IV. The two experiments which we will proceed to discuss are as follows:

Experiment I. Electron-electron colliding beam experiment with two intersecting 500-Mev electron beams. The detectors for the scattered electrons are

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[1] S. D. Drell, Ann. Phys. **4**, 75 (1958).

[2] Vacuum polarizations due to heavier particles than electrons are discussed in the Appendix. Their contributions to the cross sections are found to be negligible.