Nucleon-Nucleon Spin-Orbit Interaction and the Repulsive Core*

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Recent proposals to explain the phenomenological repulsive core and spin-orbit interaction in nucleon-nucleon scattering in terms of a vector meson field are discussed. Estimates of the mass of the vector meson made on the basis of the Bryan potential may need some revision on account of insufficiently studied possibilities of modifying that potential. Estimates of interaction constants on the basis of the Signell-Zinn-Marshak potential and the replacement of a two-body relativistic problem by a one-body problem do not appear applicable. Estimates based on a covariant matrix element but neglecting wave function distortion in the analysis of 300-Mev data are shown to be quite uncertain. Accordingly the evidence for a vector meson mass of $3m_\pi$ or $4m_\pi$ also appears to have little weight.

Masses $3m_\pi$ and $4m_\pi$ are shown to lead to central-field potential energy tails which extend into the one-pion-exchange potential region and appear therefore to be improbably large. They also lead to repulsive cores which do not fit in with the usual phenomenological hard cores as naturally as the larger heavy-photon masses. Brief mention is made of possible means of detecting the vector meson and of the effects of its finite mean life.

I. INTRODUCTION

REQUIREMENTS of relativistic invariance of predictions following from a Hamiltonian description of a two-body system are known[1] to require in certain cases the presence of spin-orbit interaction terms. While the derivations referred to have a relationship to the more familiar considerations regarding a Dirac particle in a four-vector or scalar field, they are distinct from these one-body theories, being concerned with an essentially different problem. Attempts at applying the two-particle $\mathbf{L}\cdot\mathbf{S}$ terms, derived along lines just mentioned, to the calculation of the fine structure of nuclear levels have been made.[2,3] High-energy *p-p* scattering has indicated,[4,5] although it has not proved, that $\mathbf{L}\cdot\mathbf{S}$-type potentials are probably present. Support for the presence of a large V_{LS} in the interaction energy has been obtained in the work of Signell and Marshak,[6] Signell, Zinn, and Marshak,[6] and Gammel and Thaler.[7] In the work of Signell and Marshak[6] the range parameter of V_{LS} for the asymptotic dependence at large r was unreasonably short.[8] The shortened range used by Signell, Zinn, and Marshak[6] is free from the theoretical objections[8] regarding the asymptotic behavior at large r. While the Signell, Zinn, and Marshak note might produce a feeling of optimism regarding the applicability of the general Chew-Low approach when supplemented by a phenomenologically postulated spin-orbit interaction having for its dominant spatial decay factor $\exp(-2\mu r)$, it may be mentioned that the question of the effect of changing the factor $\exp(-\mu r)$ to $\exp(-2\mu r)$ has been independently investigated by Hull *et al.*[9] with the conclusion that the fits to data are far from satisfactory.

Although the employment of the Chew-Gartenhaus potential by Marshak and Signell appeared to be preferable from a theoretical standpoint to the more purely phenomenological approach of Gammel and Thaler,[7] the fits to polarization obtained by the latter have been much the better. The spin-orbit potential used by Gammel and Thaler is located just outside the repulsive core and decreases much faster with r than $\exp(-2\mu r)$. It has been determined by empirical adjustment without a theoretical bias. Polarization has more to do with the spin-orbit potential than the differential cross section. The indications are therefore that V_{LS} is more correctly determined by the fits of Gammel and Thaler than by those of Signell, Zinn, and Marshak and that if the data are properly represented by means of potentials, the V_{LS} has a pronouncedly short-range character. This tentative conclusion is further supported by the marked success of Bryan[10] in fitting *p-p* data by a potential with a very short-range V_{LS} which is again located just outside the hard-core phenomenologic potential. This success reactivated verbal proposals made by the writer at the time of early work on K mesons to apply the general possibilities[1,2] of obtaining a V_{LS} as a correction term needed to restore relativistic covariance of a two-body central interaction. An inter-

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[1] G. Breit, Phys. Rev. **51**, 248 (1937); **51**, 778 (1937); **53**, 153 (1938).

[2] See the first part of reference 1; G. Breit and J. R. Stehn, Phys. Rev. **53**, 459 (1938).

[3] Cabell A. Pearse, Phys. Rev. **106**, 545 (1957).

[4] L. Wolfenstein, Bull. Am. Phys. Soc. **1**, 284 (1956); Phys. Rev. **76**, 541 (1949); **82**, 308 (1951). In the first reference an analysis of the scattering amplitude in terms of components in directions related to the spin has been made. It suggests the existence of an $\mathbf{L}\cdot\mathbf{S}$ potential although, as emphasized by Wolfenstein, it does not prove its existence.

[5] G. Breit, Phys. Rev. **106**, 314 (1957); see Appendix which contains a proof that the tensor interaction does not produce first-order effects on the polarization while the spin-orbit does. An independent verification has been given by M. S. Wertheim in his Yale dissertation, 1956 (unpublished).

[6] P. S. Signell and R. E. Marshak, Phys. Rev. **109**, 1229 (1958); P. S. Signell, R. Zinn, and R. E. Marshak, Phys. Rev. Letters **1**, 416 (1958).

[7] J. L. Gammel and R. M. Thaler, Phys. Rev. **107**, 291, 1337 (1957).

[8] M. L. Goldberger, Y. Nambu, and R. Oehme, Ann. Phys. **2**, 226 (1957); G. Breit, Phys. Rev. **111**, 652 (1958).

[9] M. H. Hull, K. D. Pyatt, C. R. Fischer, and G. Breit, Phys. Rev. Letters **2**, 504 (1959).

[10] R. A. Bryan, Bull. Am. Phys. Soc. **5**, 35 (1960).

action through a vector field giving rise to a repulsion yields a V_{LS} with the required sign to agree with empirical indications of nucleon-nucleon scattering data and also with those of the nuclear shell model. Since the evidence regarding the spin of the K meson has soon settled down against its having a spin 1, this proposal was not published at the time. The success of Bryan's work and especially the relatively good agreement of the phase parameters obtained by him with those obtained in a phenomenologic search at Yale[11] have recently prompted the writer to publish a short note[12] in which it is proposed to regard the repulsive core and the spin-orbit potentials as originating in a nucleon-nucleon interaction through a vector field. The phenomena of nucleon-antinucleon scattering and of form factors in electron-nucleon scattering appeared to give some support to the qualitative view proposed. A fit to the Bryan potential indicated a mass of the vector meson between $9m_\pi$ and $12m_\pi$, where m_π is the pion mass. Shortly before presenting these considerations at a meeting[12] and after the preparation of the paper in press,[12] the writer became aware of a preprint of a paper by Sakurai[13] containing a related proposal. These estimates of Sakurai are based on the Signell-Zinn-Marshak potential and make use therefore of what, according to evidence mentioned,[7–10] appears to be a V_{LS} with a too large range constant. His estimates are furthermore based on a replacement of the actual two-body problem by a supposedly equivalent one-body problem. The result of this replacement is a discrepancy of a factor 3 between the correct and the approximate formula. While it is correct to replace a two-body by a single-body Hamiltonian through the usual procedure of the separation of center of mass, the relativistic correction terms such as V_{LS} are not in general derivable from the single-particle Hamiltonian by the same procedure as for a single particle.

In a second note Sakurai[14] performs a calculation in which the vector field interaction is treated covariantly in first order employing undistorted plane waves in the calculation of the transition matrix element. The factor 3 is now correctly obtained and the discrepancy with the earlier estimate is explained in terms of the magnetic interactions familiar from the case of two electrons.[15] While the two-electron interaction shows that the factor 3 must be applied to the "equivalent" one-body result in order to obtain the correct one, the explanation cannot be regarded as complete for two reasons. In the first place the case under discussion is not the electromagnetic one. Another lack of completeness lies in the employment of undistorted wave functions in the presence of strong distorting effects caused by the vector meson and the pion interactions. The derivation in the first two references listed in reference 1 is free of this objection, the V_{LS} terms being obtained there as correction terms in the Hamiltonian, independently of whether the central-field potential is considered to be valid only in first order or not.

In his second note Sakurai employs p-p data at 310 Mev and on the basis of Wolfenstein's representation of the scattering matrix, denoted by Wolfenstein as M, compares his first-order formula with the phenomenological phase-parameter analysis of Stapp, Ypsilantis, and Metropolis[16] as modified by Cziffra, MacGregor, Moravcsik, and Stapp[17] and by MacGregor, Moravcsik, and Stapp.[17] He arrives on this basis at the conclusion that the mass of the heavy meson must be close to $3m_\pi$. This value is in agreement with the speculations[18] regarding pion-pion interactions and the existence of a three-pion bound state which have been advanced in connection with the electromagnetic form factor derived from electron-nucleon scattering experiments. It would indeed be very nice if these considerations could be related to those of nucleon-nucleon scattering. One of the objects of the present note is to locate the origin of the difference in the two estimates of the vector meson mass. Another object is to discuss the question of the mass from the viewpoint of information regarding the large-distance central-field nucleon-nucleon potential. This discussion shows the importance of obtaining better agreement between theoretical calculations of the central-field potential tail. Some attention is also given to modifications in the theory which may arise on account of the finite mean life of the heavy meson.

II. SEMIQUANTITATIVE CONSIDERATIONS

The question arises as to the reason for the large difference between the mass of the vector meson derived[12] on the basis of Bryan's phenomenological

[11] G. Breit, *Proceedings of the International Conference on Nuclear Forces and the Few Nucleon Problem, University College, London, July 8–11, 1959* [Pergamon Press, New York (to be published)]. This report was based on work in collaboration with M. H. Hull, Jr., K. D. Pyatt, Jr., C. R. Fischer, K. Lassila and T. Degges; M. H. Hull, G. Breit, K. Lassila, K. D. Pyatt, and H. Ruppel, Bull. Am. Phys. Soc. **5**, 268 (1960).

[12] G. Breit, Proc. Natl. Acad. Sci. (U. S.) **46**, 746 (1960). A preliminary account of the work has been read at the Annual Meeting of the National Academy of Sciences, April 27, 1960. In the fifth line after Eq. (2) of the paper quoted the value of $q^2\kappa$ for $\kappa=12m_\pi c/\hbar$ should have been 2.85×10^6 Mev; in line 7 of the paragraph following that of Eq. (2) the approximate values of the core potential should have been 1180, 295, 20, 1.5 Mev at $x=0.5$, 0.6, 0.8, 1.0. The writer is grateful to Mr. K. Lassila for locating the error which led to the previously incorrectly given values which did not fit the relative magnitudes of V_{LS} and V_{core} for $y/x=12$ and $y/x=9$. These corrections do not change the conclusions of the paper quoted.

[13] J. J. Sakurai, Ann. Phys. (to be published).

[14] J. J. Sakurai, Phys. Rev. (to be published). The writer is indebted to Dr. Sakurai for a prepublication copy of the manuscript.

[15] G. Breit, Phys. Rev. **34**, 553 (1929).

[16] H. P. Stapp, T. J. Ypsilantis, and N. Metropolis, Phys. Rev. **105**, 302 (1957).

[17] P. Cziffra, M. H. MacGregor, M. J. Moravcsik, and H. P. Stapp, Phys. Rev. **114**, 880 (1959); M. H. MacGregor, M. J. Moravcsik, and H. P. Stapp, Phys. Rev. **116**, 1248 (1959).

[18] G. F. Chew, R. Karplus, S. Gasiorowicz, and F. Zachariasen, Phys. Rev. **110**, 265 (1958); W. R. Frazer and J. R. Fulco, Phys. Rev. Letters **2**, 365 (1959); Geoffrey F. Chew, Phys. Rev. Letters **4**, 142 (1960).

potential and that obtained by Sakurai. It is possible that Bryan's potential is only a phenomenological device and that too much significance should not be attached to it. It is even conceivable that the trend towards a short range in the phenomenological V_{LS} which has occurred in the work of Gammel and Thaler and that of Bryan is in some sense fortuitous. It is also possible that the actual interaction cannot be properly described by a static potential. In the latter case all of the discussed arguments may be meaningless and all speculations regarding the vector meson may be pointless. For the sake of definiteness it will be assumed, however, that the potential picture has sufficient sense to make qualitative considerations possible. On this view, the obvious difference between the two estimates of the mass of the heavy photon is that in the Gammel-Thaler and Bryan calculations the distortion of the wave function by the potential used is taken into account while in Sakurai's estimate this distortion is neglected. It is necessary, therefore, to discuss the legitimacy of the assumption that the wave function is sufficiently undistorted.

In such a discussion it is useful to remember that the exact phase shift is related to the wave functions by the formula

$$\sin\delta_L = -(k/E)\int_0^\infty V\mathfrak{F}_L F_L dr, \tag{1}$$

where V is the potential, E the energy in the center-of-mass system and $\mathfrak{F}/r$, F/r are, respectively, the distorted and undistorted wave functions normalized so that asymptotically at large r, $F_L \sim \sin(kr - L\pi/2)$, $\mathfrak{F}_L \sim \sin(kr + \delta_L - L\pi/2)$, and $k/(2\pi)$ is the wave number. Here $L\hbar$ is the orbital angular momentum and relativistic corrections are neglected. The undistorted wave function approximation is obtained from (1) by replacing $\sin\delta$ by δ and $\mathfrak{F}_L$ by F_L. The first replacement is relatively innocent in many cases and is easily corrected. The second has to do with the effect of V on $\mathfrak{F}_L$. Eq. (1) is directly applicable only to the cases of uncoupled phase shifts and will thus not be used for phase parameters of the ${}^3P_2 - {}^3F_2$ coupled state and similar cases.

If the vector meson has a mass $3m_\pi$, then the range constant which enters V_{LS} is $\hbar/(3m_\pi c) \cong 1.43/3 = 0.48$ f, $x = \frac{1}{3}$, a value close to that of the phenomenological core radius. In Bryan's work the core radius of the central potential in triplet odd states is $x = 0.38$. From this viewpoint alone the employment of the undistorted wave function approximation does not appear safe because inside the repulsive core $\mathfrak{F}_L = 0$. In Fig. 1 are shown, for $E_{\mathrm{lab}} = 300$ Mev, graphs for F_L, $(1+y)e^{-y}/y^3$, and $F_L^2(1+y)e^{-y}/y^3$, with $y = 3x$. It is seen that roughly 45% of the undistorted wave function approximation integrand falls in $x < 0.4$. This error would by itself be not of a major character because a 45% error could possibly be compensated for by an adjustment of the interaction constant. Since the error is connected with the vanishing

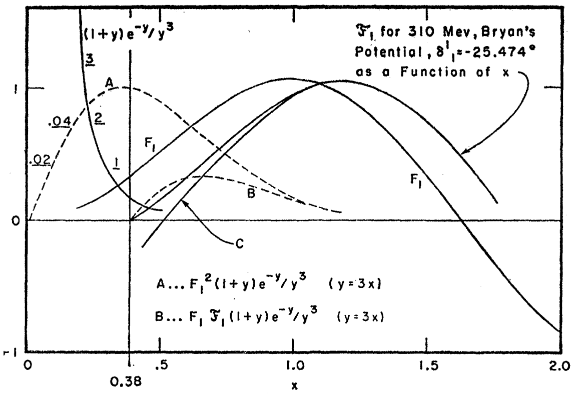

FIG. 1. Comparison of distorted and undistorted wave functions and of contributions of V_{LS} at 300 Mev for ${}^3P_1$ state. The regular (undistorted) wave function is shown as F_1, the distorted function for the Bryan potential at 310 Mev as $\mathfrak{F}_1$, the distorted function for phase shift $-26.93°$ but without potential effects as $F_1 \cos\delta_1{}^1 + G_1 \sin\delta_1{}^1$ as C. The similarity of the latter two curves is indicative of approximate independence of estimate on details of calculation. Values of $F_1{}^2(1+y)e^{-y}/y^3$ and of $F_1\mathfrak{F}_1(1+y)e^{-y}/y^3$ for $m_{hp}/m_\pi = 3$ are shown in curves A and B, respectively. Areas under these curves give the effect on the phase shift without and with partial account of wave distortion.

of $\mathfrak{F}_L$ inside the core, it implies, however, a much larger unreliability of the undistorted wave function approximation. In fact $\mathfrak{F}_L$ must have a node at the core radius $x \cong 0.4$ and hence the relatively large contributions in the region $0.4 < x < 0.7$ are seriously cut down by the employment of the distorted wave function. There is, therefore, no apparent reason for trusting the undistorted wave approximation in this case. The curve marked $F_1 \cos\delta_1{}^1 + G_1 \sin\delta_1{}^1$ shows what $\mathfrak{F}_1$ would be if there were no potential in the region $x > 0.4$ and if the phase shift for ${}^3P_1$ had approximately the value required by empirical fitting of data.[11] Its node is seen to occur not far from $x = 0.4$, although it is not claimed that it is a good approximation to $\mathfrak{F}_1$. It, nevertheless, illustrates the qualitative difference between the distorted and undistorted waves. In Fig. 1 there is shown also the exact function $\mathfrak{F}$ for the ${}^3P_1$ state calculated for the Bryan potential and a bombarding energy of 310 Mev, as well as the quantity $(F_1\mathfrak{F}_1)(1+y)e^{-y}/y^3$. Comparing the latter with $F_1{}^2(1+y)e^{-y}/y^3$, a reduction factor of $\sim\frac{1}{4}$ is estimated in the absolute value of the right-hand side of Eq. (1) as the result of the replacement of the approximate by the exact integrand. The reduction factor is caused mainly by the removal of the region in the core and by the depression of the integrand at the core radius. It is thus not likely to be insensitive to the details of the calculation. Its approximate magnitude depends, however, on the employment of a reasonably hard core. If the core were eventually shown to be truly soft, the undistorted wave function approximation could conceivably be better than indicated by the above estimate. The present estimate of the effect of wave distortion is not strongly dependent on the employment of Bryan's potential as may be seen by

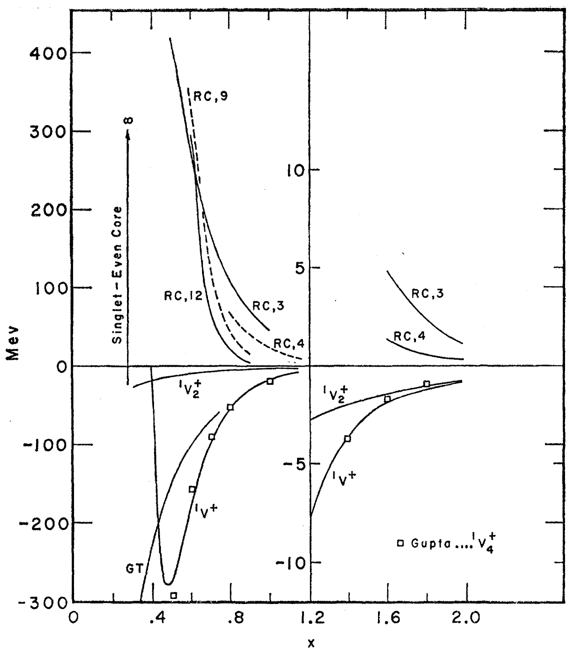

FIG. 2. Comparison of singlet-even potentials. Curves marked $^1V^+$ and $^1V_2^+$ are, respectively, Bryan's total potential and the one-pion exchange potential. Curves marked RC are repulsive cores calculated on the vector meson hypothesis. The number following RC is the vector-meson mass expressed in terms of the pion mass as a unit. The Gammel-Thaler potential is shown in curve marked GT. Values of Gupta's fourth-order potential are plotted as points marked by large squares. Different ordinate scales are used for the left and right parts of the figure. The curves RC, 3 and RC, 4 correspond to Sakurai's values of the interaction constant without taking into account the reduction of the first-order matrix element by about a factor 4 caused by wave distortion. On account of this reduction the ordinate of RC, 3 should be multiplied by about 4.

comparing $\mathfrak{F}_1$ with $F_1 \cos\delta_1^1 + G_1 \sin\delta_1^1$. The latter is determined by the phenomenological phase shift search alone. Its employment involves the assumption that the *shape* of $\mathfrak{F}_1$ is mainly determined by δ_1^1 and that the position of the node of $\mathfrak{F}_1$ is the main origin of the phase shift. It should also be mentioned that had one used $\mathfrak{F}_1^2$ in place of $F_1\mathfrak{F}_1$, an even stronger additional reduction of ~ 1.5 would have been obtained. A reason for employing $\mathfrak{F}_1^2$ is that the change from F_1 to $\mathfrak{F}_1$ includes effects of other potentials than V_{LS}. The distortion effect has, therefore, been estimated conservatively.

The phenomenological potentials in the region $0.4 < x < 1.0$ are large and from this viewpoint there is again no apparent reason for expecting the undistorted wave approximation to be valid. Thus, for example, for $x = 0.4$ Bryan's potential for $^3P_0{}^1$, $^3P_1{}^1$, $^3P_2{}^2$ has the values 1422, -790, -1225 Mev, respectively, and at $x = 0.6$ the corresponding numbers are 412, 167, -251 Mev. For $E_{\text{lab}} = 300$ Mev, i.e., $E = 150$ Mev these values are larger than the incident energy. It will be noted that the potentials have different signs and that it is not possible to claim that the effect of repulsion in the core is compensated in all cases by an attraction outside of it. Similarly Fig. 5 of the paper by Signell and Marshak[6] shows large values for the central part of the triplet-odd Gartenhaus potential, although the values for the Gammel-Thaler potential are small. On the other hand, Fig. 7 of the same paper shows large values of V_{LS} in all cases. If one is to believe the phenomenological potentials at all, the assumption of negligible wave function distortion implicit in Sakurai's improved estimate must be considered, therefore, unjustifiable by ordinary considerations. An additional although possibly not major source of error is the assumption that the tensor potential produces only first-order effects. Additional information can be obtained through the comparison of the repulsive core derived from the adjustment of parameters in V_{LS}. In Fig. 2 are plotted graphs showing the repulsive cores obtained with adjustments of parameters to represent V_{LS}. In each case

$$V_{\text{core}} = (q^2/r)e^{-\kappa r} = q^2\kappa e^{-y}/y, \tag{2}$$

$$y = \kappa r, \quad \kappa = m_{hp}c/\hbar. \tag{2.1}$$

Here m_{hp} is the mass of the heavy photon. The graphs for $y = 12x$ and $y = 9x$ have been obtained from the adjustments reported on earlier.[12] The graphs for $y = 3x$ and $4x$ have been obtained employing Sakurai's values[14] which in his notation are $f_v^2/(4\pi) = 7$ and 10, respectively, for $y/x = 3$ and 4. The relation $q^2 = \hbar c(f_v^2/4\pi)$ was used in the translation to ordinary units. In the same figure are shown graphs of the central potential according to Gammel and Thaler and Bryan for singlet even states and in the right part of the figure there is shown for comparison the one-pion exchange part (OPEP) of the singlet even potential. The figure shows that the large values of m_{hp} fit in naturally with the repulsive-core picture, giving a steep increase at $x \cong 0.6$. This is especially so for the largest m_{hp}. The figure contains also a plot of the fourth-order potential calculated by Gupta.[19] Around $x \cong 0.7$ the superposition of the repulsive core and of Gupta's curve has clearly much in common with the phenomenologic potential. On the other hand, values of $y/x = 3$ and 4 do not fit in with a superposition-of-potentials picture in as natural a manner. The apparent agreement as just brought out is perhaps overstated. The fact that $V^{(4)}$ is so much larger than $V^{(2)}$ for $x \cong 0.5$ probably indicates that the $V^{(2n)}$ with $n > 2$ are far from negligible. Some support for this view is found in the fact that Gupta's $V^{(4)}$ is much too deep for Bryan's $^3V_c^-$. Nevertheless, Gupta's calculation does give values such that if the large attraction is modified by the steep repulsive potential at small r which follows for the large masses of the vector meson, a reasonable reproduction of the phenomenological potentials is obtained. It may be mentioned that the repulsive core for mass $3m_\pi$ but without correction for wave distortion combined with Gupta's $V^{(4)}$ brings about agreement at $x \cong 0.8$. In view of the omission of wave distortion and other $V^{(2n)}$ this agreement is probably accidental. For $1.6 < x < 2.0$ the repulsive core for $m_{hp} = 3m_\pi$ gives values larger than the OPEP and for

[19] S. N. Gupta, Phys. Rev. 117, 1146 (1960).

$m_{hp}=4m_\pi$ the values are comparable with the OPEP at $x=1.6$, i.e., $r=2.3$ f. This does not fit in naturally with the evidence[20] regarding the OPEP being the main contributor to the potential for $r>2.9$ f and very probably so for $r>1.6$ f$(x>1.2)$. For ${}^3V_c^-$ one has to compare OPEP values of 0.4, 0.3, 0.25 Mev at $x=1.6$, 1.8, 2.0, respectively, with 1.4, 0.6, 0.23 Mev for the repulsive core for $m_{hp}=4m_\pi$, which makes this as well as the lower value of m_{hp} even less likely. In these comparisons no allowance has been made for the omission in Sakurai's estimates of the modification of $\mathfrak{F}$ inside the phenomenological repulsive core region and the associated effect of the node of $\mathfrak{F}$ at the core radius. These modifications make it necessary to use a larger q^2 and hence an even larger repulsive potential of heavy vector meson origin.

The empirical evidence regarding the probable absence of significant contributions to the potential at larger distances except for the OPEP is in agreement with the meson-theoretic calculations of Sugawara and Okubo.[20] According to these the fourth-order potential is largely canceled at the larger distances by an additional effect arising from a unitary transformation of the wave function. While at the smaller distances there are questions regarding convergence which make the fourth-order calculations questionable, these questions should be less serious in the tail region. The combination of phenomenological and theoretical evidence appears to be, therefore, against the extension of the repulsive core into the tail region. There is some disagreement, however, among the meson-theoretic predictions regarding the fourth-order potential in the tail region. The absolute values of the fourth-order potential calculated by Gupta are appreciably larger in the region $1.6<x<2.0$ than those of Sugawara and Okubo. These values are seen in Fig. 2 to be more nearly such as to be compensated by the $4m_\pi$ repulsive core tail. It should be remembered, however, that the repulsive core tail does not include a correction for the effect of wave function distortion. According to estimates presented above this correction may amount to a factor $\simeq 4$ and the potential tail, therefore, still appears too strong. It should also be mentioned that Gupta's published calculation does not include velocity-dependent corrections. If these should prove negligible and if continuation of the empirical analysis[20] should indicate the absence of an appreciable tail in excess of the OPEP, then the conclusion would have to be the best mass for fitting the $x=1.8$ region is between 4 and perhaps $9m_\pi$.

III. DISCUSSION

While the arguments presented appear to be excluding the possibility of $y/x=3$ and to be making $y/x=4$ unlikely, they may not be regarded as conclusive. In the first place it is conceivable that the expansion of the potential in powers of the interaction constant converges so poorly that the large repulsive effects are balanced by additional as yet uncalculated attractions in the interval $0.8<x<1.2$. This possibility cannot be denied on purely logical grounds but it raises the natural question as to why the heavy-photon interaction should dominate over the pion-exchange interactions at $x<0.4$. The $1/r$ singularity is relatively mild and the objection to the exclusion of the smaller m_{hp} appears from this viewpoint to be a forced one.

Secondly the evidence[20] regarding the dominance of the OPEP at the larger distances is not specifically concerned with the central part of the potential. It has not been explicitly shown that disagreements with data will result from the employment of the relatively large tails following from the $m_{hp}/m_\pi=3$ and 4 possibilities. There is some unpublished evidence in work done in collaboration with Dr. M. H. Hull, Jr., and K. Lassila, however, that the mathematical form of the actual potential is approximately that of the OPEP. On the whole, the argument against the masses $3m_\pi$ and $4m_\pi$ is more one of plausibility than of absolute necessity.

It may also be mentioned that for ${}^3V_c^-$ Gupta's $V^{(4)}$ gives too much attraction in comparison with Bryan at $x=0.6$ and 0.8 by such amounts that the addition of the repulsive core for $m_{hp}/m_\pi=3$ or 4 is about right to bring about agreement with Bryan's ${}^3V_c^-$. This would appear to speak in favor of the smaller m_{hp}. It may also be relevant that Bryan's values of the phase shift for ${}^1S_0$ are not especially good so that ${}^1V^+$ is not a good test case. There is no apparent reason, however, for expecting convergence of the potential in powers of the interaction constant g^2 in the region $x\cong 0.7$ since $|V^{(4)}|\gg|V^{(2)}|$. It is probable, therefore, that this agreement is accidental. It appeared only fair to record it especially in view of the fact that most of the arguments in the present note are against the small masses.

The main attraction of the supposition that the masses are small lies in the possibility of identifying the heavy vector meson with a bound state of a three-pion system. It would be strange if this mass were as large as $3m_\pi$ because some energy must go into binding. If $m_{hp}<3m_\pi$ the relationship to the phenomenological potential becomes even less plausible.

The core radius used by Chew and Ball in their phenomenologic treatment of antinucleon-nucleon scattering is small and appears to fit in better with the larger masses of the vector meson than with the smaller ones. By itself this argument is, of course, not conclusive, there being probably many possibilities of fitting the available data.

On the other hand, the values of m_{hp} derived[12] on the basis of Bryan's phenomenological potential may well be too large. While the present evidence is that a V_{LS} with a short range fits scattering data better than one with a longer range, it is not known that a somewhat longer range than that obtained by Bryan could not be used with a readjustment of other parameters. If this

[20] G. Breit and M. H. Hull, Jr., Nuclear Physics **15**, 216 (1960); M. Sugawara and S. Okubo, Phys. Rev. **117**, 605 (1960); **117**, 611 (1960). The latter two references contain a meson-theoretic calculation indicating that at large distances the fourth-order potential may be canceled by another effect.

should prove to be the case, a search for the heavy meson would not require as large energies of antinucleons as previously estimated.[12] The energy which has to be supplied in the laboratory system in a nucleon-antinucleon collision in order to reach the threshold for the production of a heavy photon pair is

$$E-M_pc^2=2[(m_{hp}/M_p)^2-1]M_pc^2.$$

For $m_{hp}/m_\pi=12$, 10, 8, 6, 4 the values of this threshold energy are, respectively, 4.0, 2.2, 0.73, -0.41, -1.22 Bev. A nominal pion mass of $270m_e$ was used in these estimates. Only for the first three cases does energy have to be supplied to produce the reaction.

Since the heavy meson, if it exists, may have only temporary stability and since the most probable possibility is that it is neutral, a search for it in antinucleon-nucleon collisions may have to be made by observing pion multiplicities. At the threshold of the reaction $\mathfrak{N}+\mathfrak{N}_{\text{anti}}\rightarrow hp+hp'$, new pion multiplicities may appear and the yield-energy curves of those already present may show cusp phenomena. If the mass of the heavy photon is small enough to make the reaction exothermic, no threshold phenomena would be observed at any energy.

Another caution regarding a too literal interpretation appears appropriate in connection with the contribution of $x<0.37$ to the V_{LS} effects. If the phenomenological core is interpreted literally as a hard core, then the type of estimate made in connection with Fig. 1 is appropriate. If, however, the role of the core is only that of determining the location of the node of the radial function[8] and if inside it the potential is not strongly repulsive, then the part of curve A in Fig. 1 for $x<0.4$ need not be wholly omitted and the reduction factor may be not as marked as $\frac{1}{4}$. This consideration favors somewhat the smaller heavy-meson masses but is hard to carry through quantitatively. Qualitatively it fits in with the difficulty of accounting for the binding energy of H^3 encountered by Blatt and Derrick.[21] Although a number of causes have been previously cited[12] which might rectify the situation, a preponderance of attractions originating in pion effects at small x changing the interpretation of the phenomenological repulsive core with an elimination of the repulsive feature but retention of effects on $\mathfrak{F}$ of the type caused by having a node at the core radius would be of help in producing the empirically required increase in the tightness of binding of H^3.

A word of caution appears appropriate regarding the possibly too short mean life of the vector meson to make it directly useful in the proposed explanations. Thus, if Fig. 1 of Carruthers and Bethe[22] may be taken as an indication of the resonance width (100-Mev half-value breadth) of the compound state formed by the two pions, then the mean life $\tau\cong0.9\times10^{-13}$ cm/c. The characteristic length 0.9×10^{-13} cm is sufficiently small to make a picture in terms of an interaction with a single particle questionable. Similar information regarding the T$=0$, $J=1$ three-pion state appears unavailable, but if it is truly a bound state then the considerations of Nambu[23] would apply and the mean life would not be necessarily too short according to Nambu's estimates. For a heavy vector meson with a mass such as would follow from Bryan's potential there would also be virtual nucleon-antinucleon pair formation which would result in the disintegration processes mentioned by him. On account of the large interaction constant resulting from the empirical fits, the probability of virtual pair formation should be larger than in Nambu's case and the mean life should be shorter. It is, therefore, likely that heavy photons with large masses cannot be considered in the nuclear interaction problem in the simple manner attempted so far and that if they participate in nucleon-nucleon interactions their essential instability will have to be incorporated in the theory.[24] For this among other reasons the quantitative considerations presented in this note may not be regarded as more than a rough indication regarding an actual situation, the general features of which are being speculated on.

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[21] J. M. Blatt, *Proceedings of the International Conference on Nuclear Forces and the Few Nucleon Problem, University College, London, July, 1959* [Pergamon Press, New York (to be published)]. G. H. Derrick and J. M. Blatt, Nuclear Phys. 8, 310 (1958), and preprint on further work by the same authors.

[22] P. Carruthers and H. A. Bethe, Phys. Rev. Letters 4, 536 (1960).

[23] Y. Nambu, Phys. Rev. **106**, 1366 (1957). The writer regrets having overlooked in a previous publication[12] Nambu's suggestion of a heavy vector meson as a possible explanation of the repulsive core. As mentioned,[12] Jastrow has also suggested that the core may be caused by a special type of meson without going into the transformation properties of the meson field.

[24] The possibility that the phenomenologically introduced heavy photon might not be an elementary particle has been mentioned previously[12] and the possibility of virtual nucleon pair formation causing complications has been realized also. It has not been sufficiently emphasized, however, that the nucleon-nucleon force considerations are necessarily affected by such processes.

Note added in proof.—The Gammel-Thaler (GT) spin-orbit potential has been loosely stated above to have a short range. Since its mathematical form differs from that expected for the vector meson interaction, a more precise statement appears advisable. Comparing the GT and heavy photon spin-orbit potentials on the basis of equality of the logarithmic derivative of the potential with distance, a match is obtained at $x=0.71$ for $m_{hp}/m_\pi=3.6$ and for $x=0.86$ for $m_{hp}/m_\pi=4.0$. The values of x are reasonably close to the maximum of curve B of Fig. 1 and lend, therefore, some support to Sakurai's mass estimate. However, for $m_{hp}/m_\pi=4$ at $x=1.6$ the exponent of the GT potential type match requires increasing the GT exponent by 13% indicating a larger mass. A related circumstance not taken into account above is the presence of a short region next to the core in which Bryan's V_{LS} is constant. This makes his average range longer and the mass smaller. Conclusions about the mass thus depend on whether small or large x are used, the larger x favoring the larger masses as though the heavy photon charge had a volume distribution. Since unpublished work in collaboration with M. H. Hull, Jr. and K. Lassila gives preliminary indications of a shorter spin-orbit range than the GT and a longer one than Bryan's, the effective m_{hp} is probably between 4 pion masses and the larger estimates.