

Effective Charge of Heavy Ions in Various Media*

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Effective-charge data for heavy ions, as obtained from range-energy curves in oxygen, argon, aluminum, and nickel, are analyzed in terms of a Fermi-Thomas representation of the atom. It would appear that this gives a superior parametrization in terms of nuclear charge. In addition, it appears to properly account for the increase in charge observed for metallic as compared to gaseous absorbers.

INTRODUCTION

CONSIDERING the importance of charge state information in certain phases of low-energy nuclear physics, an attempt has been made to investigate apparent differences observed for a given ion species traversing a variety of media. Knowledge of the actual charge distribution as a function of energy is required if one wishes to integrate the incident flux in an experiment using heavy ions as projectiles. In this instance, a direct observation of charge-state population utilizing magnetic analysis is obviously the most desirable source of the information. Several measurements of this type have been reported for nitrogen,¹⁻³ oxygen,^{4,5} neon,⁴ and fission fragments.⁶ Similarly, knowledge of the charge state is necessary if one wishes to construct range-energy curves for unmeasured materials. Hopefully, the average charge required for these computations would be simply related to the directly observed equilibrium distributions. The reverse process, namely derivation of the effective charge from range measurements, has been carried out for several combinations.^{5,7} There has been some indication^{6,7} from both types of measurements that the charge state, and hence the rate of energy loss, is dependent upon the type of absorber. This effect is not unexpected and has been treated theoretically by Bohr and Lindhard⁸ for the case of fission fragments.⁹

MEASUREMENTS

Insofar as interaction with matter is concerned, the so-called effective charge of an ion can be computed from empirical range-energy relationships. It is defined by the ratio of the specific energy loss in a given medium

to that of a proton or an alpha particle of the same velocity in the same medium. This, of course, assumes these latter to be fully stripped at the velocities in question. This comparison has been carried out for ions of boron, carbon, nitrogen, oxygen, fluorine, and neon in nuclear emulsions,^{10,11} aluminum,⁵ nickel, and oxygen.⁷ Further, in connection with other work still in progress, the authors have constructed a range curve for fluorine ions in argon. These last data have been obtained and treated in the same fashion as in reference 7. Comparison in this case was made to published range curves for protons in argon.¹²

The interconversion of this type of data with relative population measurements is, unfortunately, not necessarily simple. Since the specific energy loss depends upon the square of the ionic charge, the shape of the equilibrium distribution must be taken into account in evaluating the average. This, however, considers only the interactions between the incident ion and the electrons of the absorber. Conceivably, other mechanisms for energy loss may also be involved. For heavy ions at low velocities, appreciable contributions may be expected from nuclear scattering. Any such additional factors will serve to increase the apparent value of the effective charge. In the range data for aluminum, nickel, oxygen, and argon absorbers it is felt that geometrical considerations rule out any significant deviations due to such scattering. The same obviously is not true for the nuclear emulsion data, however.

A direct comparison of the two types of charge measurements is available in three instances. Papineau¹³ has constructed range curves for nuclear emulsions on the basis of empirical charge-state data. Comparison of these curves to experiment would seem to indicate that the assumptions of equivalence may not be too far in error. Northcliffe⁶ has shown that for oxygen ions in aluminum, in the energy region covered, the effective charge can be predicted to within experimental error by the results of a direct population measurement. Finally, the effective charge for nitrogen ions in nickel may be compared to the average charge as determined by

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¹ H. L. Reynolds, D. W. Scott, and A. Zucker, *Phys. Rev.* **95**, 671 (1954).

² H. L. Reynolds and A. Zucker, *Phys. Rev.* **95**, 1353 (1954).

³ K. G. Stephens and D. Walker, *Phil. Mag.* **45**, 543 (1954).

⁴ E. L. Hubbard and E. J. Lauer, *Phys. Rev.* **98**, 1814 (1955).

⁵ L. C. Northcliffe (to be published).

⁶ N. O. Lassen, *Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd* **26**, No. 512 (1951).

⁷ P. G. Roll and F. E. Steigert, *Nuclear Phys.* **17**, 54 (1960).

⁸ N. Bohr and J. Lindhard, *Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd* **28**, No. 7 (1954).

⁹ For an excellent review of this problem in the kev region the reader is referred to S. K. Allison, *Revs. Modern Phys.* **30**, 1137 (1958).

¹⁰ P. G. Roll and F. E. Steigert, *Nuclear Phys.* **16**, 534 (1960).

¹¹ H. H. Heckman, B. L. Perkins, W. G. Simon, F. M. Smith, and W. H. Barkas, *Phys. Rev.* **117**, 544 (1960).

¹² W. Whaling, *Handbuch der Physik*, edited by S. Flügge (Springer-Verlag, Berlin, 1958), Vol. 34.

¹³ A. Papineau, *Compt. rend.* **242**, 2933 (1956).

current measurements.¹ The latter give slightly but systematically lower values, as is expected. The differences are well within the experimental errors involved, however, and are not considered significant.

Traditionally, charge-state data are presented in the form of fractional charge as a function of ion velocity. Nevertheless, in view of the considerations outlined above and to emphasize the root mean square nature of the derived values, it is probably more reasonable to display the data as the square of the fractional charge. Occasionally this requires manipulating familiar relationships into rather strange forms. It is felt, however, that the nature of the data justifies use of this convention.

The particular function of particle velocity to use as the abscissa is somewhat predicated by the form in which the data will be utilized. Usually the mode of presentation is only of importance when one wishes to combine data from different ions to construct a universal curve. The problem, obviously, is to explicitly remove the species dependence by means of a suitable reference velocity. While the actual processes of capture and loss of electrons by an ion traversing matter are quite involved, reasonable estimates of the equilibrium charge can be made on rather simple premises.^{8,14} To first order, at least, one may assume that the two cross sections are equal for a given orbital electron when its velocity is approximately that of the ion under consideration. For a particle of nuclear charge Z , this might suggest the use of the K -electron velocity, $(c/137)Z$, as a valid parameter, as long as only the first two electrons are involved. On the other hand, for velocities such that many orbits could be involved, a statistical approach might prove preferable. In this case the Fermi-Thomas velocity, $(c/137)Z^{2/3}$, would be suggested.

The range data for nuclear emulsions extend from fractional charges of the order of unity down to about 0.5. Considering this spread of values, both methods have been applied to these data.^{10,11} The results would seem to indicate a separation into two regions, with the demarcation occurring at about 1.5 times the K -electron velocity. The slower ions appear to group better when plotted in terms of Fermi-Thomas velocities, the more energetic when plotted in terms of the K -electron velocity. The change-over would correspond to a fractional charge of the order of 0.95. While qualitatively the results are as expected, the high fractional charge to which the transition region corresponds is a little surprising. In other absorbers experimental technique restricted the low value of fractional charge to about 0.7, with most measurements extending down to only 0.8 or 0.9, depending on species. Since this is clearly in the region of tenuous applicability of the Fermi-Thomas model, the data have been invariably plotted parametric in K -electron velocity. As pointed out by Northcliffe,⁵ deviations on the part of the lighter

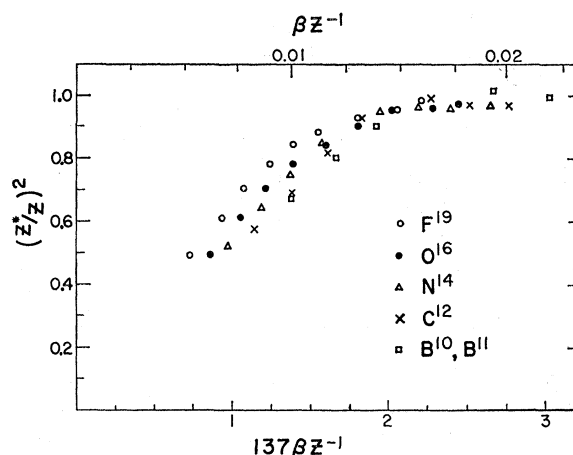


FIG. 1. The square of the fractional effective charge for various ion species in oxygen absorber as a function of the ratio of the ion velocity to the K -electron velocity.

ions may be ascribed to the greater experimental errors involved. More important, though, is the fact that for purposes of constructing a range-energy curve, these deviations are less important considering the longer ranges involved.

To illustrate the differences between the two modes of presentation, the effective charge for oxygen absorber is displayed in Figs. 1, 2. It is apparent that it is the lighter ions which are most sensitive to the exponent associated with the nuclear charge. To avoid crowding, some of the high-energy data have been arbitrarily omitted. These points are all in the region of relatively complete stripping, where Fig. 1 is clearly the better representation. As in the emulsion data, it would appear that a Fermi-Thomas description is somewhat more universal up to rather large fractional charges. The author's data for nickel and Northcliffe's for aluminum have been similarly treated with equivalent results. It

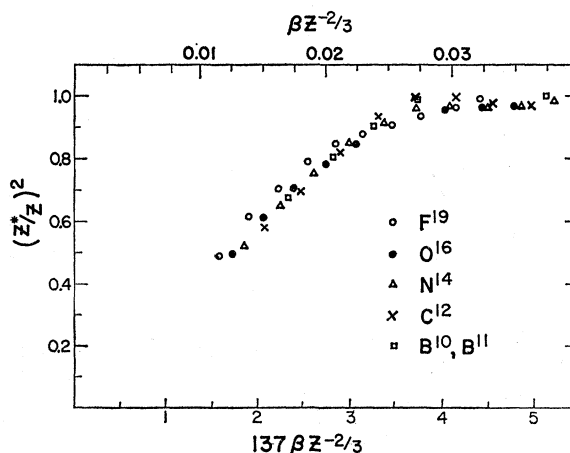


FIG. 2. The square of the fractional effective charge for various ion species in oxygen absorber as a function of the ratio of the ion velocity to the Fermi-Thomas velocity.

¹⁴ R. L. Gluckstern, Phys. Rev. **98**, 1817 (1955).

is on the basis of this improved grouping that the authors have chosen to present their data in terms of a Fermi-Thomas description.

VARIATION OF CHARGE WITH ABSORBER

In practice, capture and loss cross sections can be treated in reasonable detail only for idealized cases. One must assume a diffuse absorber and an ion which is either a clearly statistical atom or a simple Bohr atom. The former alternative has been most recently investigated by Gluckstern.¹⁴ The agreement obtained with experimental data for this case would suggest that the simple functional relationships obtained by Bohr and Lindhard⁸ are reasonable and only err in requiring a scaling factor. Moreover, use of such a simple picture allows one to estimate the deviations resulting from weakening the basic assumptions invoked above. Specifically, for large velocities, the capture cross section may be expected to take the form¹⁵

$$\sigma_e \sim 4\pi a_0^2 Z^{\frac{1}{2}} (V_0/V)^6, \quad (1)$$

where a_0 is the hydrogen Bohr radius, Z the ionic nuclear charge, V the ionic velocity, and V_0 the velocity of the electron in the first Bohr orbit of hydrogen ($V_0 = c/137$).

The requirement of diffuseness is a demand that the average time between collisions be long compared to de-excitation times. This obviously involves almost every parameter at hand, and in the absence of suitable estimates will only be clearly satisfied in the limit of rarified gases. Relaxation of this restriction would imply that now electron loss can take place from excited orbits, and capture can occur without all electrons in their

lowest states. This residual excitation would thus tend to favor electron loss and hinder electron pickup. A new equilibrium would then be approached with a somewhat higher value of net charge. Using only the functional relationships, this shift in net charge can be estimated to be⁸

$$\Delta Z^* = \frac{2}{5} Z^* (V \tau \Omega \rho / 2 V \tau \Omega \rho + 1), \quad (2)$$

where Z^* is the effective ionic charge for the diffuse reference absorber, V the ion velocity, τ the de-excitation time, Ω the capture (and hence also loss) cross section at equilibrium, and ρ the atomic density of scattering centers. The last three quantities all refer to the nondiffuse absorber. Recognizing the inverse velocity dependence of Ω , the combination of low velocity and high density would lead to a relative shift in charge of magnitude

$$\Delta Z^* = 0.2 Z^*. \quad (3)$$

In Fig. 3 have been displayed the data obtained for fluorine ions in various absorbers. Only a single species is shown to avoid possible ambiguity concerning the proper reference velocity. This eliminates misleading overlap caused by systematic variations among the different species. Fluorine was chosen simply because of the additional data available in the argon absorber. Other species give similar results. There is an apparent splitting into two distinct but convergent groupings. These would correspond to solid (high density) and gaseous (low density) absorbers, respectively. The low-velocity extension for the case of solid absorber can be visualized from the dashed curve. This is part of the equivalent plot for nitrogen ions. This curve also serves to illustrate the systematic displacements which occur among the various ion species. Lower energy information is available for this single case as a result of combining the range data in nickel from references 1 and 7.

The dotted curve represents the usual Fermi-Thomas prediction⁸ for the case of many electrons,

$$Z^* = (V/V_0) Z^{\frac{1}{2}}. \quad (4)$$

While the deviations from this are large and not entirely unexpected, it is apparent that scaling along the velocity axis by a factor of about two would give a quite reasonable description of the low-energy data.

The relative vertical displacement between these two curves may be investigated if the gaseous absorber may indeed be considered as diffuse. In spite of the approximate nature of any magnitude estimates involving τ and Ω , this limit is probably satisfied at the pressures used. These varied from a few millimeters of mercury at high energies to several centimeters for the slower particles. In Fig. 4 is plotted the relative spacing as obtained from Fig. 3. Obviously there is considerable compounding of experimental errors involved in the process of first differentiating the empirical range curves, forming ratios, and then taking differences between the resulting fractions. In the interest of avoid-

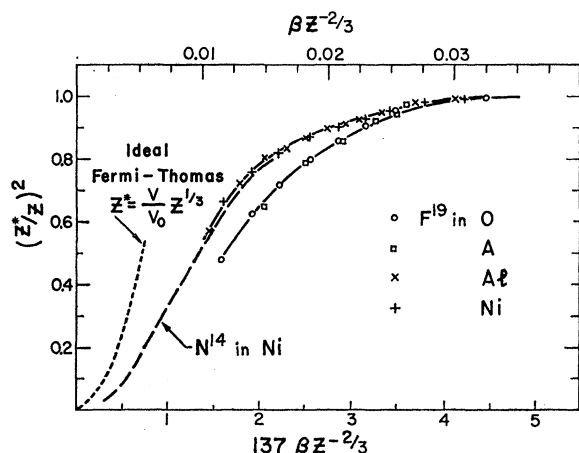


FIG. 3. The square of the fractional effective charge for fluorine ions in various absorbers as a function of the ratio of the ion velocity to the Fermi-Thomas velocity. The solid curves are considered most reasonable fits to the experimental data. The dashed curve is for nitrogen ions in nickel absorber, and may be used to visualize low-energy continuations. The dotted curve is that expected for an ideal Fermi-Thomas case.

¹⁵ N. Bohr, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd 18, No. 8 (1948).

ing even further manipulations it was therefore considered more reasonable to translate Bohr and Lindhard's estimates into relationships in terms of the data than the reverse. The dashed line is predicted by Eq. (3). The solid line corresponds to Eq. (2) for the condition

$$V\tau\Omega\rho \sim (4 \times 10^{-7})V^{-6}.$$

This would imply a velocity dependence for the equilibrium cross section of the form

$$\Omega \propto V^{-6},$$

in qualitative agreement with Eq. (1). The typical error bar shown brackets the extreme values reconcilable with the original range curves. The alternate solutions shown are obviously not precluded. They would imply, however, systematic displacements of the parent curves relative to the experimental points.

CONCLUSIONS

Even though most of the data here considered are at energies where one might normally expect severe deviations from the predictions of a statistical model of the atom, it would appear that, qualitatively at least, there may be considerable merit in such a description. To begin with, use of the Fermi-Thomas velocity as a reference parameter gives an improved systematization of the effective-charge data in terms of the nuclear charge of the ion. Secondly, the magnitude and velocity dependence of the displacement in effective charge for dense relative to diffuse media are approximately as expected. There is admittedly still the question as to whether the effective charge as defined by range measurements is equivalent to or at least simply related to the effective charge as used in the capture and loss cross section. Some reassurance is available on this score from

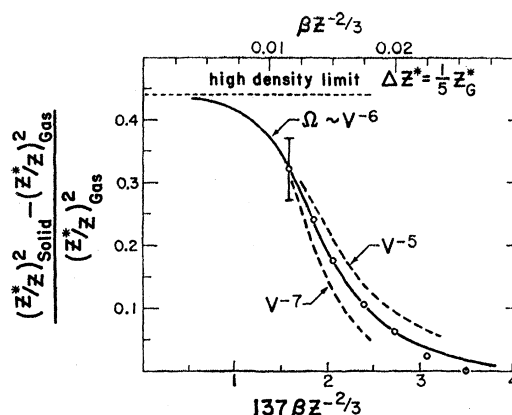


FIG. 4. Ratio of the displacement of the square of fractional effective charge for fluorine ions in a metallic relative to a gaseous absorber, to the square of the fractional effective charge in the gas, as a function of the ratio of the ion velocity to the Fermi-Thomas velocity. The dotted curve indicates value expected for limiting case of high density and low velocity. The solid curve indicates variation expected for a capture cross section of the form $(\propto V^{-6})$. The dashed curves indicate the shape expected for other functions of the velocity.

comparison to empirical population density measurements. However, even making allowance for possible residual differences, the data in Fig. 3 show rather patently a grouping in terms of the atomic density rather than the nuclear charge of the absorber. The reverse would be expected if the variations were the result of additional scattering processes.

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