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Double Beta Decay. II

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General formulas are given for the differential probability of allowed double β decay of nonoriented nuclei holding for any initial angular momentum and all possible final angular momenta according to the theory of Feynman and Gell-Mann. Six different combinations of reduced matrix elements occur. The decay probability for transitions $0 \rightarrow 2$ is small of second order compared with that for transitions $0 \rightarrow 0$. A lower limit for the half-life of the transition $0 \rightarrow 2$ of ${}_{20}Ca_{28}$ is calculated as a function of the intermediate nuclear energy using j-j shell-model configurations. A slight generalization of the coupling constants suitable for two-neutrino decay is considered.

I. INTRODUCTION

IN a former paper,[1] formulas for the probability of double β decay were given according to the theory of the Fermi interaction of Feynman and Gell-Mann,[2] which only admits double β decay with two antineutrinos. These formulas hold for allowed transitions of nonoriented nuclei of initial angular momentum $J_i=0$ to the possible final angular momenta $J_f=0, 1, 2$. Due to the approximation made, the probability for transitions $0 \rightarrow 2$ vanishes. Since these transitions are of practical interest, e.g., for ${}_{20}Ca_{28}$, it is desirable to consider higher terms of the expansion in order to get a nonvanishing value for the decay probability. Moreover, for a deeper insight into the mechanism of double β decay, especially into the coupling of angular momenta, it seems useful to give the decay formulas in a generalized form.

For single β decay, Lee-Whiting succeeded in deriving a general formulation for arbitrary forbiddenness.[3] Rosen has formulated the theory of neutrinoless double β decay in a similar manner,[4] but without considering angular correlations. A corresponding treatment of the two-neutrino case seems to be too complicated.[4] The role of certain forbidden transitions for neutrinoless decay was considered by Janouch.[5]

II. GENERAL FORMULAS

Assuming a Fermi interaction according to Feynman and Gell-Mann,[2] the differential probability for allowed double β decay of a nonoriented nucleus of any initial angular momentum J_i becomes

$$\begin{aligned} d^3\lambda = \frac{G^4}{30\pi^7} F(Z,W_1)F(Z,W_2) \sum_{mm'} \{ [& P_{mm'}{}^1(q,D)\mathfrak{A}_m\mathfrak{A}_{m'}{}^* \\ & + P_{mm'}{}^2(q,D)\mathfrak{B}_m{}^-\mathfrak{B}_{m'}{}^{-*}](1-v_1v_2\cos\theta) \\ & + [P_{mm'}{}^3(q,D)\mathfrak{B}_m{}^+\mathfrak{B}_{m'}{}^{+*} + P_{mm'}{}^4(q,D)(\mathfrak{C}_m\mathfrak{C}_{m'}{}^* \\ & + \mathfrak{D}_m\mathfrak{D}_{m'}{}^* + \mathcal{E}_m\mathcal{E}_{m'}{}^*)](1+\tfrac{1}{3}v_1v_2\cos\theta)\} \\ & \times dW_1 dW_2 \sin\theta d\theta/2, \qquad (1) \end{aligned}$$

where $q \equiv \epsilon - W_1 - W_2$ and $D \equiv W_2 - W_1$. Since the electrons are indistinguishable, D is restricted to $D \geqq 0$ (or $D \leqq 0$). This fact, however, becomes important only for integration over W_1 and W_2. Equation (1) was derived taking into account the negative sign in certain terms due to the anticommutation rules of the lepton operators.[6]

The $P_{mm'}{}^{\alpha}$'s are given as integrals over the energy difference of the antineutrinos, $\Delta \equiv q_2 - q_1$. One has

$$\begin{aligned} P^{1,2} &\equiv \frac{15}{128}(I^1 \pm I^2 + I^3 \pm I^4), \\ P^{3,4} &\equiv \frac{15}{128}(I^1 \pm I^2 - I^3 \mp I^4); \end{aligned} \qquad (2)$$

$$\begin{aligned} I_{mm'}{}^{1,2} \equiv & \int_{-q}^{q} \frac{(q^2-D^2)^2 d\Delta}{(\eta_m - D - \Delta)(\eta_{m'} - D \mp \Delta)} \\ & + \int_{-q}^{q} \frac{(q^2-D^2)^2 d\Delta}{(\eta_m + D - \Delta)(\eta_{m'} + D \mp \Delta)}, \\ I_{mm'}{}^{3,4} \equiv & \int_{-q}^{q} \frac{(q-D^2)^2 d\Delta}{(\eta_m - D - \Delta)(\eta_{m'} + D \mp \Delta)} \\ & + \int_{-q}^{q} \frac{(q^2-D^2)^2 d\Delta}{(\eta_m + D - \Delta)(\eta_{m'} - D \mp \Delta)} \end{aligned} \qquad (3)$$

Δ should be restricted to $0 \leqq \Delta \leqq q$ (or $-q \leqq \Delta \leqq 0$). The extension of the domain of integration is due to the appearance of sums like

$$\int_0^q f(\Delta)d\Delta + \int_0^q f(-\Delta)d\Delta = \int_{-q}^{q} f(\Delta)d\Delta.$$

All the four $P_{mm'}{}^{\alpha}$'s are even functions of D; $P_{mm'}{}^3$ and $P_{mm'}{}^4$ vanish for $D=0$, $P_{mm'}{}^1$ and $P_{mm'}{}^2$ do not.

$\mathfrak{A}_m$, etc., are the following six combinations of

[1] L. Meichsner, Phys. Rev. **117**, 489 (1960), hereafter referred to as part I. See also there for the notation used here.
[2] R. P. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1959).
[3] G. E. Lee-Whiting, Can. J. Phys. **36**, 1199 (1958).
[4] S. P. Rosen, Can. J. Phys. **37**, 780 (1959).
[5] F. Janouch, J. Exptl. Theoret. Phys. (U.S.S.R.) **36**, 335 (1959) [translation: Soviet Phys.—JETP **36**(9), 231 (1958)]; Czech. J. Phys. **B10**, 1 (1960).
[6] W. H. Furry, Phys. Rev. **56**, 1184 (1939).

reduced matrix elements[7]:

$$\mathfrak{A}_m\equiv\left(\frac{2J_f+1}{2J_i+1}\right)^{\frac{1}{2}}\{\langle f\|\mathbf{1}\|m\rangle\langle m\|\mathbf{1}\|i\rangle+3^{\frac{1}{2}}(2J_m+1)^{\frac{1}{2}}\times W(11J_iJ_f;0J_m)\langle f\|\boldsymbol{\sigma}\|m\rangle\langle m\|\boldsymbol{\sigma}\|i\rangle\},$$

$$\mathfrak{B}_m^{\pm}\equiv\left(\frac{2J_f+1}{2J_i+1}\right)^{\frac{1}{2}}\{\langle f\|\mathbf{1}\|m\rangle\langle m\|\boldsymbol{\sigma}\|i\rangle-\langle f\|\boldsymbol{\sigma}\|m\rangle\langle m\|\mathbf{1}\|i\rangle\pm6^{\frac{1}{2}}(2J_m+1)^{\frac{1}{2}}\times W(11J_iJ_f;1J_m)\langle f\|\boldsymbol{\sigma}\|m\rangle\langle m\|\boldsymbol{\sigma}\|i\rangle\},$$

$$\mathfrak{C}_m\equiv\left(\frac{2J_f+1}{2J_i+1}\right)^{\frac{1}{2}}(20)^{\frac{1}{2}}(2J_m+1)^{\frac{1}{2}}\times W(11J_iJ_f;2J_m)\langle f\|\boldsymbol{\sigma}\|m\rangle\langle m\|\boldsymbol{\sigma}\|i\rangle,\tag{4}$$

$$\mathfrak{D}_m\equiv\left(\frac{2J_f+1}{2J_i+1}\right)^{\frac{1}{2}}2^{\frac{1}{2}}\{\langle f\|\mathbf{1}\|m\rangle\langle m\|\boldsymbol{\sigma}\|i\rangle+\langle f\|\boldsymbol{\sigma}\|m\rangle\langle m\|\mathbf{1}\|i\rangle\},$$

$$\mathcal{E}_m\equiv\left(\frac{2J_f+1}{2J_i+1}\right)^{\frac{1}{2}}\{3^{\frac{1}{2}}\langle f\|\mathbf{1}\|m\rangle\langle m\|\mathbf{1}\|i\rangle-(2J_m+1)^{\frac{1}{2}}\times W(11J_iJ_f;0J_m)\langle f\|\boldsymbol{\sigma}\|m\rangle\langle m\|\boldsymbol{\sigma}\|i\rangle\}.$$

where the W's are Racah coefficients.[8] The given expressions can be different from zero only for $J_f=J_i$, $J_i\pm1$, $J_i\pm2$. Moreover, we have

$$\begin{aligned}&\mathfrak{A}_m=\mathcal{E}_m=0 && \text{for } J_f\neq J_i;\\ &\mathfrak{B}_m^{\pm}=\mathfrak{D}_m=0 && \text{for } J_f=J_i\pm2 \text{ and for } 0\to0;\\ &\mathfrak{C}_m=0 && \text{for } 0\to0,\ 0\to1,\ \tfrac{1}{2}\to\tfrac{1}{2},\ 1\to0.\end{aligned}\tag{5}$$

These equations hold since either the reduced matrix elements or the Racah coefficients vanish, or both. For $J_i=0$ one has

$$\begin{aligned}0\to0:\quad&\mathfrak{A}_m=\langle f\|\mathbf{1}\|m\rangle\langle m\|\mathbf{1}\|i\rangle+3^{\frac{1}{2}}\langle f\|\boldsymbol{\sigma}\|m\rangle\langle m\|\boldsymbol{\sigma}\|i\rangle,\\ &\mathcal{E}_m=3^{\frac{1}{2}}\langle f\|\mathbf{1}\|m\rangle\langle m\|\mathbf{1}\|i\rangle-\langle f\|\boldsymbol{\sigma}\|m\rangle\langle m\|\boldsymbol{\sigma}\|i\rangle;\\ 0\to1:\quad&\mathfrak{B}_m^{\pm}=3^{\frac{1}{2}}(\langle f\|\mathbf{1}\|m\rangle\langle m\|\boldsymbol{\sigma}\|i\rangle-\langle f\|\boldsymbol{\sigma}\|m\rangle\langle m\|\mathbf{1}\|i\rangle)\\ &\qquad\pm6^{\frac{1}{2}}\langle f\|\boldsymbol{\sigma}\|m\rangle\langle m\|\boldsymbol{\sigma}\|i\rangle,\\ &\mathfrak{D}_m=6^{\frac{1}{2}}(\langle f\|\mathbf{1}\|m\rangle\langle m\|\boldsymbol{\sigma}\|i\rangle+\langle f\|\boldsymbol{\sigma}\|m\rangle\langle m\|\mathbf{1}\|i\rangle);\\ 0\to2:\quad&\mathfrak{C}_m=(20)^{\frac{1}{2}}\langle f\|\boldsymbol{\sigma}\|m\rangle\langle m\|\boldsymbol{\sigma}\|i\rangle.\end{aligned}\tag{6}$$

From Eq. (1) it is to be seen that the angular correlations appear only through the factors $1-v_1v_2\cos\theta$ and $1+\frac{1}{3}v_1v_2\cos\theta$. Equations (1)–(5) show that the energy distribution and angular correlations depend on the change of angular momentum.

[7] It should be noted that the factor $[(2J_f+1)/(2J_i+1)]^{\frac{1}{2}}$ is not incorporated in $\mathfrak{B}_m^{\pm}$ in part I.

[8] M. E. Rose, *Elementary Theory of Angular Momentum* (John Wiley & Sons, Inc., New York; Chapman & Hall, Ltd., London, 1957), Chap. VI and Appendix I.

III. EXPANSIONS

The integrals of Eqs. (3) can be evaluated in an elementary way. The results are best expressed as expansions in powers of q. One has

$$\begin{aligned}P_{mm'}{}^{1,3}=&\frac{15}{4}q^5\sum_{n=0,\text{even}}^{\infty}\frac{q^n}{(n+1)(n+3)(n+5)}\\ &\times\sum_{k=0,\text{even}}^{n}\left[\frac{1}{(\eta_m-D)^{k+1}}\pm\frac{1}{(\eta_m+D)^{k+1}}\right]\\ &\times\left[\frac{1}{(\eta_{m'}-D)^{n-k+1}}\pm\frac{1}{(\eta_{m'}+D)^{n-k+1}}\right],\\ P_{mm'}{}^{2,4}=&\frac{15}{4}q^5\sum_{n=0,\text{even}}^{\infty}\frac{q^n}{(n+1)(n+3)(n+5)}\\ &\times\sum_{k=0,\text{odd}}^{n}\left[\frac{1}{(\eta_m-D)^{k+1}}\pm\frac{1}{(\eta_m+D)^{k+1}}\right]\\ &\times\left[\frac{1}{(\eta_{m'}-D)^{n-k+1}}\pm\frac{1}{(\eta_{m'}+D)^{n-k+1}}\right].\end{aligned}\tag{7}$$

The necessary and sufficient condition for this expansion, $q/(\eta_m\mp D)<1$, is always satisfied, since we have

$$\frac{q}{\eta_m}\leqq\frac{q}{\eta_m-|D|}\leqq\frac{q}{\eta_m-(\epsilon-2)+q}\leqq\frac{\epsilon-2}{\eta_m}<1,$$

due to $0\leqq|D|\leqq W-2$, $W+q=\epsilon$, $0\leqq q\leqq\epsilon-2$, and $\eta_m>\epsilon-2$. The last relation follows from the fact that the energy of the whole system in the intermediate state must be greater than ϵ_i. Owing to the good convergence of the expansion in consequence of $[(n+1)(n+3)(n+5)]^{-1}$, it is not necessary in order to break off after a few terms that $q/(\eta_m-D)\ll1$ hold. Assuming $\epsilon_m\geqq\epsilon_i$ and ϵ not greater than that for ${}_{20}Ca_{28}$, i.e., $\epsilon\lesssim10$, one has $(\epsilon-2)/\eta_m\lesssim0.8$. In this case breaking off after $n=2$ gives sufficient accuracy in the whole energy range.

A further expansion in powers of D, considering terms up to the third order, gives

$$\begin{aligned}P_{mm'}{}^1=&\frac{1}{\eta_m\eta_{m'}}q^5+\left(\frac{1}{\eta_m\eta_{m'}{}^3}+\frac{1}{\eta_m{}^3\eta_{m'}}\right)\left(\frac{1}{7}q^7+q^5D^2\right)\\ &+\left(\frac{1}{\eta_m\eta_{m'}{}^5}+\frac{1}{\eta_m{}^3\eta_{m'}{}^3}+\frac{1}{\eta_m{}^5\eta_{m'}}\right)\left(\frac{1}{21}q^9+q^5D^4\right)\\ &+\left(\frac{3}{\eta_m\eta_{m'}{}^5}+\frac{1}{\eta_m{}^3\eta_{m'}{}^3}+\frac{3}{\eta_m{}^5\eta_{m'}}\right)\frac{2}{7}q^7D^2+\cdots,\end{aligned}$$

$$P_{mm'}{}^{2}=\frac{1}{\eta_m{}^2\eta_{m'}{}^2}\frac{1}{7}q^7+\left(\frac{1}{\eta_m{}^2\eta_{m'}{}^4}+\frac{1}{\eta_m{}^4\eta_{m'}{}^2}\right)\left(\frac{1}{21}q^9+\frac{3}{7}q^7D^2\right)+\cdots, \tag{8}$$

$$P_{mm'}{}^{3}=\frac{1}{\eta_m{}^2\eta_{m'}{}^2}q^5D^2+\left(\frac{1}{\eta_m{}^2\eta_{m'}{}^4}+\frac{1}{\eta_m{}^4\eta_{m'}{}^2}\right)\left(\frac{3}{7}q^7D^2+q^5D^4\right)+\cdots,$$

$$P_{mm'}{}^{4}=\frac{1}{\eta_m{}^3\eta_{m'}{}^3}\frac{4}{7}q^7D^2+\cdots.$$

The necessary and sufficient condition for this, $D^2/\eta_m{}^2<1$, is always satisfied, but convergence is not so good as for the previous expansion. Nevertheless, breaking off after a few terms is admissible, since the maximum errors are at small q, where $d^3\lambda$ is small anyway.

Combining Eqs. (1) and (8) we get

$$\begin{aligned}
d^3\lambda=&(G^4/30\pi^7)F(Z,W_1)F(Z,W_2)p_1W_1p_2W_2\\
&\times\{q^5|\textstyle\sum_m\eta_m{}^{-1}\mathcal{A}_m|^2(1-v_1v_2\cos\theta)\\
&+(1/7)q^7[2\,\mathrm{Re}(\textstyle\sum_m\eta_m{}^{-1}\mathcal{A}_m\sum_m\eta_m{}^{-3}\mathcal{A}_m{}^*)\\
&+|\textstyle\sum_m\eta_m{}^{-2}\mathcal{B}_m{}^-|^2](1-v_1v_2\cos\theta)\\
&+q^5D^2[2\,\mathrm{Re}(\textstyle\sum_m\eta_m{}^{-1}\mathcal{A}_m\sum_m\eta_m{}^{-3}\mathcal{A}_m{}^*)(1-v_1v_2\cos\theta)\\
&+|\textstyle\sum_m\eta_m{}^{-2}\mathcal{B}_m{}^+|^2(1+\frac{1}{3}v_1v_2\cos\theta)]\\
&+(1/21)q^9[2\,\mathrm{Re}(\textstyle\sum_m\eta_m{}^{-1}\mathcal{A}_m\sum_m\eta_m{}^{-5}\mathcal{A}_m{}^*)\\
&+|\textstyle\sum_m\eta_m{}^{-3}\mathcal{A}_m|^2\\
&+2\,\mathrm{Re}(\textstyle\sum_m\eta_m{}^{-2}\mathcal{B}_m{}^-\sum_m\eta_m{}^{-4}\mathcal{B}_m{}^{-*})](1-v_1v_2\cos\theta)\\
&+q^5D^4\{[2\,\mathrm{Re}(\textstyle\sum_m\eta_m{}^{-1}\mathcal{A}_m\sum_m\eta_m{}^{-5}\mathcal{A}_m{}^*)\\
&+|\textstyle\sum_m\eta_m{}^{-3}\mathcal{A}_m|^2](1-v_1v_2\cos\theta)\\
&+2\,\mathrm{Re}(\textstyle\sum_m\eta_m{}^{-2}\mathcal{B}_m{}^+\sum_m\eta_m{}^{-4}\mathcal{B}_m{}^{+*})(1+\frac{1}{3}v_1v_2\cos\theta)\}\\
&+(4/7)q^7D^2\{[3\,\mathrm{Re}(\textstyle\sum_m\eta_m{}^{-1}\mathcal{A}_m\sum_m\eta_m{}^{-5}\mathcal{A}_m{}^*)\\
&+\frac{1}{2}|\textstyle\sum_m\eta_m{}^{-3}\mathcal{A}_m|^2\\
&+\frac{3}{2}\,\mathrm{Re}(\textstyle\sum_m\eta_m{}^{-2}\mathcal{B}_m{}^-\sum_m\eta_m{}^{-4}\mathcal{B}_m{}^{-*})](1-v_1v_2\cos\theta)\\
&+[\frac{3}{2}\,\mathrm{Re}(\textstyle\sum_m\eta_m{}^{-2}\mathcal{B}_m{}^+\sum_m\eta_m{}^{-4}\mathcal{B}_m{}^{+*})\\
&+|\textstyle\sum_m\eta_m{}^{-3}\mathcal{C}_m|^2+|\sum_m\eta_m{}^{-3}\mathcal{D}_m|^2+|\sum_m\eta_m{}^{-3}\mathcal{E}_m|^2]\\
&\qquad\times(1+\frac{1}{3}v_1v_2\cos\theta)\}\}dW_1dW_2\sin\theta d\theta/2.
\end{aligned}\tag{9}$$

In this approximation all possible combinations of reduced matrix elements occur.

IV. TRANSITIONS 0→2

Equations (5) and (9) show that transitions $0\rightarrow2$ are just "second forbidden" in the sense of part I. They correspond to the unique forbidden transitions of single β decay. The total decay probability is

$$\lambda(0\rightarrow2)=\frac{G^4}{30\pi^7}f_{72}(Z,\epsilon)|\sum_m\eta_m{}^{-3}\mathcal{C}_m|^2, \tag{10}$$

with

$$\begin{aligned}
f_{72}(Z,\epsilon)\equiv&\frac{4}{7}\iint_{W_1\leq W_2}dW_1dW_2\,F(Z,W_1)F(Z,W_2)\\
&\times p_1W_1p_2W_2(\epsilon-W_1-W_2)^7(W_2-W_1)^2\\
\simeq&\frac{1}{13680}G(Z,\bar{W}_1)G(Z,\bar{W}_2)(\epsilon-2)^{11}\\
&\times\left[1+\frac{2}{3}(\epsilon-2)+\frac{2}{13}(\epsilon-2)^2+\frac{1}{91}(\epsilon-2)^3+\frac{1}{2730}(\epsilon-2)^4\right];
\end{aligned}\tag{11}$$

$$\begin{aligned}
\bar{W}_{1,2}=&\tfrac{1}{2}(1\mp3^{-\frac{1}{2}})\bar{W},\\
\bar{W}=&(1/28)\{7(\epsilon-2)+40+[49(\epsilon-2)^2-112(\epsilon-2)+256]^{\frac{1}{2}}\}.
\end{aligned}\tag{12}$$

$\bar{W}_1$ and $\bar{W}_2$ are determined in the same way as the values $\bar{W}_1=\bar{W}_2=\bar{W}/2$ for $f_{50}(Z,\epsilon)$ in part I. If $\bar{W}_1<1$ results, $\bar{W}_1$ and $\bar{W}_2$ given above are to be replaced by

$$\begin{aligned}
\bar{W}_1=&1,\\
\bar{W}_2=&(1/24)\{5(\epsilon-2)+14+[25(\epsilon-2)^2+44(\epsilon-2)+100]^{\frac{1}{2}}\}.
\end{aligned}\tag{13}$$

From Eq. (10) we can calculate the half-life for $0\rightarrow2$ of ${}_{20}Ca_{28}$. The six neutrons and the two protons in the $f_{7/2}$ state can couple to $J_f=2$ in nine different ways. From these nine functions, one with isotopic spin $T=3$ and eight with $T=2$ may be constructed. We want, however, to get a lower limit for $t(0\rightarrow2)$. Therefore, we consider that linear combination with $|\langle f\|\boldsymbol{\sigma}\|m_1\rangle|$ as great as possible and find

$$|\langle f\|\boldsymbol{\sigma}\|m_1\rangle|=(3.00)^{\frac{1}{2}}.$$

With this we get

$\epsilon_{m1}-\epsilon_i$ =	0	2	4	6 Mev
$t(0\rightarrow2)$ =	0.27×10^{20}	1.4×10^{21}	1.5×10^{22}	0.84×10^{23} years.

$\langle f\|\boldsymbol{\sigma}\|m_1\rangle$ vanishes for all the other eight possible linear combinations orthogonal to the one considered.

V. TRANSITIONS 0→1

Replacing $\bar{W}_1$ and $\bar{W}_2$ of Eqs. (12) and (13), respectively, by the values holding for $f_{50}(Z,\epsilon)$, as was done for $0\rightarrow1$ in part I, gives no serious errors. For the sake of completeness we give the correct values here.

They are

$$f_{70}(Z,\epsilon):\quad \bar{W}_1=\bar{W}_2=\tfrac{1}{2}\bar{W},$$

$$\bar{W}=(1/24)\{5(\epsilon-2)+32+[25(\epsilon-2)^2-64(\epsilon-2)+256]^{\frac{1}{2}}\}; \tag{14}$$

$$f_{52}(Z,\epsilon):\quad \bar{W}_{1,2}=\tfrac{1}{2}(1\mp 2^{-\frac{1}{2}})\bar{W},$$

$$\bar{W}=(1/24)\{7(\epsilon-2)+36+[49(\epsilon-2)^2-72(\epsilon-2)+144]^{\frac{1}{2}}\}. \tag{15}$$

If $\bar{W}_1<1$ results, one has to put instead

$$\bar{W}_1=1,$$

$$\bar{W}_2=(1/20)\{5(\epsilon-2)+12+[25(\epsilon-2)^2+40(\epsilon-2)+64]^{\frac{1}{2}}\}. \tag{16}$$

VI. COUPLING CONSTANTS

Recent experiments show that the Fermi interaction differs somewhat from that proposed by Feynman and Gell-Mann[2] insofar as the V and A contributions are not equal. There might also be other slight deviations from this theory.[9] For the study of double β decay with two (anti)neutrinos, the assumption $C_V=C_V'$, $C_A=C_A'$ with arbitrary C_V, C_A and all other coupling constants vanishing is sufficient. Then one has to replace $\mathbf{1}$ and $\boldsymbol{\sigma}$ by $(2^{\frac{1}{2}}C_V/G)\mathbf{1}$ and $-(2^{\frac{1}{2}}C_A/G)\boldsymbol{\sigma}$, respectively.

[9] H. Schopper and H. Müller, Nuovo cimento **13**, 1026 (1960).

VII. ERRORS IN PART I

(a) The six neutrons and the two protons of ${}_{22}\mathrm{Ti}_{26}$ in the $f_{7/2}$ state can couple to $J_f=0$ in four (not nine) different ways. From these four functions one linear combination with $T=4$ and three (not eight) with $T=2$ may be constructed.

(b) The linear combination

$$|f\rangle=(5/6)^{\frac{1}{2}}|00_n0_p\rangle-(1/6)^{\frac{1}{2}}|02_n2_p\rangle,$$

for which $T=2$ holds, contains the smallest possible neutron and proton angular momenta, but its $\langle|\mathbf{J}_{n,p}|^2\rangle$ is not as small as possible. This is, however, true for

$$|f\rangle\simeq 0.967|00_n0_p\rangle-0.236|02_n2_p\rangle-0.088|04_n4_p\rangle-0.049|06_n6_p\rangle.$$

We now have

$$\langle f\|\boldsymbol{\sigma}\|m_1\rangle=-0.403,$$

and the half-life is $(8.4)^{-1}$ times as that for the former $|f\rangle$. Both functions $|f\rangle$ are obvious generalizations of that with $J_n=J_p=0$.

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