Proposal for Measuring the π^0 Lifetime by π^0 Production in Electron-Electron or Electron-Positron Collisions*

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(Received June 9, 1960)

The cross section for production of π^0 mesons by colliding electrons is calculated in the virtual photon approximation. This cross section is directly proportional to the inverse π^0 lifetime, and the proportionality constant is independent of the strong couplings. For a center-of-mass energy of 300 Mev and a π^0 mean life of 10^{-18} sec the total cross section is about 10^{-33} cm^2.

I. INTRODUCTION

UPPER and lower bounds are known for the mean life, τ, of the π^0 meson. The upper bound, $\tau<5\times10^{-16}$, was determined by attempting to measure the distance from the π^0 production event to the point of decay via the electron decay mode.[1] The lower bound, $\tau>5\times10^{-19}$, comes from the scattering of light by protons.[2] In this paper it is proposed to measure τ by the production of π^0 mesons by colliding electrons.

Let p_0 be the momentum of each electron, m the electron mass, μ the meson mass, and τ the meson mean life. It is shown in Sec. II that the total cross section for π^0 production is given, in natural units, by the formula

$$\sigma_T=16\left(\frac{1}{137}\right)^2\frac{1}{\mu^3\tau}f(\gamma)[\ln(p_0/m)]^2 +\text{terms of order }[\ln(p_0/m)], \quad (1)$$

where $\gamma=\mu/2p_0$. The function $f(\gamma)$ is given by

$$f(\gamma)=(2+\gamma)^2\ln(1/\gamma)-(1-\gamma^2)(3+\gamma^2). \quad (2)$$

It is of the order of unity except very near $p_0=\mu/2$. At $p_0\sim\mu$, and for $\tau\sim10^{-18}$ sec, Eq. (1) gives $\sigma_T\sim10^{-33}$ cm^2. Although the cross section is greater by an order of magnitude at several Bev, the problem of discrimination against multiple π^0 production probably restricts the energy to $p_0\sim\mu$ or less.

The mesons come out primarily forward or backward with a distribution function

$$\frac{dN}{dq}=\frac{1}{\omega}\left[p_0^2+\left(p_0-\frac{\omega+q}{2}\right)^2\right]\left[p_0^2+\left(p_0-\frac{\omega-q}{2}\right)^2\right], \quad (3)$$

where q is the momentum and ω the energy of the produced meson. The limits on q are zero and $q_m=p_0-(\mu^2/4p_0)$. The function $f(\gamma)$ in Eq. (1) is determined by integrating dN/dq between zero and q_m.

If it is not possible to measure the energy of the two photons resulting from the decay of the π^0 one must discriminate in some other way against the double bremsstrahlung process, which occurs in the same order of $\alpha=1/137$. If τ is sufficiently small, it may be possible to do this by using the angular spread of the photons (which is presumably of order m/p_0 for the bremsstrahlung). If not, the competing wide-angle cross section must be calculated and substrated from the observed events. It may be noted that the interference between the two modes of double photon production (π^0 and bremsstrahlung) is always smaller than the π^0 mode by one power of 1/137, independent of the value of τ. It is interesting that this would not necessarily be the case for strongly coupled particles, i.e., $p+\pi^-\rightarrow p+\pi^-$ directly and via the very narrow resonant Λ^0 state.

The calculation leading to Eq. (1) holds for any combination of electrons and positrons, since the exchange (or, for the electron-positron case, annihilation) contribution to σ_T is not proportional to $[\ln(p_0/m)]^2$. Furthermore, the coefficient of $[\ln(p_0/m)]^2$ in σ_T is the lifetime of the π^0 for decay into almost real photons (of mass comparable to the electron mass). The terms in σ_T of order $\ln(p_0/m)$ and unity involve virtual photons of mass $\sim p_0$, and therefore do not directly measure the π^0 lifetime. The order of magnitude of the effect of virtual photons of large mass can, for $p_0\lesssim\mu$, be estimated quite accurately by neglecting the variation of the π^0 decay matrix-element with photon mass and performing a complete electrodynamic calculation. This calculation, together with one of wide-angle double bremsstrahlung, is being done by Wilner.

II. DERIVATION OF EQ. (1)

We start from the well-known expression for the equivalent number of light quanta N_k per fast electron. This number is[3]

$$N_kdk=\frac{2\alpha}{\pi}\frac{dk}{k}\left(\frac{p_1^2+p_0^2}{2p_0^2}\right)\ln(\bar{p}/m), \quad (4)$$

* This work is supported in part by funds provided by the U. S. Atomic Energy Commission, the Office of Naval Research, and the Air Force Office of Scientific Research.

[1] G. Harris, J. Orear, and S. Taylor, Phys. Rev. **106**, 327 (1957).

[2] M. Jacob and J. Mathews, Phys. Rev. **117**, 855 (1960). F. E. Low, *1958 Annual International Conference on High-Energy Physics at* CERN, edited by B. Ferretti (CERN Scientific Information Service, Geneva, 1958), report of G. F. Chew.

[3] R. B. Curtis, Phys. Rev. **104**, 211 (1956); R. H. Dalitz and D. R. Yennie, Phys. Rev. **105**, 1598 (1957).

where $\alpha = 1/137$, p_1 and p_0 are the final and initial electron momenta, m is the electron mass, k the equivalent photon energy (or electron energy loss), $\bar{p}$ an average electron momentum of the order of $(p_1 p_0/k)$. To the extent that $\ln(\bar{p}/m) \gg 1$, we do not need the precise value of $\bar{p}$. Since the validity of Eq. (4) depends on just that inequality, we shall set $\bar{p} = p_0$.

In our problem we have two incident electrons. Again to logarithmic accuracy, each electron goes forward, and longitudinal photons and exchange scattering may be neglected.

The cross section for π^0 production will therefore be, in the center-of-mass system of the two electrons,

$$\sigma_T = \left(\frac{2\alpha}{\pi}\right)^2 \left(\ln\frac{p_0}{m}\right)^2 \int \frac{dk_1}{k_1}\frac{dk_2}{k_2}\frac{(p_1^2+p_0^2)(p_2^2+p_0^2)}{4p_0^4} \times \sigma(k_1,k_2), \quad (5)$$

where $\sigma(k_1,k_2)$ is the cross section for production of a π^0 by two oppositely directed photons of momenta k_1 and k_2, respectively.

To calculate $\sigma(k_1,k_2)$, let $\langle|M|^2\rangle_{\rm av}$ be the polarization-averaged square of the invariant matrix element connecting a pion to two photons.

The pion lifetime is given in terms of $\langle|M|^2\rangle_{\rm av}$ by

$$\frac{1}{\tau} = 2\pi \int_{2\pi} \frac{d\mathbf{k}}{(2\pi)^3}\frac{\delta(2k-\mu)}{(2k)^2(2\mu)} \times 4\langle|M|^2\rangle_{\rm av} = \frac{1}{8\pi\mu}\cdot\langle|M|^2\rangle_{\rm av} \quad (6)$$

where μ is the pion mass. The cross section $\sigma(k_1,k_2)$ is given by

$$\sigma(k_1,k_2) = \frac{(2\pi)^4}{2}\int \frac{d\mathbf{q}}{(2\pi)^3}\frac{\delta(\mathbf{k}_1+\mathbf{k}_2-\mathbf{q})\delta(k_1+k_2-\omega)}{(2k_1)(2k_2)2\omega} \times \langle|M|^2\rangle_{\rm av}. \quad (7)$$

Since $\mathbf{k}_1$ and $\mathbf{k}_2$ are in opposite directions we must have $q = k_1 - k_2$ and $\omega = k_1 + k_2$, or $4k_1k_2 = \mu^2$. Equation (7) then simplifies to

$$\sigma(k_1,k_2) = \frac{\pi}{\mu^2}\delta(4k_1k_2-\mu^2)\langle|M|^2\rangle_{\rm av} = \frac{8\pi^2}{\mu\tau}\delta(4k_1k_2-\mu^2) \quad (8)$$

whose invariant expression is

$$\sigma = \frac{8\pi^2}{\mu\tau}\delta((k_1+k_2)_\lambda{}^2+\mu^2). \quad (9)$$

We substitute (8) into (5) and obtain

$$\sigma_T = \frac{32\alpha^2}{\mu^3\tau}\left(\ln\frac{p_0}{m}\right)^2 \int dk_1 dk_2 \times \delta(4k_1k_2-\mu^2)\frac{(p_1^2+p_0^2)(p_2^2+p_0^2)}{p_0^4}. \quad (10)$$

Next, re-introduce the meson energy and momentum as independent variables according to

$$k_1 = (\omega+q)/2, \quad (11)$$

$$k_2 = (\omega-q)/2, \quad (12)$$

and

$$4k_1k_2 = \omega^2 - q^2, \quad \partial(k_1k_2)/\partial(\omega,q) = \tfrac{1}{2}:$$

$$\sigma_T = \frac{16\alpha^2}{\mu^3\tau}\left(\ln\frac{p_0}{m}\right)^2 \int d\omega dq \times \delta(\omega^2-q^2-\mu^2)\frac{(p_1^2+p_0^2)(p_2^2+p_0^2)}{p_0^4}$$

$$= \frac{16\alpha^2}{\mu^3\tau}\left(\ln\frac{p_0}{m}\right)^2 \times \frac{1}{2}\int \frac{dq}{\omega}\frac{(p_1^2+p_0^2)(p_2^2+p_0^2)}{p_0^4}, \quad (13)$$

where

$$p_1 = p_0 - (\omega+q)/2 \quad \text{and} \quad p_2 = p_0 - (\omega-q)/2. \quad (14)$$

The momentum spectrum of produced mesons is given by Eq. (13). The limits on q are determined by the condition that p_1 and p_2 be greater than zero, that is $-q_m \leqslant q \leqslant q_m$ where

$$q_m = p_0 - (\mu^2/4p_0). \quad (15)$$

The final result for σ_T, Eqs. (1) and (2), is obtained by carrying out the integral in Eq. (13) between the limits given by Eq. (15).

I would like to thank Professor Feshbach for a helpful conversation.