

Search for a Second π^0

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The energy distribution of neutrons from the charge exchange reaction $\pi^- + p \rightarrow \pi^0 + n$ has been studied, for π^- mesons stopped in liquid hydrogen, to investigate the possible existence of a second π^0 with mass within a few Mev of the mass of the "ordinary" π^0 . No neutron group corresponding to such a second π^0 was seen. The sensitivity of the measurement was such that a second group of relative intensity above 10–20% would have been seen for any second π^0 with a mass in the range between about $\frac{3}{4}$ Mev and 2 Mev away from the mass of the ordinary π^0 .

The data also give a lower limit to the π^0 lifetime: $\tau > \sim 5 \times 10^{-21}$ sec.

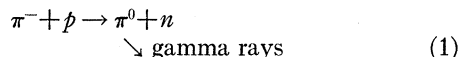
INTRODUCTION AND SUMMARY

THE " π meson" is considered to consist of an isotopic-spin triplet, the π^+ , π^0 , π^- mesons. It is well known, however, that insofar as the π meson is intimately associated with the isotopic-spin- $\frac{1}{2}$ nucleons, there "must exist" an isotopic-spin *singlet* boson in addition to the triplet. This second kind of π^0 , with isotopic spin $T=0$, may be called the π_0^0 .

The question may be asked whether the mass of the π_0^0 might be close to that of the π^0 . The theory of elementary particles does not presently permit a decisive theoretical answer to this question. We consider then the experimental evidence on the mass or masses of the neutral pi meson.

During the first ten years or so following the detection and identification of the π^0 , measurements of its mass were made and reported with an accuracy of about $0.3m_e$ ¹ (m_e =mass of the electron). However, the experiments on which this result was based (see references given in 1) were analyzed on the assumption that only a single π^0 exists. A second group of π^0 's could have escaped detection if sufficiently weak—for mass differences up to 1 or 2 Mev, a second group of relative intensity 20% or more could have escaped detection.

In the course of some thoughts on methods of experimentally studying the lifetime of the π^0 ,² it became clear that a study of the Q value of the reaction



could provide a sensitive means of investigating whether two different neutral pi mesons of relatively close mass (i.e., within a few Mev) might in fact be produced in this reaction. In order to investigate this possibility, as well as to determine a lower limit for the lifetime of the π^0 , the experiment reported here was carried out.

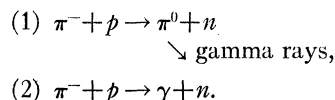
In this experiment, the Q value of reaction (1) was determined by measuring the energy spectrum of neutrons produced in reaction (1), using π^- mesons

brought to rest in hydrogen. The neutron spectrum was determined by a time-of-flight technique. The result of the experiment is that no neutron group corresponding to a second π^0 was seen. The data are such that within a confidence limit of 90% a second group of relative intensity above 10%–20% would have been seen for any second π^0 with a mass in the range between about $\frac{3}{4}$ Mev and 2 Mev away from the mass of the ordinary π^0 .³ (The detection system made use of the decay γ rays from the π^0 ; it is assumed that any second π^0 in the above mass range would have decay characteristics similar to those of the ordinary π^0 .)

APPARATUS

The energy of the neutrons produced by bringing π^- mesons to rest in a thin liquid hydrogen target (2 cm) was measured by a time-of-flight technique. The experimental arrangement is shown in Fig. 1.

The capture of π^- mesons in hydrogen leads to the reactions



In each case, one or two gamma rays are produced essentially at the same time as the neutron (except in a very small fraction of events, in which only "internal conversion" electrons appear, and no γ rays). The "zero time" signal was obtained from the scintillation counter γ_3 , which in conjunction with the counters γ_1 (anticoincidence) and γ_2 (coincidence) detected γ rays by conversion in a Pb converter $\frac{1}{4}$ inch thick.

The neutron signal was obtained from the scintillation counters n_1 and n_2 . These were identical counters, two being used simply to increase the detection solid angle.

The "zero time" and neutron signals were delayed in lengths of RG63/U cable several hundred feet long,

¹ Cohen, Crowe, and Dumond, *Fundamental Constants of Physics* (Interscience Publishers, New York, 1957), p. 47.

² K. Lande and W. Selove, Atomic Energy Commission Report NYO-8546, April, 1958 (unpublished).

³ The "ordinary" π^0 detected in this work is to be identified as the neutral member of the $T=1$ triplet. Recent work on the consistency of low-energy pion-nucleon interactions, e.g., M. Cini, R. Gatto, E. L. Goldwasser, and M. Ruderman, *Nuovo cimento* **10**, 243 (1958), indicates that the various low-energy measurements are consistent with the idea that only a single π^0 is present; the situation was less clear when the present work was begun.

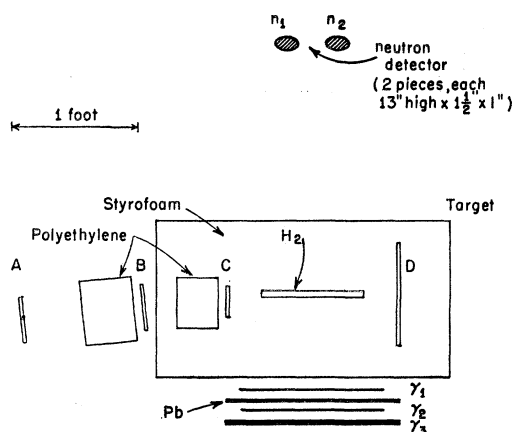


FIG. 1. Experimental arrangement, top view. All counters plastic scintillator. A, B—4 in. \times 4 in. \times $\frac{1}{4}$ in.; C—3 in. \times 6 in. \times $\frac{3}{8}$ in.; D—10 in. \times 10 in. \times $\frac{3}{8}$ in.; γ_1 , γ_2 —14 in. \times 14 in. \times $\frac{1}{8}$ in.; γ_3 —17 in. \times 17 in. \times $\frac{1}{8}$ in. The liquid hydrogen target proper was contained in a stainless steel container 10 in. \times 10 in. \times $\frac{3}{4}$ in., with a 0.001-inch wall. This container, with a liquid hydrogen reservoir, was enclosed in a Styrofoam case, with a heat shield at liquid nitrogen temperature.

while the coincidence and pulse size tests, described below, were made. The time between acceptable signals was then measured simultaneously with a vernier chronotron⁴ and an oscilloscope.

Events were time analyzed when the following requirements were met (see Fig. 2):

1. that a γ ray be detected in coincidence (10 nanoseconds)⁵ with an incoming π^- ;
2. that a neutron signal occur in an interval from 0.1 μ sec ahead to 0.5 μ sec following the γ -ray signal;
3. that the neutron signal be of proper size.

The π^- beam was monitored by counters A and B, connected to a fast coincidence circuit (4 nsec). Since many of the pions going through A and B would not stop in the hydrogen in the target, a third pion counter, C, was located just ahead (5 inches) of the target. This greatly reduced the accidental rate between incoming particles and the γ -ray telescope; it also helped eliminate background from pions scattered into the γ telescope after going through A and B.

Gamma rays were identified by requiring a coincidence between γ_2 and γ_3 and no signal from γ_1 . A two-counter telescope was used rather than a single counter to reduce background from fast neutrons produced by π^- capture in the material surrounding the hydrogen target. The "zero time" signal was taken from γ_3 , which was especially designed to provide very good time resolution; an independent measurement showed that for signals from γ rays, this counter had an effective resolution of about 1 nsec despite its rather large size (17 in. \times 17 in.).

The totality of coincidence requirements on the

"zero time" signal can be written as $(AB)C\bar{\gamma}_1\gamma_2\gamma_3$, the bar indicating anticoincidence. It can be seen from Fig. 2 that the fivefold coincidence circuit C_2 sufficed to fulfill requirement 1. The output of C_2 was a rectangular pulse 0.6 μ sec in duration. This pulse was used in fulfilling requirement 2.

The neutrons produced in reaction (1) have a kinetic energy of about 0.4 Mev. The pulse size resulting from recoil protons produced in the detector by these neutrons is not far above the pulse size from individual photoelectrons in the photomultipliers.⁶ In order to reduce accidentals, each piece of the neutron detector was viewed by two photomultiplier tubes, and a coincidence was required between these two tubes. In order to make the detection efficiency as high as possible, and at the same time obtain good time resolution, the neutron detector was divided into two pieces, as indicated in Fig. 1. In this way optimum optical coupling to the photomultipliers was obtained with the 6810A tubes used.

A further reduction in the over-all accidental rate was obtained during most of the data taking by using those neutron signals known to be of proper size for a 0.4-Mev neutron (requirement 3 above). The auxiliary fast pulse-height-analyzer circuit was used for this purpose.

If the output of C_3 fell within the 0.6- μ sec gate from C_2 , then C_4 gave an output signal which was used to form a trigger pulse for the oscilloscope. The delays were set to accept neutron signals which appeared from -0.1μ sec to $+0.5 \mu$ sec with respect to the "zero time" signal. Thus, the random background at negative times was also measured.

The neutron and "zero time" signals, combined in a distributed mixer, were displayed on one trace of a

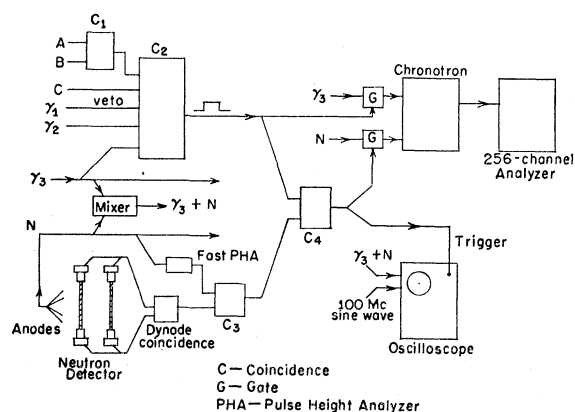


FIG. 2. Simplified block diagram of electronics. Auxiliary amplifiers and delays are not shown. See text for further discussion.

⁶ In a separate experiment the pulse size from neutrons in the fractional Mev range was measured; see M. Gettner and W. Selove, Rev. Sci. Instr. **31**, 450 (1960). The relative detection efficiency for different neutron energies was also obtained for the detector used in the present experiment.

⁴ H. W. Lefevre and J. T. Russell, Rev. Sci. Instr. **30**, 159 (1959).

⁵ 1 nanosecond (nsec) = 10^{-9} sec.

Tektronix 517A oscilloscope, in which a T57 dual-beam cathode-ray tube had been installed. A 100-Mc/sec sine wave,⁷ which was displayed on the other trace, was photographed with each pair of pulses. The time interval between the two pulses was taken as the interval between the peaks. The same set of pulses scanned by several people showed a reading error of ± 0.9 nsec.

γ_3 ("zero time" signals) and neutron signals were time analyzed by the chronotron at the same time they were recorded on the oscilloscope. The chronotron functioned by allowing the "zero time" signal and the neutron signal to circulate in loops of slightly different lengths, with the "zero time" signal in the longer loop, until the signals were in coincidence. The time between pulses is the number of circulations made by the neutron pulse multiplied by the difference in electrical length between the two loops. An R.C.L. 256-channel analyzer sorted the events into channels according to the number of circulations made, and recorded the number of events in each channel.

To reduce background events, narrow gates were used at the chronotron inputs. A "zero time" pulse was allowed to circulate only if requirement (1) above had been met. The neutron pulse was gated into the chronotron only if all three requirements listed above had been met.

Pulses of known time separation were regularly injected in place of the neutron and "zero time" signals to calibrate the chronotron. These signals were formed by using a mercury pulser to produce a pulse similar in shape and duration to the γ_3 and neutron pulses, and dividing the pulser output into two signals, one the size of a typical "zero time" signal and the other the size of a typical 0.4-Mev neutron pulse. The "neutron" signal was delayed various amounts by cables of known length. The channel number corresponding to each cable length was then recorded. The chronotron was calibrated in this manner approximately every hour.

Since the calibration signal simulated an actual event, it appeared on the oscilloscope. Periodically the calibration signals were photographed for each of the delay cables used. The scanning of these photographs showed the time interval for the calibration signals remained constant, within the reading error of ± 0.9 nsec; therefore, no systematic drifts in the oscilloscope-plus-calibration system were present. The values of the delays measured with the oscilloscope were used to calculate the channel width of the chronotron.

PROCEDURE AND RESULTS

The basic procedure by which we studied the energy spectrum of neutrons in the vicinity of 0.4 Mev was to measure the time-of-flight distribution for several

different neutron flight distances. In this way, any effect of signal-time dependence on pulse height was minimized, and the necessity for a precise measurement of the internal time delays in the equipment was avoided. Also, by examining flight time spectra at different distances any spurious bump due either to instrumentation or to scattering could be properly identified.

We made a check of the position of true zero on our time-of-flight scale, by using events in which the two γ 's from π^0 decay were detected, one in the γ detector and one in the neutron detector. For this measurement, a $\frac{1}{8}$ -in. thick lead converter was placed in front of the neutron detector to increase its γ detection efficiency, and an attenuator was inserted in the neutron signal line, to give signals of size comparable to those from 0.4-Mev neutrons. The attenuator delay, for signals of the actual shape occurring, was measured separately.

The time of flight of the 8.8-Mev neutrons from the radiative capture reaction $\pi^- + p \rightarrow \gamma + n$ was also measured with this arrangement. Using the oscilloscope data, the position of this group when corrected for the attenuator delay agreed with the position of the same group found in data taken with pulses about one hundred times smaller. (The pulse size spectrum for proton recoils from neutrons extends down to zero.) The mean position of the groups differed by 0.1 ± 0.6 nsec, showing that there was no measurable dependence of signal time on pulse height.

From the 8.8-Mev neutron time of flight at 2 feet, a value of 142 ± 4 Mev was calculated for the π^- mass. This is in good agreement with the accepted value of 139.63 ± 0.04 Mev.¹

With the lead converter and attenuator removed, neutron time-of-flight spectra were measured at distances of 2 ft, $2\frac{1}{2}$ ft, and 3 ft. During most of the data taking, the neutron signal pulse-height acceptance limits were set approximately a factor of $1\frac{1}{2}$ beyond the values which gave the best signal-to-background ratio for the 0.4-Mev group. (A small fraction of the 2-ft oscilloscope data was taken with no pulse-height restriction applied.) With the help of the chronotron these limits were set, in a short time, while the experiment was in progress. Subsequent measurement of neutron pulse heights in this detector, both by analysis of the oscilloscope data and in a separate experiment carried out with monoenergetic neutrons, confirmed that the pulse-height limits had been set as desired with respect to the 0.4-Mev neutrons.

To obtain time values for the γ -ray and 8.8-Mev groups, the mean of each distribution was calculated; the uncertainty assigned corresponds to the calculated standard deviation of the mean.

The time distribution for the 0.4-Mev neutron group was not symmetric. The asymmetry observed agrees approximately with the calculated effect of the relatively large angular intervals subtended by the hydrogen target and the neutron detector at each other's position.

⁷ This signal was obtained from a frequency doubler driven by the 50-Mc/sec signal from a Tektronix type 180 time-mark generator.

TABLE I. Pulse times, in nsec.

Distance	γ time	Oscilloscope		Flight times		Chronotron
		Neutron times 0.4 Mev	8.8 Mev	0.4 Mev	8.8 Mev	flight time 0.4-Mev neutron
2 ft	174.1 ± 0.5	246.0 ± 0.5	188.6 ± 0.2	71.9 ± 0.7	14.5 ± 0.4	71.9 ± 0.8
$2\frac{1}{2}$ ft	174.4 ± 0.5	261.0 ± 0.5		86.6 ± 0.7		90.3 ± 1.1
3 ft	174.2 ± 0.4	279.5 ± 0.5		105.3 ± 0.6		104.5 ± 0.9

The time assigned to the 0.4-Mev group was the most probable value, corresponding to the time of flight of neutrons having the most probable flight path. The uncertainty assigned represents an estimate of the precision with which the distribution defines a most probable value, and is approximately σ/\sqrt{n} , where n is the number of events above background in the group and 2σ is the width which includes the central $\frac{2}{3}$ of the events.

The neutron flight time is the difference between the measured times of the γ -ray and neutron groups, plus the calculated flight time of the γ ray to the neutron counter. The measured time of the γ -ray group was also corrected for the delay in the attenuator used when this group was measured. This procedure does not take account of any systematic signal time dependence on pulse height; but as discussed above, the oscilloscope data, which were used for the final results, showed no evidence of such a dependence.

Taking the effects of the finite size of the target and neutron detector into account, the most probable flight paths for detected 0.4-Mev neutrons were calculated to be 24.3 ± 0.25 in., 30.3 ± 0.25 in., 36.2 ± 0.25 in. The error stated is the estimated uncertainty in the location of the hydrogen target in its Styrofoam container. Flight time spectra from a 2-ft run are shown in Fig. 3.

The flight times obtained are listed in Table I. The γ -ray times shown were obtained from the oscilloscope data and have been corrected for the γ -ray flight time to the neutron detector and for the attenuator delay of 11.0 ± 0.3 nsec. The small dispersion in the γ -time values reflects the good stability of the oscilloscope system.

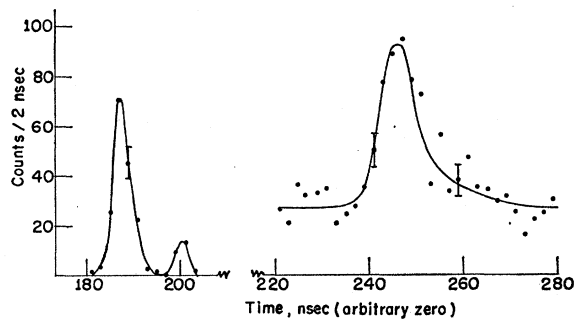


FIG. 3. Data from a run at 2 ft, as obtained from the oscilloscope. The left-hand part shows the " γ " and 8.8-Mev groups, from a short run, with a Pb converter in front of the neutron detector. The right-hand part shows the 0.4-Mev group, for a run of longer duration. Standard deviation is shown for a few points.

The three flight time measurements of the charge exchange neutrons, when plotted as a function of flight path, should ideally define a straight line passing through the origin and having a slope equal to the inverse of the particle velocity. A least squares calculation which took into account the uncertainties in the time and distance measurements was used to determine the slope and intercept of the line best fitted by the experimental points. The uncertainty in the calculated quantities was also obtained.

The values of the slope and intercept were separately calculated for the chronotron and oscilloscope data. The values of the intercept were 5.0 ± 4 nsec and 2.0 ± 4 nsec, respectively. We conclude that no large undetected systematic errors were present. The χ^2 test for each determination gave a probability of about 20%.

The inverse velocity obtained was 33.1 ± 1.6 nsec/ft for the chronotron data, 33.9 ± 1.4 nsec/ft for the oscilloscope data. Since the oscilloscope and chronotron data were obtained from the same group of events, they are not statistically independent; therefore, the two values of the inverse velocity cannot be combined in a meaningful manner. The value obtained from the oscilloscope data was used to calculate the $\pi^- - \pi^0$ mass difference because the oscilloscope system showed better stability (and had better resolution) than the chronotron.

The value of 33.9 ± 1.4 nsec/ft corresponds to a $\pi^- - \pi^0$ mass difference of $(9.0 \pm 0.5)m_e$ (m_e = electron mass). We believe this value to be free of systematic error than our earlier reported value of $(9.6 \pm 0.5)m_e$,⁸ which was based on a time-of-flight measurement at a single distance. Using a method similar to ours, values of $(9.05 \pm 0.09)m_e$ ⁹ and $(9.01 \pm 0.08)m_e$ ¹⁰ have been reported.

To investigate the possible existence of a neutron group from a second π^0 , all the data from the three distances were combined in a single plot, shown in Fig. 4. Since the oscilloscope system had much better stability than the chronotron, the scope data were combined using smaller time intervals. For this reason the oscilloscope data were used to determine the

⁸ M. Gettner, L. Holloway, D. Kraus, K. Lande, E. Leboy, and W. Selove, Phys. Rev. Letters **2**, 471 (1959).

⁹ R. Haddock, A. Abashian, K. Crowe, and J. Czirr, Phys. Rev. Letters **3**, 478 (1959).

¹⁰ P. Hillman, W. C. Middlekoop, T. Yamagata, and E. Zavattini, Nuovo cimento **14**, 887 (1959).

maximum possible intensity of any second slow neutron group present.

The shape of the curve shown in Fig. 4 is consistent with the presence of a single neutron group. The 9-nsec width at half maximum of the observed group can be accounted for by contributions of 8 nsec (as measured in a separate experiment) from photomultiplier and scintillator time jitter, 3 nsec from the distribution of flight paths, and 1 nsec from the reading error. On both sides of the peak, the distribution of experimental points is consistent with a uniform random background.

We can set an upper limit to the strength with which a second slow neutron group, from a second π^0 , could be present and yet undetected in this experiment. We have calculated this upper limit for several possible mass values of a second π^0 , in the manner described below. The results, for a second π^0 heavier than the "ordinary" one, are shown in Fig. 5, which gives the maximum intensity of a second group, relative to the main group, as a function of the mass difference $\Delta\mu$ between the ordinary π^0 and a second π^0 . This curve corresponds to a confidence level of 90%.

The maximum possible intensity of a second group of neutrons which could be present in the main group itself was calculated by finding the minimum number of counts needed to increase the observed width by a clearly visible amount over the calculated width of 9 nsec. The curve in Fig. 5 goes to 1.0 for a mass difference of about 0.3 Mev—if there were two π^0 's present within this mass difference, that fact could not be detected in this experiment whatever the relative intensity.

An upper limit on the intensity of any second slow neutron group distinct from the main group can be set in the following way. First, we note that half of the

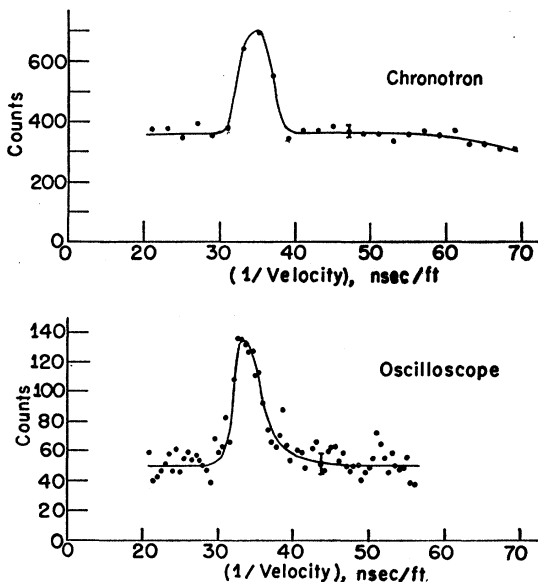
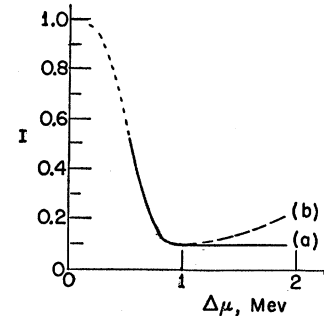


FIG. 4. Combined data for all flight distances.

FIG. 5. Maximum intensity I of a second π^0 heavier than the ordinary one by $\Delta\mu$ Mev. I is given relative to the strength of the main group. (a) Maximum number of counts; (b) maximum number of neutrons produced. See text.



events in the main group are contained in a $2\frac{1}{2}$ -nsec/ft interval, the other half lying in an interval of roughly 7 nsec/ft. Next, we note that a second group of such intensity as to correspond to one standard deviation (σ) on background, in a $2\frac{1}{2}$ -nsec/ft interval, would correspond to about 4% of the intensity of the main group. We can then choose some convenient deviation from the flat background to set a criterion for detectability of any second group. A deviation of about $2\frac{1}{2}\sigma$ represents a reasonable criterion, for example, by the following argument. The chance that in any one $2\frac{1}{2}$ -nsec/ft interval the observed count would be larger than the average value by more than $2\frac{1}{2}\sigma$ is about 1 in 160. There are roughly 10 such $2\frac{1}{2}$ -nsec/ft intervals in the flat background region in Fig. 4; consequently, the chance that any one would be larger by more than $2\frac{1}{2}\sigma$ is about 6%. On the other hand, if there were a true second group, of size corresponding to $2\frac{1}{2}\sigma$ (so corresponding to 10% of the main group), the chance that the observed count would then be larger than the average background value by $2\frac{1}{2}\sigma$ would be 50%. Thus, the relative odds that a true 10% second peak would give a count more than $2\frac{1}{2}\sigma$ above average background would be about 8 to 1 (50% divided by 6%). In this way we can say that with about 90% confidence any second peak more than 10% as large as the main peak would have been detected.

In the entire region of flat background, we conclude from this analysis that any second group present must contain a number of counts less than about 10% of the main group. In order to give a corresponding figure concerning the maximum possible neutron intensity in any such second group, the preceding figure must be modified to account for the decrease in detection efficiency for lower energy neutrons¹¹ and the decrease in the number of lower energy neutrons which could escape the target unscattered.

We have calculated these effects for neutrons of lower energy than 0.4 Mev; the results are shown in Fig. 5, curve (b). For a second π^0 heavier than the ordinary one, we conclude from our data that the intensity of the corresponding neutron group must be less than a

¹¹ The relative detection efficiency for neutrons of various energies was measured in a separate experiment. Over the range of energies covered by the oscilloscope data in Fig. 4, the relative efficiency varied by somewhat less than a factor of 2.

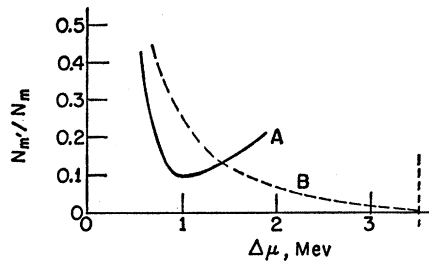


FIG. 6. Upper limit, $N_{m'}/N_m$, for production of a second π^0 (mass m') relative to production of the ordinary π^0 (mass m), as a function of the mass difference $\Delta\mu = m' - m$. A—this experiment ($\sim 90\%$ confidence limit); B—Cassels *et al.* (see reference 12).

fraction f of the main group intensity, where f ranges from about 10% for a mass difference of $\frac{3}{4}$ Mev to about 20% for a mass difference of 2 Mev.

For any second π^0 lighter than the ordinary π^0 , our data indicate a similar upper limit, in the range between $\frac{3}{4}$ and 2 Mev from the ordinary π^0 .

A recent study of the $\pi^- + p$ charge exchange reaction at rest, with the specific purpose of searching for a second π^0 , has been recently reported by Cassels, Jones, Murphy, and O'Neill.¹² In that study the $\pi^- - \pi^0$ mass difference is investigated by measuring the angular correlation of the decay γ rays from the π^0 . Their results enable them to set an upper limit on the intensity of any second group of π^0 's decaying into two gamma rays. The results from this experiment, and from the work of Cassels *et al.*, are shown in comparable terms in Fig. 6, for a second π^0 heavier than the ordinary one. For a second π^0 lighter than the ordinary one, both

¹² J. M. Cassels, D. P. Jones, P. G. Murphy, and P. L. O'Neill, Proc. Phys. Soc. (London) 74, 92 (1959).

experiments indicate an upper intensity limit with a $|\Delta\mu|$ dependence similar to, and somewhat tighter than, that shown in Fig. 6.

Finally, we may remark on the lower limit for the π^0 lifetime which can be derived from the present data. The shape of the peak in Fig. 4 (oscilloscope data) is in good agreement with the shape calculated taking into account the intrinsic time resolution of the equipment, the finite geometry effects, and the self-absorption in the target and in the detector. From the sharpness of the rise of the peak we can then conclude, from the uncertainty-principle relation $\Delta E \Delta t \sim \hbar$, that the lifetime is greater than $\sim 5 \times 10^{-21}$ sec. The true lifetime is of course believed to be much greater than this, and there are other measurements, of an indirect type, indicating a larger value for the lower limit.¹³

ACKNOWLEDGMENTS

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¹³ See, e.g., L. G. Hyman, R. Ely, D. H. Frisch, and M. A. Wahlig, Phys. Rev. Letters 3, 93 (1959).