Two-Nucleon L·S Potential in Pseudoscalar Meson Theory*

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Nonstatic corrections to the two-nucleon potential of Brueckner and Watson and of Gartenhaus are computed within the framework of the γ_5 theory. These terms appear as spin-orbit corrections of order μ/M to the static potentials.

The S matrix is calculated in second and fourth order for a reduced form of the relativistic theory. The potential is then chosen so as to duplicate this S matrix to the required order in the coupling constant and μ/M. We consider to what extent our reduction of the γ_5 theory changes its character.

The resulting potentials are given in analytic form for no cutoff in momentum space and in numerical form for the Gaussian cutoff employed by Gartenhaus. We give also some additional static corrections to previous potentials. A qualitative comparison is made with the experimental observations in nucleon-nucleon scattering, the fine structure in the splitting of the He^5 nucleus, and the contribution of the nonstatic potential to the magnetic moment of the deuteron.

1. INTRODUCTION

THE fine-structure splittings of the levels in the nucleus have prompted many people to propose that a velocity-dependent (spin-orbit term) be included in the two-nucleon potentials. Recently Signell and Marshak[1] have shown that a good fit to the unpolarized and polarized two-nucleon elastic scattering data up to 150 Mev may be obtained by adding an empirical, charge-independent, short-range, attractive, spin-orbit potential to the Gartenhaus potential.[2] Gartenhaus used the nonrelativistic, P-wave, extended-source, Hamiltonian. He calculated in perturbation theory keeping terms to fourth order in the coupling constant. However, he omitted the so-called "ladder corrections," dropping them with the aid of "Brueckner and Watson's argument."[3] These terms lead, as is well known, to an unbound deuteron.

The Gartenhaus meson-theoretic potential gives a good fit to all of the low-energy two-nucleon data. This is both encouraging and surprising since the use of perturbation theory in meson theory calculations has dubious validity. Presumably the inclusion of a renormalized coupling constant $f^2/4\pi\approx 0.089$ and a cutoff energy $\omega_{\max}\approx 6\mu$, which are determined from real meson-nucleon scattering and photoproduction at low energies, includes certain higher order effects. The hope, of course, is that the higher order effects which are not included in this manner modify only the high-energy, short-range, behavior of the potential.

It is interesting to investigate the (first-order) nucleon velocity-dependent terms in meson theory to see if a plausible explanation for the spin-orbit term can be given. We will follow precisely the philosophy of Brueckner and Watson (BW) and Gartenhaus (G) in deriving these terms. Our Hamiltonian will be a nonrelativistic reduction of the (PS) (PS) theory. The potential itself will then be computed using nonrelativistic perturbation theory. As in the case of G we drop all so-called "ladder terms." The (PS) (PS) theory in this manner leads to a spin-orbit potential of the type postulated by Signell and Marshak (SM). Our results will be an expansion in powers of the coupling constant and the ratio of the velocity of the nucleons to that of light. We shall assume that $(v/c)_{\text{nucleon}}\approx\mu/M$. We shall keep only the second- and fourth-order terms in the coupling constant. Only the zeroth order term in μ/M for the instantaneous potential and the first-order term in μ/M for the $\mathbf{L}\cdot\mathbf{S}$ potential will be retained.

Other treatments of this problem have been given: Sato, Itabashi, and Sato,[4] Klein,[5] Marshak and Okubo,[6] and most recently Sugawara and Okubo.[7] All, except Sugawara and Okubo,[7] have used a modified type of (PS) (PV) theory. These treatments[4–6] include additional terms in the coupling Hamiltonian but consider only their $\mathbf{L}\cdot\mathbf{S}$ effects. (By additional we mean terms other than the usual $\boldsymbol{\sigma}\cdot\nabla\phi$ coupling used by Gartenhaus.) Also, it may be pointed out that these earlier treatments concerned themselves only with cutoff-independent Hamiltonians. As a result, the final potentials in position space are highly singular at the origin. In order to avoid this singularity, the potentials are simply set equal to zero for all radii smaller than an arbitrary cutoff distance.

In addition to computing the $\mathbf{L}\cdot\mathbf{S}$ pieces of the potential, we have computed the corrections to the Gartenhaus static potential. We have done this for the Hamiltonian with and without cutoff. For the case of the Hamiltonian without cutoff our analytic forms for

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[4] S. Sato, K. Itabashi, and I. Sato, Progr. Theoret. Phys. (Kyoto) **14**, 303 (1955).

[5] A. Klein, R. Raphael, and N. Tzoar, Phys. Rev. Letters **2**, 433 (1959).

[6] R. E. Marshak and S. Okubo, Ann. Phys. **4**, 166 (1958).

[7] M. Sugawara and S. Okubo, Phys. Rev. **117**, 605, 611 (1960).

the potential are consistent with the results of the previous investigators.

In Sec. II we discuss our reduction of the theory to an effective nonrelativistic Hamiltonian. In Sec. III we discuss the definition of a potential and give the potentials resulting from the specific form of the interaction chosen. In Sec. IV we compare the potential with the experimental results.

II. REDUCTION OF THE (*PS*) (*PS*) THEORY

The nonrelativistic limit of the symmetric (*PS*) (*PS*) theory is defined, for our purposes, by the application of a Foldy transformation to the relativistic Hamiltonian.[8,9] That is,

$$H'=e^{iS}He^{-iS}-ie^{iS}(\partial/\partial t)(e^{-iS}), \qquad (1)$$

where

$$H=\boldsymbol{\alpha}\cdot\mathbf{p}+\beta M+g\beta\gamma_5\boldsymbol{\tau}\cdot\phi, \qquad (2)$$

and

$$S=-(i\beta/2M)(\boldsymbol{\alpha}\cdot\mathbf{p}+g\beta\gamma_5\boldsymbol{\tau}\cdot\phi). \qquad (3)$$

We find to order g^2/M^3 that

$$\begin{aligned}H'=M+p^2/2M+g^2\boldsymbol{\phi}^2/2M+(g/2M)\boldsymbol{\sigma}\cdot\nabla\boldsymbol{\phi}\\ +g^2(\partial\boldsymbol{\phi}/\partial t)^2/8M^3+(g^2/4M^2)\boldsymbol{\tau}\cdot\phi\times\phi\\ +(g/8M^2)[\boldsymbol{\sigma}\cdot\mathbf{p},\partial\boldsymbol{\phi}/\partial t]_+\\ -(1/8M^3)[(\boldsymbol{\sigma}\cdot\mathbf{p}-ig\boldsymbol{\phi})(\boldsymbol{\sigma}\cdot\mathbf{p}+ig\boldsymbol{\phi})]^2, \qquad (4)\end{aligned}$$

where

$$\boldsymbol{\phi}=\sum_{i=1}^{3}\tau_i\phi_i=\boldsymbol{\tau}\cdot\phi.$$

It has long been known, that the (*PS*) (*PS*) theory leads to an extremely strong *S*-wave scattering in Born approximation.[10,11] This is in sharp disagreement with experiment. It has been suggested that a more exact solution of the field equations results in a self-damping of the *S*-wave pions. It is argued that this damping is so strong that the *S*-wave effects may be neglected altogether. If the only term in the static theory interaction Hamiltonian is of the form $\boldsymbol{\phi}^2$, then it is possible to solve exactly the Heisenberg equations of motion for the meson field variables.[12] The scattering is reduced in this case from its value in Born approximation (BA) by approximately a factor of one hundred. The BA does not take into account properly the diminution of the wave function near the origin due to the repulsive potential simulated by the $\boldsymbol{\phi}^2$ term. Since we shall use low-order perturbation theory to determine the scattering of two nucleons, we must regard the coefficients of all the terms in the reduced Hamiltonian as subject to effective renormalization by higher-order interactions. We cannot calculate the relative renormalization of the various terms of H, but we shall adjust their coefficients so that they agree with the experimental $\pi-N$ scattering. Thus the coefficients of the terms in H are regarded as experimentally measured quantities. Meson-meson scattering would also contribute to the nucleon force and possibly to the $\mathbf{L}\cdot\mathbf{S}$ term (if it were strong enough). We omit entirely any such term in the Hamiltonian. The importance of the FW transformation is that it allows us to start with the nonrelativistic (*PS*) (*PS*) theory and modify it in what we consider a reasonable way to fit experiment. Suppression of pair terms in the γ_5 covariant theory is more difficult.[13]

We have investigated the question of how much this procedure changes by dropping of the γ_5 theory. We suppress pair terms completely by dropping $g^2\boldsymbol{\phi}^2/2M$ from the reduced Hamiltonian[4] and then compute the second and fourth order, energy shell, scattering matrix element for two nucleons. We then expand the answer in powers of μ/M and retain the terms of relative order one (velocity independent) and μ/M (velocity dependent). We then compute, within the framework of the relativistic γ_5 theory, that part of the same matrix element which is proportional to $\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2$. The relativistic matrix element was integrated over the fourth component of the virtual meson momentum, and expanded to the appropriate order in μ/M. The coefficient of $\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2$ in both cases was the same. Since the $\boldsymbol{\phi}^2$ interaction cannot lead to terms proportional to $\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2$, this comparison indicates that, except for the explicit suppression of pair terms, the nonrelativistic approach is the same as the γ_5 theory. We therefore write in place of (4)

$$\begin{aligned}H'=M+\mathbf{p}^2/2M+2M\alpha(f/\mu)^2\boldsymbol{\phi}^2+(f/\mu)\boldsymbol{\sigma}\cdot\nabla\boldsymbol{\phi}\\ +\gamma(f/\mu)^2\boldsymbol{\tau}\cdot\phi\times\phi+\beta(f/\mu)(1/4M)[\boldsymbol{\sigma}\cdot\mathbf{p},\partial\boldsymbol{\phi}/\partial t]_+\\ +(\epsilon/2M)(f/\mu)^2\boldsymbol{\phi}^2\\ -(\delta/8M^3)[(\sigma\cdot\mathbf{p}-ig\boldsymbol{\phi})(\boldsymbol{\sigma}\cdot\mathbf{p}+ig\boldsymbol{\phi})]^2, \qquad (5)\end{aligned}$$

where α, β, γ, δ, ϵ are empirical damping factors to be determined and we have used $(f/\mu)=g/2M$, the static constant, $f^2/4\pi\approx 0.10$.

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[9] J. M. Berger, L. L. Foldy, and R. K. Osborn, Phys. Rev. **87**, 1061 (1952).
[10] G. Wentzel, Phys. Rev. **80**, 802 (1952).
[11] A. Klein, Phys. Rev. **95**, 1061 (1954).
[12] S. Drell and E. M. Henley, Phys. Rev. **88**, 1053 (1953).

[13] M. Gell-Mann had suggested the following way of treating the relativistic γ_5 theory. The nucleon field operator satisfies the equation $(i\boldsymbol{\partial}-M-g\gamma_5\boldsymbol{\phi})\psi=0$. If we multiply on the left by $(i\boldsymbol{\partial}+M-g\gamma_5\boldsymbol{\phi})$, we arrive at a second-order equation;

$$[\square^2-M^2-g\gamma_\mu\gamma_5\partial_\mu\boldsymbol{\phi}-g^2\boldsymbol{\phi}^2]\psi=0.$$

It would seem that a possible method of getting rid of *S*-wave pions in the pseudoscalar theory would be to throw out the $\boldsymbol{\phi}^2$ term in this "squared" theory. The resulting "truncated" theory which is still relativistic may then be used to calculate an $\mathbf{L}\cdot\mathbf{S}$ potential. The truncated theory is quite similar to the (*PS*) (*PV*) theory. That is, the meson coupling at each vertex is $\gamma_5\boldsymbol{k}$ where k is the four-momentum of the meson in question. On the other hand, the nucleon propagators are $1/(\square^2-M^2)$. Thus, the theory is not as divergent in higher orders as the (*PS*) (*PV*) theory. We have calculated these potentials. The results are not the same as the $\mathbf{L}\cdot\mathbf{S}$ potential arrived at from the Foldy transformation, although quite similar in character. However, this truncated theory is not satisfactory since, for various processes, the resulting S matrix seems to be nonunitary. Also, the identification of the BW "ladder piece" (see Sec. III) is not clear in a relativistic expression.

Theoretical static model calculations taking into account S-wave meson rescattering effects seem to indicate that one should take $\alpha \approx 0.02$ and $\gamma \approx 0.7$,[14] if one is to fit the S-wave pion-nucleon scattering data. Owing to the uncertainty of these estimates we will, in the final numerical compilation of the potential, take α equal to zero and β equal to 1. The term proportional to β has, as we show in footnote 17, a certain flexibility to it. We may change the coefficient of the term and at the same time add new terms to the last term of (5). Since there is apparently no reason for choosing β very small or very large, we choose, for the equivalent Hamiltonian (5), to take $\beta=1$. Since the terms with coefficients ϵ and δ do not give any significant contributions unless ϵ and δ are much larger than 1, we shall neglect them.[15]

In computing all diagrams we use the value $f^2/4\pi = 0.10$, the value of the renormalized coupling constant used by Gartenhaus. In addition, we should include rescattering effects for P-wave mesons, just as the Low equation does. We shall neglect all rescatterings except insofar as they are included in the renormalized coupling constant. It is therefore to be expected that there will be a rescattering correction to our results. BW have investigated this P-wave rescattering for the static potential and find that its effects are small. A more sophisticated treatment of the nuclear force problem, perhaps from the point of view of dispersion theory, would of course include it.

III. THE POTENTIALS

We now write the second- and fourth-order potentials as

$$V=(f^2/4\pi)[A^{(2)}+(\mu/M)B^{(2)}]+(f^2/4\pi)\Delta + [f^4/(4\pi)^2][A^{(4)}+(\mu/M)B^{(4)}+(\mu/M)F\mathbf{L}\cdot\mathbf{S}], \quad (6)$$

where A, B, and F are functions of the relative distance between the two nucleons and where we have anticipated the result that the largest spin-orbit term is fourth order in f and first order in μ/M. The term $B^{(4)}$ will be neglected since it is a correction to the static potential of order $(f^2/4\pi)\mu/M$. We feel that there are enough uncertainties in the static potential itself to justify the neglect of this term. Since $f^2/4\pi \approx \mu/M$, we would expect to include $B^{(2)}$ in the final result. However, due to the symmetry properties of the diagrams, $B^{(2)}$ is equivalent to zero. In the same way, we keep only the largest velocity-dependent piece, F. Marshak and Okubo have shown that the only form such a potential, linear in the nucleon velocities and not vanishing for real free-particle scattering, can have is $\mathbf{L}\cdot\mathbf{S}$. The term

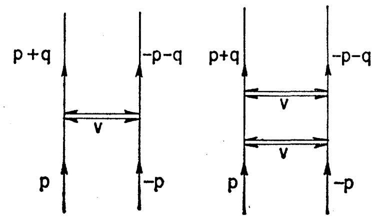


FIG. 1. First and second BA for two-nucleon potential scattering.

Δ, on the other hand, depends on the nucleon velocities, as well as their position. It is the term which gives rise to the controversial "ladder terms," and requires careful explanation.

In order to define a potential, we begin with the formal relation

$$S_{fi}=\delta_{fi}+2\pi i\delta(E_i-E_f)R_{fi}, \quad (7)$$

$$R_{fi}=\langle\phi_f|V|\psi_i^+\rangle, \quad (8)$$

$$\psi_i^+=\phi_i+1/(E_i-H_0+i\epsilon)\psi_i^+, \quad (9)$$

where V in field theory is an integral over space, of second-quantized operators, and for the case of nonrelativistic potential scattering, it is the potential we wish to define. The potential is defined so that it duplicates the S matrix to any given order for real free-particle scattering. The potential is expanded as a power series in the coupling constant:

$$V=V^{(2)}+V^{(4)}+\cdots, \quad (10)$$

$$R_{fi}=R_{fi}^{(2)}+R_{fi}^{(4)}+\cdots. \quad (11)$$

Then to fourth order,

$$R_{fi}^{(2)}=\langle f|V^{(2)}|i\rangle, \quad (12)$$

$$R_{fi}^{(4)}=\langle f|V^4|i\rangle+\sum_n \frac{\langle f|V^{(2)}|n\rangle\langle n|V^{(2)}|i\rangle}{(E_i-E_n)}. \quad (13)$$

This means that the two diagrams, for potential scattering (first and second BA), Fig. (1), must agree with the fourth-order R matrix from field theory. Equation (13) enables us to obtain the so-called "fourth-order" potential.

As is indicated by (13), iterations of the second-order potential must be subtracted out of the fourth order R matrix in defining $V^{(4)}$. In addition, if one calculates and includes in $V^{(2)}$ certain nonstatic pieces which depend on the nucleon velocities and which vanish on the energy shell [these are designated by $f^2\Delta$ in (6)], then the iteration of these terms must be subtracted from the fourth-order potential. The momentum dependence of Δ is just such as to cancel the usual pole present as E_i-E_n in Eq. (13) and yield a nonsingular function. These terms are the so-called ladder terms mentioned previously. BW have already noticed that if these terms are grouped with the fourth order, then the resulting static potential gives an unbound deuteron and, in general, disagrees with experiment. On the other hand, they show that if the term Δ is kept in

[14] S. D. Drell, M. H. Friedman, and F. Zachariasen, Phys. Rev. **104**, 236 (1956).

[15] A. Klein has included the term $\delta \approx M/\mu$ in an attempt to include the effective rescattering from the $(\frac{3}{2},\frac{3}{2})$ resonance. We ignore this. We also do not complete our Hamiltonian so as to make it a relativistic covariant because we feel that all the empirical corrections made to the FW result reflect only the failure of perturbation theory to treat correctly the γ_5 theory.

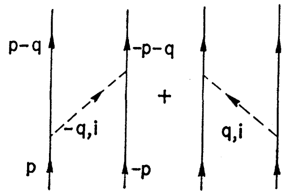
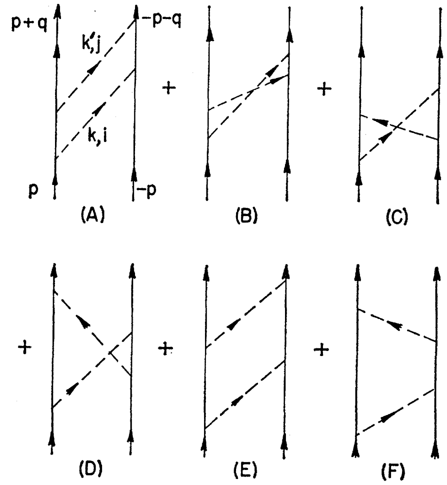


FIG. 2. Nonrelativistic second-order nucleon force diagrams. Arrows point in the direction of increasing time.

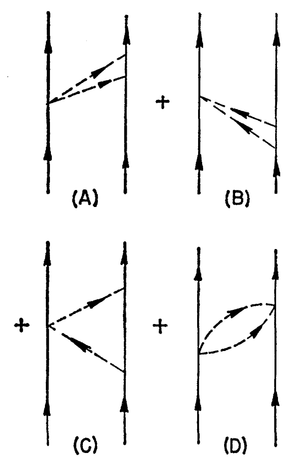
second order, then for wave functions of the potential $V-(f^2/4\pi)\Delta$ with a phenomenological hard core, the addition of the term Δ shifts things so slightly that it may be neglected. Their conclusion is that the retention of this term in the fourth-order static potential seriously overestimates it, since its origin in the fourth-order potential may be traced to the predominance of high-momentum components in the wave function for the two nucleons. These high-momentum components are in turn a consequence of the singularity in $V_{\text{static}}^{(2)}$ which in coordinate space has a singularity like $1/r^3$. The term Δ is omitted from the Gartenhaus potential. The same situation prevails with the $\mathbf{L}\cdot\mathbf{S}$ pieces. The iteration of Δ leads to a fourth-order $\mathbf{L}\cdot\mathbf{S}$ term. To be consistent with the results of BW we shall drop this $\mathbf{L}\cdot\mathbf{S}$ term from the fourth-order potential and neglect the effect of Δ on the scattering, since it vanishes on the energy shell and is presumably a small correction for a wave function satisfying the Schrödinger equation[16] for $V-(f^2/4\pi)\Delta$.

Within the context of the static model the BW argument is quite explicit. However, when one considers the possible forms the Hamiltonian (5) can take under the unitary transformation discussed in footnote 17, we must be a little more careful. Some of the new terms of order $1/M^3$ which are generated in such a transformation contribute to the $\mathbf{L}\cdot\mathbf{S}$ potential, as they must. However, the two "different" methods of computing the potential do not yield the same results unless all the diagrams of required order generated by these pieces are included. There are, in fact, a number of diagrams which are formally ladder diagrams and where the momentum dependence of the second-order pieces are just such as to cancel the pole in an intermediate state consisting of only two nucleons. These diagrams must be included, otherwise a unitary transformation[17] will cause changes in the actual potential.



+ REFLECTED DIAGRAMS

FIG. 3. "Single-action" fourth-order nonrelativistic nuclear force diagram. All vertices may contain a $\boldsymbol{\sigma}\cdot\boldsymbol{\nabla}$ or a $[\boldsymbol{\sigma}\cdot\mathbf{p},\,(\partial\phi/\partial t)]_+$ interaction.



+ REFLECTED DIAGRAMS

FIG. 4. "Double-action" fourth-order nonrelativistic nuclear force diagrams. All vertices may contain $\boldsymbol{\phi}^2$ or a $\boldsymbol{\tau}\cdot\boldsymbol{\phi}\times\boldsymbol{\phi}$ interaction.

We apply the BW argument then only to diagrams *where the pole-cancelling pieces* come from recoil correction to propagators.

We now evaluate the potential V of Eq. (6). For the sake of clarity we rename the second- and fourth-order pieces of the potentials. The subscripts indicate where the separate pieces come from. By BW we indicate the pieces already written down by Brueckner and Watson, and by Galilean we mean the piece generated by the term proportional to β in the Hamiltonian (5). The diagrams representing these potentials are given in Figs. 2, 3, and 4. The analytic form of these potentials

[16] At energies of about 150 Mev, the neglect of Δ should be reinvestigated. Previous results hold only for the deuteron.

[17] It is interesting to point out that the fourth and seventh terms of Eq. (4) give an effective vertex $\boldsymbol{\sigma}\cdot(\mathbf{v}_{\text{meson}}-\frac{1}{2}\mathbf{v}_{\text{nucleon}})$. The reduced theory must be Galilean invariant. As a result one might *expect* a term of the form $\boldsymbol{\sigma}\cdot(\mathbf{v}_{\text{meson}}-\mathbf{v}_{\text{nucleon}})$ to appear at the vertex. This is indeed the term appearing at the vertex of the reduced (PS) (PV) theory. Although the argument that the term $\boldsymbol{\sigma}\cdot(\mathbf{v}_{\text{meson}}-\mathbf{v}_{\text{nucleon}})$ must appear has been used previously, the statement is incorrect. It is possible to show that the coefficient of $\sigma\cdot v_{\text{nucleon}}$ may be any number at all relative to the coefficient $\sigma\cdot v_{\text{meson}}$ and still lead to the same physical result.

Suppose one takes the reduced Hamiltonian (4) and performs a second unitary transformation of the form e^{iS} where $S=\rho(g/8M^2)[\boldsymbol{\sigma}\cdot\mathbf{p},\phi]_+$; ρ is an arbitrary numerical factor. Such a transformation changes the coefficient of the seventh term in (4) so that it now becomes $(g/8M^2)(1+\rho)$. However, it is essential to note that such a transformation introduces new terms of order $1/M^3$ into the reduced Hamiltonian. We have demonstrated by explicit calculation that to the required order in g and (μ/M) the ambiguity arising from the unitary transformation $S=\rho(g^2/8M^2)[\boldsymbol{\sigma}\cdot\mathbf{p},\,\boldsymbol{\phi}]_+$ is not reflected in the potential. That is to say, the potential is independent of the adjustment of β by means of this unitary transformation.

are correspondingly

$$(f^2/4\pi)A^{(2)}\equiv V^{(2)}(\mathrm{BW}), \tag{14}$$

$$V^{(2)}(\mathrm{BW})=(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)[(f/\mu)^2/(2\pi)^3]\int d^3k\, v^2(k)(\boldsymbol{\sigma}_1\cdot\mathbf{k})(\boldsymbol{\sigma}_2\cdot\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{r}}/\omega^2, \tag{15}$$

$$\langle p_f|(f^2/4\pi)\Delta|p_i\rangle=(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)[(f/\mu)^2/(2\pi)^3]\int d^3k\, v^2(k)(\boldsymbol{\sigma}_1\cdot\mathbf{k})(\boldsymbol{\sigma}_2\cdot\mathbf{k}) \times[(\mathbf{p}_f{}^2-\mathbf{p}_i{}^2)/2M\omega^2][1/(\omega+(\mathbf{p}_f{}^2-\mathbf{p}_i{}^2)/2M)]e^{i\mathbf{k}\cdot\mathbf{r}}, \tag{16}$$

$$(f^2/4\pi)A^{(4)}\equiv V^{(4)}(\mathrm{BW})+V^{(4)}(\phi)+V^{(4)}(\phi^2), \tag{17}$$

$$V^{(4)}(\mathrm{BW})=-[(f/\mu)^4/(2\pi)^6]\int d^3kd^3k'\, v^2(k)v^2(k')e^{i(\mathbf{k}+\mathbf{k}')\cdot\mathbf{r}} \times[\{3(\mathbf{k}'\cdot\mathbf{k})^2+2(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)(\boldsymbol{\sigma}_1\cdot\mathbf{k}'\times\mathbf{k})(\boldsymbol{\sigma}_2\cdot\mathbf{k}'\times\mathbf{k})\}/(\omega^3\omega'^2) +\{2(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)(\mathbf{k}'\cdot\mathbf{k})^2+3(\boldsymbol{\sigma}_1\cdot\mathbf{k}'\times\mathbf{k})(\sigma_2\cdot\mathbf{k}'\times\mathbf{k})\}/(\omega^3\omega'(\omega+\omega'))], \tag{18}$$

$$V^{(4)}(\phi)=-(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)[(f/\mu)^4/(2\pi)^6]\int d^3kd^3k'\, v^2(k)v^2(k')e^{i(\mathbf{k}+\mathbf{k}')\cdot\mathbf{r}} \times[\{2\gamma(\mathbf{k}'\cdot\mathbf{k})-\gamma^2\omega\omega'\}/(\omega\omega'(\omega+\omega'))+(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\gamma^2/\omega'], \tag{19}$$

$$V^{(4)}(\phi^2)=-[(f/\mu)^4/(2\pi)^6]\int d^3kd^3k'\, v^2(k)v^2(k')e^{i(\mathbf{k}+\mathbf{k}')\cdot\mathbf{r}} \times[12M\alpha(\mathbf{k}'\cdot\mathbf{k})/\omega^2\omega'^2+24M^2\alpha^2/(\omega\omega'(\omega+\omega'))], \tag{20}$$

$$(\mu/M)(f^2/4\pi)^2F\mathbf{L}\cdot\mathbf{S}\equiv V_{\mathbf{L}\cdot\mathbf{S}^\pi}+V_{\mathbf{L}\cdot\mathbf{S}^0}+V_{\mathbf{L}\cdot\mathbf{S}}(\phi)+V_{\mathbf{L}\cdot\mathbf{S}}(\phi^2), \tag{21}$$

$$V_{\mathbf{L}\cdot\mathbf{S}^\pi}=(f/\mu)^4[(3+2\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\beta/((2\pi)^6M)]\int d^3kd^3k'\, v^2(k)v^2(k')e^{i(\mathbf{k}+\mathbf{k}')\cdot\mathbf{r}} \times[\{i(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot(\mathbf{p}\times\mathbf{k}')(\mathbf{k}\cdot\mathbf{k}')/(\omega^2\omega'^2)+i(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot(\mathbf{k}'\times\mathbf{k})(\mathbf{p}\cdot\mathbf{k}')/(\omega^2\omega'^2)], \tag{22}$$

$$V_{\mathbf{L}\cdot\mathbf{S}^0}=-(3-2\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)[(f/\mu)^4/((2\pi)^6 2M)]\int d^3kd^3k'\, v^2(k)v^2(k')e^{i(\mathbf{k}+\mathbf{k}')\cdot\mathbf{r}} \times[i(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot(\mathbf{k}'\times\mathbf{k})(\mathbf{p}\cdot\mathbf{k})(\mathbf{k}'\cdot\mathbf{k})/(\omega^3\omega'^3)], \tag{23}$$

$$V_{\mathbf{L}\cdot\mathbf{S}}(\phi)=(2\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)[(f/\mu)^4\gamma/((2\pi)^6M)]\int d^3kd^3k'\, v^2(k)v^2(k')e^{i(\mathbf{k}+\mathbf{k}')\cdot\mathbf{r}} \times[i(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot(\mathbf{k}'\times\mathbf{k})(\mathbf{p}\cdot\mathbf{k})/(\omega^2\omega'^2)-(\beta/2)i(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot(\mathbf{k}'\times\mathbf{p})/\omega'^2], \tag{24}$$

$$V_{\mathbf{L}\cdot\mathbf{S}}(\phi^2)=[(f/\mu)^4/(2\pi)^6]\int d^3kd^3k'\, v^2(k)v^2(k')e^{i(\mathbf{k}+\mathbf{k}')\cdot\mathbf{r}}[6\alpha i(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot(\mathbf{k}'\times\mathbf{k})(\mathbf{p}\cdot\mathbf{k}) \times(\omega^2+\omega'^2+\omega\omega')/(\omega^3\omega'^3(\omega+\omega'))-6\alpha\beta i(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)(\mathbf{k}'\times\mathbf{p})/(\omega\omega'(\omega+\omega'))]. \tag{25}$$

The iteration of Δ [see Fig. 3, diagrams (E) and (F)] leads to

$$V_{I\Delta}=(f/\mu)^4[(3-2\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)/(2\pi)^6]\int d^3kd^3k'\, v^2(k)v^2(k')e^{i(\mathbf{k}+\mathbf{k}')\cdot\mathbf{r}}[(\boldsymbol{\sigma}_1\cdot\mathbf{k})(\boldsymbol{\sigma}_2\cdot\mathbf{k})(\boldsymbol{\sigma}_1\cdot\mathbf{k}')(\boldsymbol{\sigma}_2\cdot\mathbf{k}')/\omega^3\omega'^2]. \tag{26}$$

The iteration of Δ [see Fig. 3, diagrams (E) and (F)] leads to an $\mathbf{L}\cdot\mathbf{S}$ piece of the form:

$$V_{I\mathbf{L}\cdot\mathbf{S}}=(f/\mu)^4[(3-2\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)/((2\pi)^6(2M))]\int d^3kd^3k'\, v^2(k)v^2(k')e^{i(\mathbf{k}+\mathbf{k}')\cdot\mathbf{r}} \times[i(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot(\mathbf{k}'\times\mathbf{k})(\mathbf{p}\cdot\mathbf{k})(2\omega'+\omega)/\omega^4\omega'^3]. \tag{27}$$

The coefficients α, β, and γ have been included to show the origin of all terms. Their numerical values are summarized as $\alpha \approx 0.01$, $\beta \approx 1$, $\gamma \approx 1$. Superscript x or 0 indicates "crossed" or "uncrossed" diagrams respectively (c.f. figure 3).

It is to be pointed out that some of the additional terms in the Hamiltonian contribute to the static potential as well as to the $\mathbf{L}\cdot\mathbf{S}$ potential. With the present choice of α all terms involving it could just as well be set equal to zero. We have included these terms for completeness only. In all numerical computations we will consider $\alpha=0$. There remain, however, two terms arising from the $(\boldsymbol{\tau}\cdot\phi\times\phi)$ term in the Hamiltonian which do contribute significantly to the fourth-order static potential. These terms combine to give a potential which is attractive in the isotopic singlet states and repulsive in the isotopic triplet states. Both the static piece and the $\mathbf{L}\cdot\mathbf{S}$ piece coming from the $\boldsymbol{\tau}\cdot\phi\times\phi$ interaction have a delta-function singularity in the relative coordinate for the case $v^2(k)=1$. Owing to their extremely singular behavior we shall omit (for the case of unity cutoff, and also for the case of a smooth momentum space cutoff) the δ-function parts. These δ-function pieces are peculiar for a number of reasons. For the case of no cutoff in momentum space it is certainly legitimate to neglect these terms since they contribute to the potential only at one point and would, presumably, be lumped in with a phenomenological hard core. When a smooth momentum space cutoff is used, this is not the case. These terms are then spread out and contribute to the potential. Two things should be noted about these terms. First, the size of the potentials generated by the δ-function pieces is extremely sensitive to the maximum momentum allowed. By increasing this cutoff slightly above the value we shall set it at later on, we can make this term have no effect in the region of interest. However, it is also true that by decreasing the value of the cutoff slightly these terms become very important. Thus, there is a serious question here. Second, we believe that they should be omitted because they do not appear in the relativistic γ_5 form of the theory. We suspect that these terms reflect the inadequacies of the Foldy transformation in treating precisely the very singular parts of the interaction energy of two nucleons, although we cannot prove this.

For the case $v^2(k)=1$ the evaluation of the integrals (14)–(24) is straightforward. We include in the following tabulation only those integrals which are not explicitly evaluated in BW. The delta-function pieces mentioned previously are dropped.

$$V^{(4)}(\phi)=(f^2/4\pi)^2\mu\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2(\gamma/\pi x^4)[(10+\gamma)K_1(2x)+8x^2K_1(2x)-x(20+2\gamma)K_1'(2x)], \tag{28}$$

$$V^{(4)}(\phi^2)=12M\alpha(f^2/4\pi)^2e^{-2x}(x+1)^2/x^4-48M\alpha^2(M/\mu)(f^2/4\pi)^2K_1(2x)/(\pi x^2), \tag{29}$$

$$V_{\mathbf{L}\cdot\mathbf{S}}{}^x=-\mu\beta(3+2\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)(\mu/M)(f^2/4\pi)^2e^{-2x}[(x+1)(x^2+x+1)/x^6]\mathbf{L}\cdot\mathbf{S}, \tag{30}$$

$$V_{\mathbf{L}\cdot\mathbf{S}}(\phi)=-\mu(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\gamma(\mu/M)(f^2/4\pi)^2e^{-2x}[(x+1)^2/x^6]\mathbf{L}\cdot\mathbf{S}, \tag{31}$$

$$V_{\mathbf{L}\cdot\mathbf{S}}{}^0=\mu(3-2\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)(\mu/M)(f^2/4\pi)^2(4/\pi^2)[(xK_0(x)+2K_1(x))/x^2]^2\mathbf{L}\cdot\mathbf{S}, \tag{32}$$

$$V_{\mathbf{L}\cdot\mathbf{S}}(\phi^2)=-(24\alpha\mu/\pi)(f^2/4\pi)^2(1-\beta/2)\{[3K_1(2x)+2xK_0(2x)]/x^4\}\mathbf{L}\cdot\mathbf{S}, \tag{33}$$

$$V_{I\mathbf{L}\cdot\mathbf{S}}=-V_{\mathbf{L}\cdot\mathbf{S}}{}^0-(3-2\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\mu(\mu/M)(f^2/4\pi)^2e^{-2x}[(x+1)(x^2+3x+3)/x^6]\mathbf{L}\cdot\mathbf{S}. \tag{34}$$

A prime on the K function indicates differentiation with respect to the entire argument.

We regard a smooth momentum-space cutoff as a necessary, if unexplained, part of the theory and therefore a careful comparison with experiment requires a recalculation of the potential in configuration space. For this reason we have numerically calculated the form of the potential when such a cutoff is used. In analogy with the Chew theory,[18] the results of using such a cutoff in k space should not depend strongly on the form of the cutoff used but may depend upon the maximum momentum allowed. A square cutoff, owing to its discontinuous nature, gives rise to oscillations in the potential. Gartenhaus used a cutoff of the form

$$v(k)=\exp(-k^2/k_M^2), \tag{35}$$

with

$$\omega_M=(k_M^2+\mu^2)^{\frac{1}{2}}=6\mu.$$

We use the same cutoff function. Figures 5 and 6 are plots of the additional static pieces in the iso-singlet and iso-triplet states, respectively. Figures 7 and 8 are the corresponding plots for the $\mathbf{L}\cdot\mathbf{S}$ pieces.[19]

Before going to a comparison with the available experimental data, we stress again the extreme simplicity of this calculation. Our Hamiltonian was chosen on the basis of a nonrelativistic reduction of the (PS) (PS) theory. The S-wave interactions were then suppressed in the nonrelativistic theory and the potential calculated with this Hamiltonian as the starting point. Correspondingly V was chosen to duplicate the fourth-order S matrix (except for the "ladder terms"), for real free nucleons. All higher order rescattering effects were neglected. A number of fourth-order velocity-dependent $\mathbf{L}\cdot\mathbf{S}$ pieces of order $f^4\mu/M$ were not included since they vanished on the energy shell and

[18] G. F. Chew, Phys. Rev. 95, 285 (1954).

[19] We can supply upon request numerical values of the potential either tabulated or punched on IBM cards.

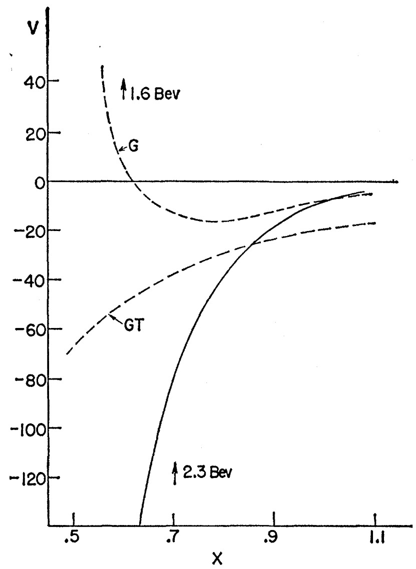


FIG. 5. Plot in the iso-singlet state ($\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2=-3$) of the static potential coming from the $\boldsymbol{\tau}\cdot\phi\times\phi$ term in the Hamiltonian, where $x\equiv\mu r$. The dotted curve marked G is the Gartenhaus static potential in the spin-triplet, orbital even states (for example, the $^3S_1$ state). If $\gamma=1$ the resulting potential is the sum of these two curves. The curve marked GT is the corresponding phenomenological potential of Gammel and Thaler.

The sum of the two curves looks similar to the GT curves. However, with the present choice of γ it would appear that the potential would be attractive all the way, which is no doubt undesirable. If γ were changed to some smaller value (in the neighborhood of 0.7) the curve might indeed give reasonable answers for the deuteron ground state.

thus would not appear in our definition of the potential until order f^6.

IV. DISCUSSION AND COMPARISON WITH EXPERIMENT

Nucleon scattering data up to 150 Mev give a reasonable indication of the type of $\mathbf{L}\cdot\mathbf{S}$ potential needed to fit the data. The data have been fitted quite well by SM. They have added to the Gartenhaus potential the following isotopic spin-independent, phenomenological, potential:

$$V_{\mathbf{L}\cdot\mathbf{S}}(r)=\frac{V_0}{(r/r_0)}\frac{d}{d(r/r_0)}\left[\frac{e^{-(r/r_0)}}{(r/r_0)}\right],\quad r>r_c$$
$$=V_{\mathbf{L}\cdot\mathbf{S}}(r)\,|_{r=r_c},\qquad r<r_c \qquad (36)$$

where

$$V_0=30\text{ Mev},\quad r_0=1.07\times10^{-13}\text{ cm},\quad r_c=1/M.$$

The experimental data at these moderate energies indicate definitely that the potential for the triplet odd states should be attractive. The situation with respect to triplet even states is not conclusive. The addition of a spin-orbit potential to the straight Gartenhaus potential immensely improves the agreement with experiment. The major reason for this improvement is that the very large Gartenhaus $^3P_0$ phase shift has been brought down from 57.3° to 15.5°. This is due to the fact that the spin-orbit potential supplies a large repulsive core in the $^3P_0$ state which wipes out the effect of the otherwise deep attractive well in that state. SM find, on the other hand, that they could just as well have chosen zero for the potential in the triplet even state.

The theoretical isotopic triplet potential, in agreement with SM, is attractive except at short distances where the cutoff adds a repulsive core. The theoretical $\mathbf{L}\cdot\mathbf{S}$ is nevertheless considerably smaller than the phenomenological potential of SM. However, it is possible, by changing the coupling constant slightly (from 0.09 to 0.12) and increasing the maximum cutoff slightly, to improve the over-all agreement considerably. The question of agreement is not a simple one. In the first place, it is evident that the spin-orbit potential is not unique. Definite predictions can be made only on the basis of a numerical solution of the coupled Schrödinger equations. Thus one potential may be as acceptable as another one which looks quite different. In support of this we point out that any $\mathbf{L}\cdot\mathbf{S}$ potential derived from meson theory will involve the exchange of at least two mesons. As a result it will always have the asymptotic form of an algebraic function times $e^{-2\mu r}$. The original SM potential did not have this behavior. However, recently Signell *et al.*[20] have shown that a

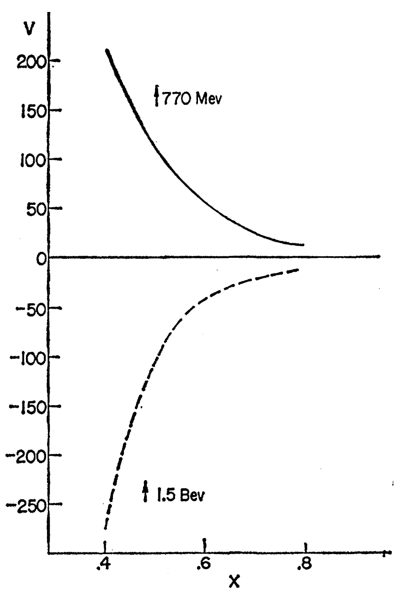


FIG. 6. Plot in the isotopic triplet state ($\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2=1$) of the static potential coming from the $\boldsymbol{\tau}\cdot\phi\times\phi$ term in the Hamiltonian where $x\equiv\mu r$. The dotted curve shown for comparison purposes is a plot of the Gartenhaus static potential in the spin triplet orbital odd states (for example, the $^3P_0$ state). If $\gamma=1$ the resulting potential is the sum of these two curves. The fact that these two terms tend to cancel one another is an important feature since the $^3P_0$ Gartenhaus phase shift is much too large to agree with experiment (see discussion of results).

[20] P. S. Signell, R. Zinn, and R. E. Marshad, Phys. Rev. Letters 1, 416 (1958).

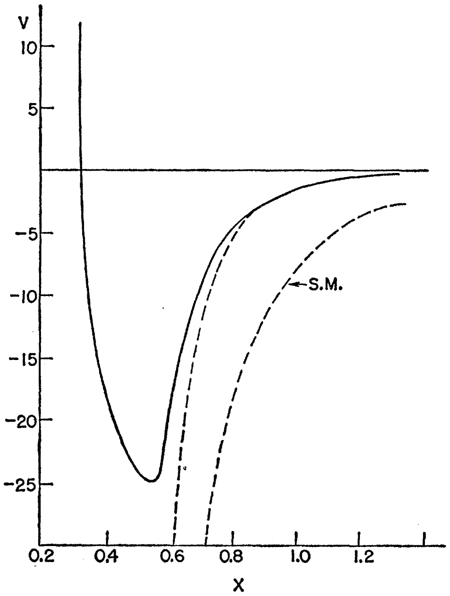


FIG. 7. Plot of the coefficient of **L·S** in the iso-triplet state, where $x=\mu r$. The unlabeled dotted curve is a plot of "unity cutoff" $[v^2(k)=1]$ and the solid curve for a Gaussian cutoff. The ordinate is in Mev and $f^2/4\pi$ was chosen as 0.1. The curve labeled SM is a plot of the phenomenological Signell and Marshak **L·S** potential in the iso-triplet state.

change in the range of their original potential to agree with the theoretical range of $1/2\mu$ results in an improved agreement with the p-p scattering data. Secondly, there remains the question of how sensitive are the predictions at these energies to changes in the core of the static potential.[21,22] The **L·S** potential in the isotopic singlet state is strongly repulsive in contrast with the results of SM. This is a definite prediction of the particular way in which we have chosen the pieces of the fourth-order potential. If the BW argument had been ignored, then the potential would have been attractive in both isotopic states. The fact that it is repulsive, at present, does not seem to be in obvious contradiction with the experimental results on two-nucleon scattering.

It is well known that such a spin-orbit potential will change the magnetic moment of the deuteron,[23] since the velocity-dependent potential introduces an extra coupling with an external electromagnetic field. If we are considering the deuteron, the appropriate **L·S** potential is the isotopic singlet. The intrinsically negative potential proposed by SM causes a shift in the deuteron's magnetic moment of approximately

$$\Delta\mu_{\mathbf{L}\cdot\mathbf{S}}=-0.056 \text{ nuclear magneton.}$$

This is an undesirable shift since it precludes the possibility of matching the experimental results even if the probability for finding the deuteron in a D state were reduced to zero. On the other hand, our potential gives a positive shift which depends on the third power of the cutoff distance in coordinate space. Since the shift is positive and of the order of 0.1 nuclear magneton, it is possible to match the experimental value by increasing the amount of D state present or changing the cutoff radius slightly.

Another and more qualitative comparison with experiment may be found in the shell-model theory of spin-orbit splitting in the nucleus. Calculations to date in this area are very approximate since the nucleon wave functions are not well known. If we write the potential as

$$V_{\mathbf{L}\cdot\mathbf{S}}=[V_1(r)+V_2(r)\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2]\mathbf{L}\cdot\mathbf{S},$$

both the V_1 and V_2 parts give first-order splittings for a single particle outside a closed shell, the $V_1(r)$ through a direct expectation value and an exchange term and the $V_2(r)$ term through an exchange integral only, since it is evident that the closed shell has no net isotopic spin.

We have made a rough comparison with experiment by calculating the $P_{\frac{3}{2}}-P_{\frac{1}{2}}$ splitting to be expected in He^5. The energy difference is given by the formulas of Blanchard and Avery.[24] We choose harmonic oscillator wave functions for the nucleons, taking radial wave

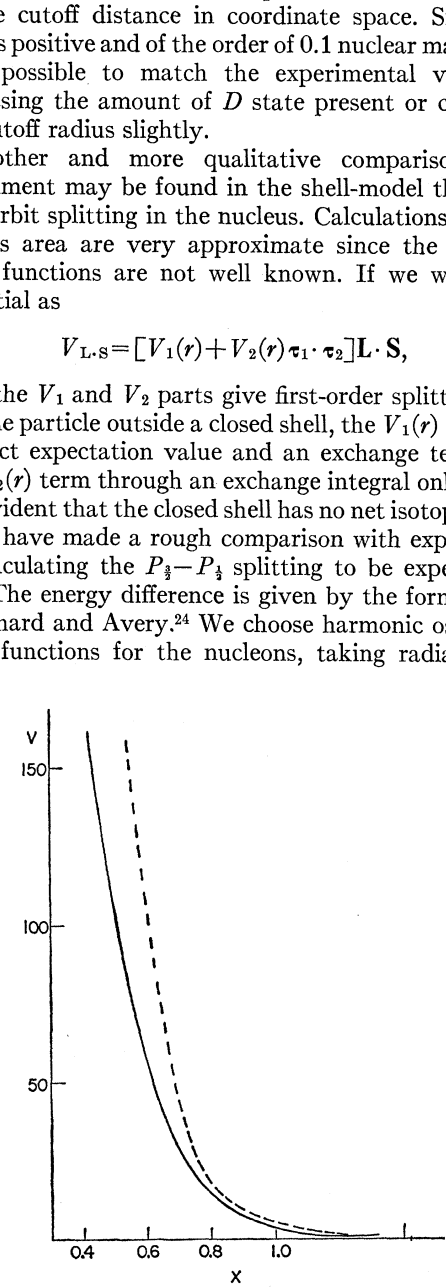


FIG. 8. Iso-singlet $(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2=-3)$ **L·S** potentials, where $x=\mu r$. Ordinate is in Mev. Dotted curve is analytic form, solid curve is for the Gaussian cutoff (see text).

[21] The pieces of the static potential arising from the fifth term in the Hamiltonian of Eq. (5), Figs. 5 and 6, present interesting possibilities along these lines. In the iso-triplet state this repulsive potential automatically cuts down on the $^3P_0$ phase shift. Thus the **L·S** potential, for the purpose of cutting down on the $^3P_0$, does not, *a priori*, have to be as big as the phenomenonological **L·S** potential of SM.

[22] S. Otsuki, Progr. Theoret. Phys. (Kyoto) **20**, 171 (1958).

[23] H. Feshbach, Phys. Rev. **107**, 1626 (1957).

[24] C. H. Blanchard and R. Avery, Phys. Rev. **81**, 35 (1951).

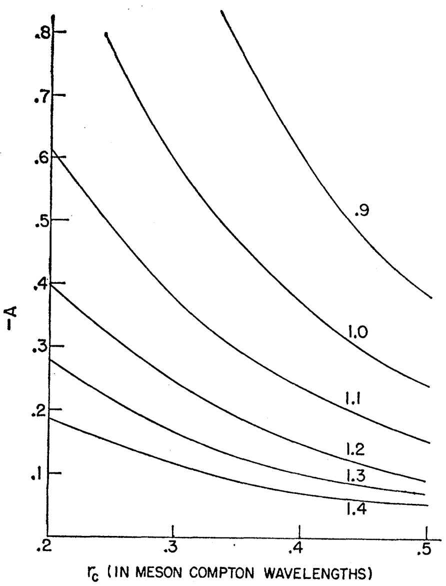


FIG. 9. Fine-structure splitting in He^5 (see Sec. IV of text).

functions

$$f=(\alpha/\sqrt{\pi})^{\frac{3}{2}}(4\pi)^{\frac{1}{2}}\exp(-\alpha^2r^2/2), \qquad S\text{-state nucleons};$$

$$g=(2\alpha/\sqrt{3})(\alpha/2\sqrt{\pi})^{\frac{1}{2}}2\alpha r\exp(-\alpha^2r^2/2), \qquad P\text{-state nucleons}.$$

For the purpose of this crude estimate we approximate our two-nucleon spin-orbit potential by

$$\begin{aligned} V_{\mathbf{L}\cdot\mathbf{S}}&=(V_1'+V_2'\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)e^{-2x}/x^5, & r>r_c; \\ &=0, & r<r_c. \end{aligned} \tag{37}$$

The resultant splitting is

$$P_{\frac{3}{2}}-P_{\frac{1}{2}}\approx[V_1'+V_2']\times 0.75A,$$

where $V_1'+V_2'\approx 20$ Mev and A is a dimensionless function of the cutoff radius, r_c, and the size of the nucleus, α^{-1}. A is plotted in Fig. 9. It is encouraging that agreement with observed value of the splitting (3.5 Mev) corresponds to the choice $1/\alpha\approx 1.2/\mu$ and $r_c\approx 0.3$, a result not incompatible with other shell-model calculations. The splitting is quite sensitive to the cutoff in r space and therefore no quantitative results follow. The reason for this trouble is the rather sharp singularity of the approximate $V_{\mathbf{L}\cdot\mathbf{S}}$ at the origin.

ACKNOWLEDGMENTS

We would like to thank Dr. M. Gell-Mann for suggesting this problem and for his continuing aid and encouragement during the course of the computation. We would like to especially thank Dr. R. P. Feynman with whom all parts of the final manuscript were discussed. Thanks is also due to the Hughes Aircraft Company for the use of the IBM 709 Computer and for their help in preparing the manuscript. We would also like to thank Dr. Sugawara and Dr. Okubo for a preprint of their work and for some interesting comments on this problem, and Dr. R. F. Christy for a number of informative discussions.

Note added in proof. The recent work[7] of Sugawara and Okubo implies that there are no $\mathbf{L}\cdot\mathbf{S}$ pieces of the required order in the γ_5 theory, with "pairs suppressed." The only $\mathbf{L}\cdot\mathbf{S}$ pieces which arise come from the inclusion of additional pair terms in the relativistic coupling terms. These results do not contradict the results of this computation. Sugawara and Okubo "damp" pairs by neglecting all negative-energy diagrams. That is to say, they omit all intermediate states (in "old fashioned" perturbation theory) which contain an antinucleon. Although it is true that the large matrix elements of order one between nucleon and antinucleon states should be damped, it is by no means clear that the v^2/c^2 corrections to these vertices should be treated in the same way. Our point is that writing the matrix elements for these negative-energy diagrams as $g^2[1+O(v^2/c^2)]$ and then retaining the (v^2/c^2) term will give an $\mathbf{L}\cdot\mathbf{S}$ potential. If these pieces had been included by Sugawara and Okubo, their answer would be in agreement with the analytic potential computed here, including, of course, the piece $V_{I\mathbf{L}\cdot\mathbf{S}}$. The differences between the two approaches then arise from two sources. First, we do not suppress pair terms by suppressing all negative-energy states, but instead we suppress S-wave terms in the nonrelativistic Hamiltonian (5). Second, we employ the BW argument (and what we think is a consistent extension of it) while Sugawara and Okubo do not use this argument. This means that even the static potentials will disagree.