Quasi-Elastic Proton-Proton Scattering at 158 Mev*

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(Received June 15, 1960)

The cross section in impulse approximation for a quasi-elastic proton-proton scattering reaction with a bound target proton depends on the probability of finding inside the nucleus a proton with a momentum equal to the momentum transfer required by kinematics. This momentum transfer is determined if the incident energy and the energies and angles of the two outgoing partners of a proton pair are measured. Events involving s and p protons can be experimentally distinguished by noting that more energy is required to remove an s proton from the nucleus than a p proton. Thus the momentum eigenfunctions of each state can be explored experimentally by varying the momentum transfer, keeping constant the energy sum of a proton pair. Two methods of doing this to bring out the difference between scattering from s and p protons are discussed. They are (1) examination of the distribution of events as a function of the energy sharing between the two outgoing protons, at fixed scattering angles, and (2) examination of the angular distribution of events having fixed sharing. Born approximation calculations are presented for predicting the regions of maximum sensitivity to differences between s and p protons. These predictions are used in the design of two experiments on the reaction $C^{12}(p,2p)B^{11}$ whose results are reported. They show good qualitative agreement with the predictions for p protons, namely that the sharing will tend to be unequal if the scattering angle of each proton is near 45° [type (1) experiment] and that the angular distribution of events of equal sharing will have a minimum at 45° [type (2) experiment]. The predicted behavior (opposite to that for p protons) for events involving s protons is not observed, and this fact is discussed in the light of possible inadequacies of the theory or of the experiment.

I. INTRODUCTION

THE first direct demonstration of proton-proton scattering with the target protons located inside a complex nucleus was carried out by Chamberlain and Segrè.[1] Bombarding a lithium target with 340-Mev protons, they detected both product protons in coincidence and measured their angular correlation. This type of $(p,2p)$ reaction is usually called "quasi-elastic proton-proton scattering"; the incident proton interacts with a proton bound in the target nucleus, and in first order approximation the interaction is thought to proceed as though the target nucleon were free, except that it has an initial momentum and that energy is required to remove it from the nucleus.

Wilcox and Moyer[2] measured the energy distribution of one of the coincident protons. Tyrén, Hillman, and Maris[3] measured the sum of the energies of both partners of a proton pair and showed the existence of definite group structure on a plot of number of proton pairs vs energy sum. The existence of these groups is interpreted in terms of the protons being located in different states of the shell model. This should make it possible to select target protons in a certain state of the target nucleus by selecting the energy sum of the proton pairs. Extensive measurements at 153 Mev with the most loosely bound protons in carbon have recently been reported by Gooding and Pugh.[4] The work presented here was carried out at the same time as the Harwell experiments—158-Mev protons and a carbon target were used. There is very little overlap in the two studies.

* Supported by the joint program of the Office of Naval Research and the U. S. Atomic Energy Commission.
[1] O. Chamberlain and E. Segrè, Phys. Rev. **87**, 81 (1952).
[2] J. M. Wilcox and B. J. Moyer, Phys. Rev. **99**, 875 (1955).
[3] H. Tyrén, P. Hillman, and Th. A. J. Maris, Nuclear Phys. **7**, 10 (1958).
[4] T. J. Gooding and H. G. Pugh, Nuclear Physics (to be published). We are indebted to the authors for making a preprint available to us.

The ultimate hope of quasi-elastic scattering experiments is to obtain detailed information about the nuclear momentum distribution inside nuclei, and possibly to study "off the energy shell" proton-proton scattering. At present most of the effort is going into studying the validity of the reaction model. We will first discuss which of the many possible experiments with a given target are most likely to yield the desired information, and then report our experimental results.

II. DISCUSSION OF POSSIBLE QUASI-ELASTIC PROTON-PROTON SCATTERING EXPERIMENTS

In such experiments, a proton of momentum $\mathbf{k}_0$ is incident on a nucleus of mass A (we set $\hbar=M=c=1$). Two protons emerge with momenta $\mathbf{k}_1$ and $\mathbf{k}_2$, the residual nucleus (mass $A-1$) recoils with a momentum $-\mathbf{q}$ (not measured directly) and an excitation energy $E_x=E_B-E_0$. E_0 represents the binding energy of the most loosely bound proton and E_B represents the net decrease of kinetic energy in the reaction, which is the energy absorbed by the target nucleus if the nuclear recoil energy can be neglected. Conservation of energy and momentum require

$$\mathbf{k}_0=\mathbf{k}_1+\mathbf{k}_2-\mathbf{q},$$
$$\frac{k_0^2}{2}=\frac{k_1^2}{2}+\frac{k_2^2}{2}+\frac{q^2}{2(A-1)}+E_B. \quad (1)$$

We will find it useful to define an "energy sharing parameter"

$$x^2=\frac{k_1^2}{k_2^2} \quad \text{or} \quad \frac{k_2^2}{k_1^2}, \quad (2)$$

where $x^2\leq 1$.

In the spirit of the impulse approximation, the above reaction is the result of a proton-proton interaction in

which the target proton is bound. If it is assumed that the incoming and outgoing protons have no other interactions with nucleons in the target nucleus, individually or collectively, then the momentum of the target proton before the collision is $+\mathbf{q}$, the remainder of the nucleus moving with a momentum $-\mathbf{q}$, a motion which is unaffected by the interaction. E_B represents the binding energy of the target proton, which determines the excitation energy E_x of the residual nucleus according to $E_x=E_B-E_0$.

The state of the target nucleus is represented, according to the single-particle model, by a product wave function of individual nucleon wave functions. Neglecting spins, the cross section for scattering of a proton of momentum $\mathbf{k}_0$ by a bound proton of momentum $\mathbf{q}$ resulting in two protons of momenta $\mathbf{k}_1$ and $\mathbf{k}_2$ is given in Born approximation by

$$\sigma(\mathbf{k}_0\mathbf{k}_1\mathbf{k}_2)=\frac{1}{4\pi^2}\left|\int e^{-i\mathbf{k}_1\cdot\mathbf{r}}V(\mathbf{r})e^{i\mathbf{k}_0\cdot\mathbf{r}}d\mathbf{r}\right|^2 \times\left|\int e^{-i\mathbf{q}\cdot\mathbf{r}}\Psi(\mathbf{r})d\mathbf{r}\right|^2. \quad (3)$$

The first term in this result represents the cross section for free proton-proton scattering (laboratory system) in the "off the energy shell" region specified by Eq. (1). The second term represents the probability of finding the target proton with momentum $\mathbf{q}$. As will be discussed later, important corrections must be made to this treatment; however, Eq. (3) will be used as a guide to the type of experiment which will verify the theory of quasi-elastic proton-proton scattering, and which will give information on the nature of the single-particle wave functions in the nucleus.

In carbon, there are two tightly bound $1s$ and four less tightly bound $1p$ protons present. Using three-dimensional harmonic oscillator wave functions[5] and assuming that the proton-proton "off the energy shell" matrix element is constant, the following quasi-elastic proton-proton cross sections are obtained:

$$1s \quad \sigma(\mathbf{k}_0,\mathbf{k}_1,\mathbf{k}_2)\propto\exp(-q^2/\beta^2), \quad (4)$$

$$1p \quad \sigma(\mathbf{k}_0,\mathbf{k}_1,\mathbf{k}_2)\propto\frac{q^2}{\beta^2}\exp(-q^2/\beta^2), \quad (5)$$

β represents a characteristic nuclear momentum which is related to the corresponding characteristic nuclear size a by $\beta=1/a$. If we measure $\mathbf{k}_0$, $\mathbf{k}_1$, and $\mathbf{k}_2$ in the experiment, then q^2 and E_B are determined by Eq. (1). To measure the detailed differential cross section with a beam of known energy, it is thus necessary to measure both the angle *and* energy of the two partners of a proton pair to completely determine the problem.

[5] P. M. Morse and H. Feschbach, *Methods of Theoretical Physics* (McGraw-Hill Book Company, Inc., New York, 1953), Sec. 12.3.

Since s and p protons have different binding energies, selection of pairs according to different values of E_B is expected to allow separation of events originating from s and p protons. It is sufficient to measure $\sigma(\mathbf{k}_0,\mathbf{k}_1,\mathbf{k}_2)$ as a function of $q^2(k_0^2,E_B,\theta_1,\theta_2,x^2)$ to determine the Fourier transform of the nuclear wave function. Keeping the incident energy $k_0^2/2$ and the energy E_B constant, q^2 can be varied by changing the scattering angles θ_1 or θ_2, or by varying the sharing parameter x^2. In practice it is desirable to vary q^2 by more than one method in order to have a consistency check on the validity of the reaction model.

Two types of experiments in which q^2 is varied for a fixed E_B are the following:

(1) Two detectors are kept at fixed scattering angles θ_1 and θ_2 and the energy of each partner of a proton pair is measured. The apparatus can be biased so that only proton pairs corresponding to a fixed value of E_B are detected,[4] or alternately all proton pairs are accepted and then divided into groups of equal E_B for analysis. For each group of fixed E_B a study is made of the number of pairs as function of the energy sharing parameter x^2.

(2) Two detectors are each biased to accept only energies corresponding to fixed E_B and fixed x^2. An angular distribution is then taken. In theory, carrying out experiment (1) by accepting all proton pairs at a large number of angles will automatically give the information of experiment (2) for a wide range of values of E_B and x^2. However, in our experience it seems best to separate the two types of measurement to prevent very long runs or poor statistical accuracy in sensitive regions. The rate of collecting proton pairs is severely limited by the counting rate in each detector for both accidental and dead-time reasons, since only a very small fraction of the protons counted in each detector correspond to the desired pair protons. To obtain sufficient statistical accuracy it is thus advantageous to

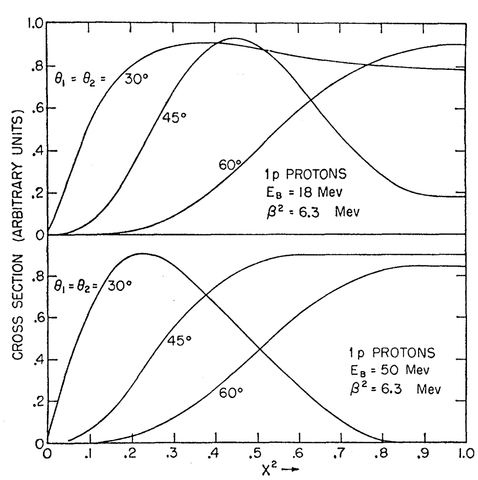

FIG. 1. Calculated cross sections for p protons as a function of the energy sharing parameter x^2 for two binding energies E_B and three scattering angles $\theta_1=\theta_2$.

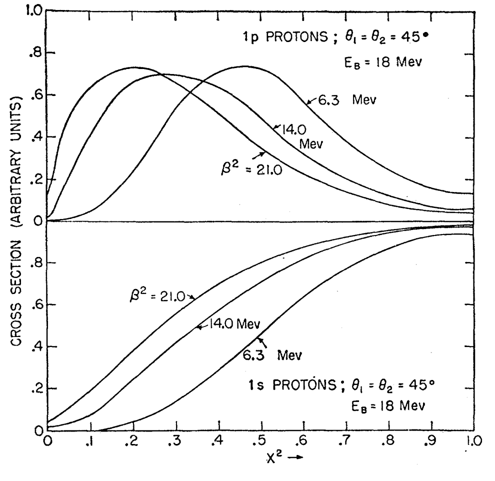

FIG. 2. Calculation of the effect of varying the nuclear momentum parameter β^2 on the cross sections for $1s$ and $1p$ protons, if $E_B=18$ Mev and $\theta_1=\theta_2=45°$.

limit the proton pairs that are detected to those that have strong angular (or energy) characteristics and, as will be discussed below, this requires a sharp selection of x^2 and E_B or θ and E_B. For the same practical reasons, it is desirable to have similar counting rates in both detectors. This can be accomplished by using detectors of similar efficiency at the same scattering angles θ_1 and θ_2, and by varying both scattering angles together, so that θ_1 remains equal to θ_2. (σ depends only on q^2—whether q^2 is varied by keeping θ_1 constant and varying θ_2 only, or varying both simultaneously, is immaterial.) Using an incident energy of 160 Mev, predictions of Eq. (3) for the results of Type (1) experiments are plotted in Figs. 1 and 2 for several values of the variables. In Fig. 1 the cross section for scattering from $1p$ protons given by Eq. (5) is plotted as a function of the sharing parameter x^2 for three scattering angles $\theta_1=\theta_2$ and for 2 values of the binding energy E_B assuming a momentum constant $\beta^2=6.3$ Mev. In Fig. 2 are plotted the cross sections for scattering from $1s$ and $1p$ protons [Eqs. (4) and (5)] at $\theta_1=\theta_2=45°$ and $E_B=18$ Mev, as a function of the sharing parameter x^2 for various values of β^2.

The general shape of the $1s$ proton cross section vs x^2 curve (Fig. 2) is characteristic for other angles besides 45°; the smooth rise toward high values of x^2 reflects the fact that for s protons zero momentum is the most probable and they therefore tend to behave like free protons, with all kinematic relations smeared out by the nuclear momentum distribution.

The cross section vs x^2 curves for $1p$ protons are quite similar to the corresponding curves of $1s$ protons, except in the vicinity of the scattering angles $\theta_1=\theta_2=\theta_{\min}$ which correspond to $q^2=0$. Since p protons have zero probability for this momentum value, the cross section must vanish at $\theta_1=\theta_2=\theta_{\min}$. The value of $\theta_{\min}$ as a function of E_B is inferred from Eq. (1):

$$\cos\theta_{\min}=\frac{1}{\sqrt{2}}\frac{k_0}{(k_0^2-2E_B)^{\frac{1}{2}}}. \quad (6)$$

This gives minimum angle values of 41° and 30° for binding energies of 18 and 50 Mev, respectively. The cross sections for $1s$ protons have maxima at these angles. The curve for $E_B=50$ Mev at 45° looks very much like an s proton curve, but the characteristic p proton behavior appears at $\theta_{\min}=30°$.

Some predictions for the results of experiments of Type (2) are shown in Fig. 3 for both $1s$ and $1p$ protons. The characteristic behavior of the angular distributions is closely connected with the discussion of Fig. 1 and 2. The sharp cross section dip predicted at $\theta_{\min}$ for $1p$ protons occurs only for values of $x^2=1$ when $q^2=0$. The large-angle cross sections are sensitive to the value of β: Since they can be measured with fair accuracy, Type (2) experiments should be more useful than Type (1) experiments in obtaining information on nuclear momentum distributions.

The above results are based on an idealized reaction model and a Born approximation calculation. Using as a guide the comparison of Born approximation calculations with the results of elastic scattering experiments,[6] we expect in fact a pronounced smearing out of all characteristic angular and energy features. This corresponds to the neglect of refraction, reflection, and absorption in the approximation used. The neglect of secondary interactions, varying from small energy losses to complete absorption, is not expected to seriously affect the qualitative features of the results for the most loosely bound p protons except for the absolute cross-section value, but might affect more seriously the predictions for s protons. s protons having a larger value

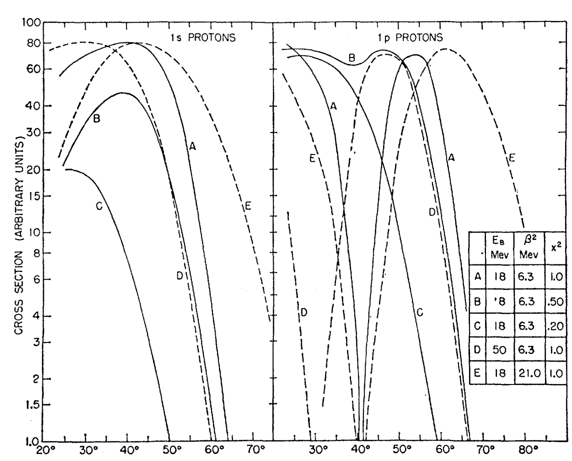

FIG. 3. Calculated angular dependence of cross sections for $1s$ and $1p$ protons for various values of E_B, β^2, and x^2.

[6] G. Gerstein, J. Niederer, and K. Strauch, Phys. Rev. **108**, 427 (1957).

of E_B cannot be distinguished from p protons having lost some small energy in a secondary interaction. The purity of the shell-model states will also affect the accuracy of the calculation. The assumption of a constant proton-proton scattering matrix element should affect the over-all shape of the cross-section curves, but not the details such as the minima for $1p$ protons. Maris[7,8] and Riley[9] have discussed some of these problems, and have made more accurate calculations, but mostly in a form not directly applicable to our experiment. Other more complete calculations are reported in progress.[10]

The effect of these corrections on the simple calculations shown in Figs. 1–3 is such as to reinforce the previous conclusion that in order to detect the characteristic features of the theoretical predictions, it is advisable to sharply select θ and E_B in a Type (1) experiment, or x^2 and E_B in a Type (2) experiment. A great deal of information will be lost by experimentally integrating over various parameters such as E_B or x^2. It is on this basis that the carbon measurements reported in the next section were carried out, and that further work is being planned.

III. EXPERIMENT TYPE (1): ENERGY SHARING

A. Apparatus

The unpolarized proton beam of the cyclotron was focussed by two quadrupole magnets to a spot about $\frac{3}{8}$ in. in diameter (Fig. 4). A flat carbon target, surface-ground to a thickness of 0.020 in., was used. Two very

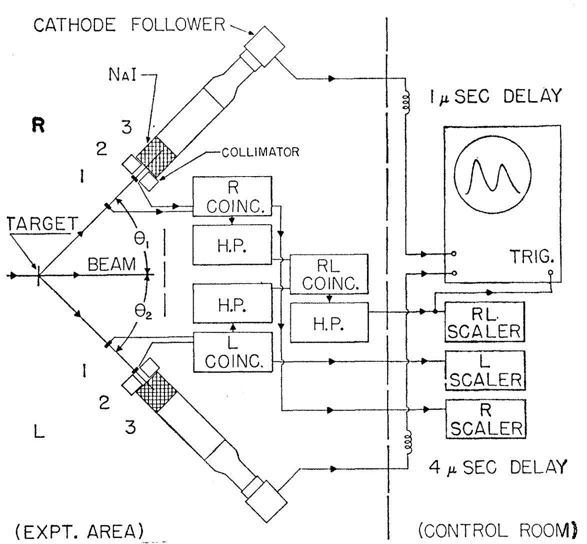

FIG. 4. Block diagram of the experimental apparatus.

[7] Th. A. J. Maris, P. Hillman, and H. Tyrén, Nuclear Phys. **7**, 1 (1958).
[8] Th. A. J. Maris, Nuclear Phys. **9**, 577 (1958).
[9] K. F. Riley, Nuclear Phys. **13**, 407 (1959).
[10] J. Nuttall and K. F. Riley; quoted in reference 4.

similar telescopes on movable beams were set at 44° each to the beam direction. The telescopes will be called R and L; each consisted of three counters: No. 1 of thin scintillating plastic, $\frac{1}{2}$ in. wide by 1 in. high, coupled to an RCA 6810A photomultiplier through a short light pipe; No. 2 of similar construction but $\frac{5}{8}$ in. wide by $1\frac{1}{8}$ in. high, and No. 3 a total energy counter. The latter consisted of a thallium-activated sodium iodide crystal, 3 in. in diameter by about 3 in. deep, coupled to a Dumont 6363 photomultiplier (3-in. photocathode) through a cylindrical Lucite light pipe 7 in. long. The thickness of the defining scintillators was such that a proton of 25.3-Mev energy would just reach the sodium iodide crystal. The first counter was 10 in. from the target, the second 4 in. behind the first, and a brass collimator located between counters 2 and 3 prevented almost all the protons that had not passed through the second scintillator from reaching the third.

Positive $R3$ and $L3$ output pulses of the order of 1 volt were obtained from the eighth dynode of the 6363 photomultiplier, the two remaining dynodes connected to the anode serving as a collector. After introducing a 3-μsec delay between the $L3$ and $R3$ pulses, they were mixed, amplified and displayed on an oscilloscope. The oscilloscope trace was triggered by a coincidence between pulses from counters $R1$, $R2$, $L1$, $L2$ as shown in Fig. 4. The accidental rate was obtained by introducing a time delay between the R and L coincidences corresponding to the travel time of the beam around the cyclotron. The coincidence circuits were operated with an 8-μsec resolving time. Scalers recorded the number of particles passing through the R and L telescopes, respectively, and the number of R-L coincidence counts.

The oscilloscope was a specially constructed instrument of high resolution, linearity, and stability. Its display was photographed by a Dumont oscillograph movie camera, the film moving continuously. The film was subsequently read on a standard microfilm reader which had a transparent ruled scale affixed to its screen. The system from the cathode followers through the microfilm reader was tested prior to the run and the linearity, resolution, and long-term stability were all found to be better than $\frac{1}{2}\%$. It was possible to observe either total-energy pulse $R3$ or $L3$, or the sum of the two, on a hundred-channel pulse-height analyzer; this was used to check the phototube and electronic gain stability during the run.

With the cyclotron beam directly incident on each telescope, an energy width of 1.4% (full width at half maximum) was observed.

B. Data Taking and Analysis

At the beginning of the run the system was calibrated to give the relation between pulse height and energy for each telescope. This was done with the direct beam degraded to known energies with CH_2 absorbers, using

the Berkeley range-energy tables.[11] Points on this calibration were repeated during the run. In addition the free proton-proton peak (see below) served as a valuable check on the stability of the system.

A pulse pair was accepted for further analysis only when neither member seemed to ride on another pulse. Of a total of about 10 000 pairs recorded on the film, 5000 pairs were accepted for analysis. A small correction was applied to the right pulse for the residuum of the left pulse which had preceded it.

The right and left pulses of each pair were separately converted to energy using the experimental calibration curves. These curves proved to be linear except at small energies since a zero-height pulse corresponded to 25.3 Mev, not 0 Mev, due to the thickness of counters 1 and 2. Within the accuracy of the experiment, the recoil energy of the target nucleus was negligible. The energy sum of the energy quotient of each pair were computed. The quotient was taken in whichever way gave a result less than 1 (a study of the data had shown that the R/L and L/R distributions were the same). The data were then arranged in matrix form, the row index of each event being its sum and the column index, the quotient. From this matrix the distributions to be discussed were drawn.

C. Results

Figure 5 shows the distribution of events as a function of their energy sum. The abscissa represents E_B, the "energy lost to the nucleus," that is (neglecting the nuclear recoil energy), the incident energy less the energy sum of the pair. The dotted curve represents the normalized accidental contribution.

The events fall into four main groups. The first peak at $E_B=0$ is due to scattering from hydrogen contamination of the target. This small contamination was a happy accident, as it provided a continuous stability check on the equipment, and also gave an experimental resolution curve for both the energy sum and the energy sharing parameter (which should be $x^2=1$) of both pulses of a coincidence pair. The energy sum resolution of 1.9% (full width at half maximum) is in good agreement with the energy resolution of the individual telescopes. The distribution in x^2 [Fig. 6(A)] is consistent with the geometry and the accuracy of the calibration procedure.

The sharp peak, centered at 17 Mev and extending from about 13 to 25 Mev, corresponds to protons from the $C^{12}(p,2p)B^{11}$ reaction in which the residual nucleus is left in its ground and/or in low-lying excited states which cannot be resolved from each other. We postulate that this peak corresponds to the scattering of the incident proton with one of the p protons in the carbon nucleus, since these are the least tightly bound. The distribution (as a function of x^2) of the events included under this peak is shown in Fig. 6(B). It indeed has the general shape predicted in Fig. 1 for the scattering from a p proton. A second broad peak due to carbon events extends from 26 to 42 Mev. The sharing parameter distribution for events under this peak is shown in Fig. 6(C) and differs little from Fig. 6(B). The remaining events of Fig. 5 have been somewhat arbitrarily divided into two groups. The sharing distribution of the

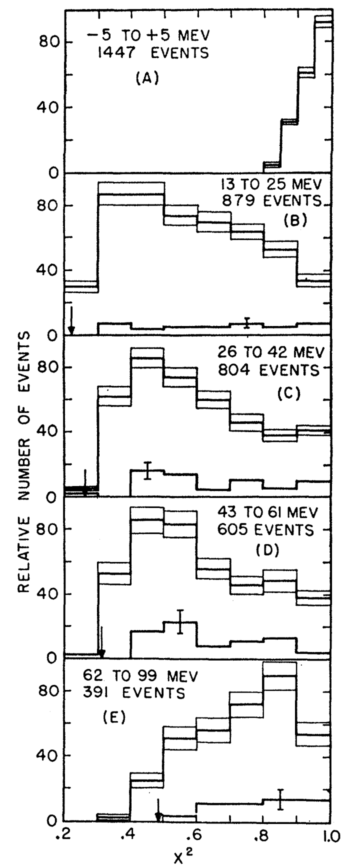

FIG. 6. Measured distribution of events as a function of x^2 for various groups of Fig. 5. The range of E_B defining each group is indicated, as is the total number of events in that group. The arrow on the left of each diagram indicates the average detection threshold for events in that diagram. The envelopes on each histogram represent the statistical error.

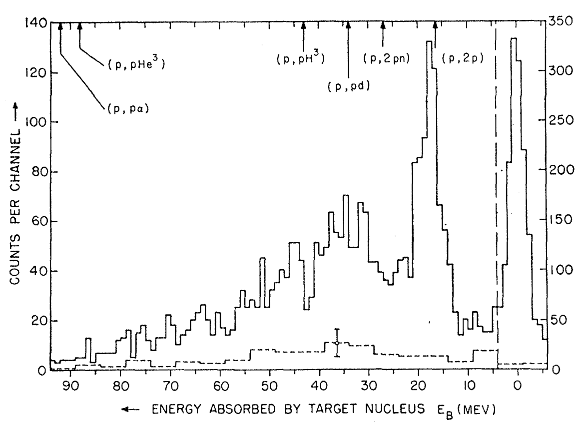

FIG. 5. Measured distribution of pair events as a function of E_B. Note the change of vertical scale at 4 Mev. An arrow indicates the point to the left of which the indicated competing reaction can contribute to the spectrum. The dotted spectrum is the accidental contribution of the pair distribution.

[11] M. Rich and R. Madey, University of California Radiation Laboratory Report UCRL-2301 (unpublished).

group from 43 to 61 Mev is much the same as those of the preceding two groups whereas that of the lowest group, from 62 to 99 Mev, is broadly peaked toward equal sharing. It is possible that this is at least partially the result of a decreased detection efficiency as the energy of the protons approaches the telescope threshold. All the measurements presented in Fig. 6 have a definite cutoff value for x^2 due to the finite range of the defining counters. The average cutoff point is shown on each diagram by an arrow. In addition there is some evidence (Sec. *D* below) of reduced scanning efficiency for low values of x^2 so that the peaks in the middle three diagrams of Fig. 6 may be somewhat lower than they should be. The general features of the energy sum distribution of Fig. 5 agree with the corresponding distribution reported by Tyrén *et al.*[3] and Gooding and Pugh[4] with somewhat poorer energy resolution. Gooding and Pugh also present energy sharing results for many scattering angle combinations for proton pairs lying under the 17-Mev peak from carbon. Their result closest to ours was obtained at $\theta_1=\theta_2=40°$ and within the limited statistics available, does not seem to show the minimum at $x^2=1$ of Fig. 6(B) obtained at $\theta_1=\theta_2=44°$.

D. Experimental Errors

We have considered the following experimental errors:

1. Accidental coincidences, measured as explained in Sec. III*A*, are plotted in Figs. 5 and 6 normalized to the same number of incident protons as the real coincidence counts. The effect of accidentals is seen to be small.

2. Film scanning bias: Because of the relatively large number of pulse pairs rejected by the scanner on the basis of our acceptance criterion, the possibility of scanning bias must be considered. About $\frac{1}{4}$ of the data was read by a second scanner. A careful examination of the events missed by either scan failed to show any bias which might seriously influence the sharing diagrams of Fig. 6, except for a relatively larger inefficiency for pairs with small values of x^2. Bias shared by both scanners would not have been detected.

3. Energy calibration: A fact which was insufficiently appreciated at the time the data were taken is that a small uncertainty in the calibration will cause the apparent energy sum of pairs having the same actual energy to vary with the value of x^2. Protons really lying at 21 Mev in Fig. 5 might appear to lie at 21 Mev for $x^2=1.0$ but at 23 Mev for $x^2=0.4$. When we drew sharing distributions dividing the 17-Mev peak of Fig. 5 more finely we observed sharp changes in the sharing spectrum which were due to this difficulty. However, the uncertainties in the calibration should have a negligible effect on the sharing parameter diagrams as presented in Fig. 6.

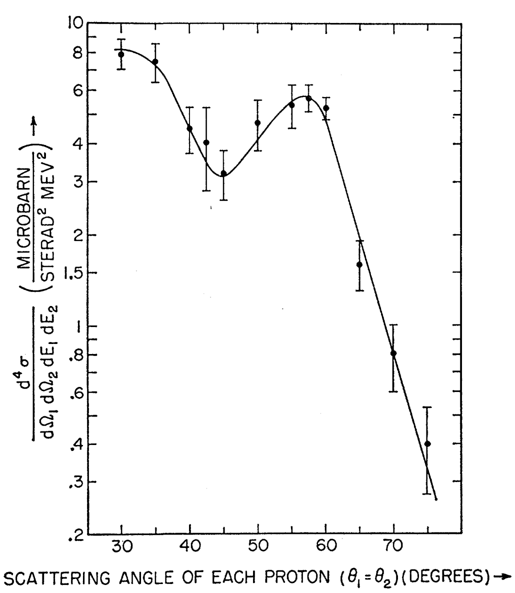

FIG. 7. Measured angular dependence of the cross section for scattering events with $E_B\approx 17$ Mev and $x^2\approx 1$. The line through the experimental points serves merely to guide the eye. The error in the absolute cross section is $\pm 10\%$. The data have been corrected for accidental coincidences.

IV. EXPERIMENT TYPE 2: ANGULAR DISTRIBUTION

A. Apparatus

The same telescopes were used as in the previous experiment, the total-energy crystal being replaced by a thin scintillator of $2\frac{1}{4}$ in. diameter viewed by an RCA 6810A through a 7-in. light pipe, and placed in anticoincidence with the defining counters. Thus each telescope was a single-channel range telescope, CH_2 absorber being placed between counters 1 and 2 to determine the lower detection threshold, and between counters 2 and 3 to determine the energy spread accepted.

The coincidence circuits of the previous experiment were replaced by transistor circuits. The input pulses to these circuits were delay-line clipped to give the *RL* circuit a resolving time (full width of the relative delay curve) of 4 millimicroseconds. The shape of the delay curve was checked from time to time during the run, using delay lines remotely inserted by relays. A Faraday cup monitor was used during part of the run, a helium-nitrogen ionization chamber later on.

B. Data Taking and Analysis

The incident beam energy was first measured by the range method[11] as 158.2 Mev. Each telescope was then set to detect protons of energies from 65 to 73 Mev, corresponding to events in the highest carbon peak of Fig. 5, with $0.9\leq x^2\leq 1.0$. Measurements of the accidental coincidence rate, made as explained previously, were interleaved with measurements of the total coincidence rate. The beam was steady, so that the correction for accidentals was reliable. This correction has been made on Fig. 7, which shows the observed angular distribution. There were about $\frac{1}{3}$ as many accidental counts as total counts.

At each angle a small correction was made in the telescope absorber to take into account the variation in the energy of the recoiling nucleus, as the angle between the telescopes was varied. The sensitivity to this correction proved, however, to be negligible, as was shown by purposely omitting it on one occasion.

There was a 10% background (target out) rate in the individual telescopes (presumably due to air scattering) but none at all in the *RL* rate.

C. Results

The measured angular distribution shows a dip at about 45° as predicted by the simple theory (Fig. 3) on the assumption that the incoming proton interacts with a p proton of carbon. The fact that the dip is relatively shallow is probably due to the neglect of diffraction and refraction in the calculations of Fig. 3, and the experimental necessity of detecting events in a finite range of x^2. The absolute cross section on Fig. 7 has an error of $+10\%$ due mostly to the uncertainty in the absolute charge measurement of the Faraday cup. It has been corrected for nuclear absorption in the telescopes. The angular resolution of each telescope was 2.0 degrees (full width at half height of the beam profile).

V. DISCUSSION

Those proton pairs corresponding to the narrow peak at $E_B=17$ Mev in Fig. 5 will be considered first. The position and width of this peak indicates that it includes proton pairs emitted such that the residual B^{11} nucleus is left in its $I=\frac{3}{2}$ ground state, and possibly in the first excited $I=\frac{1}{2}$ state at 2.13 Mev.[12] The present experimental resolution does not permit an estimate of the relative contribution of these two residual states to the observed width. In pure j-j coupling only the ground state can be excited when a $p_{\frac{3}{2}}$ proton is removed by a direct proton-proton collision. In pure L-S coupling both the $I=\frac{1}{2}$ and $\frac{3}{2}$ states can result from such an interaction.

The pairs belonging to the 17-Mev peak have the characteristic properties predicted for p protons by the Born approximation calculations of Sec. II. There is a strong tendency for the two correlated protons to share their energy unequally when both are emitted near 45° to the incident beam direction [Fig. 6(B)] and it is relatively unlikely for two protons of equal energy to be scattered 45° each to the incident direction (Fig. 7).

Figure 5 shows that proton pairs corresponding to values of E_B larger than 21 Mev are emitted with a rather broad distribution of binding energy peaked around 35 Mev. There is evidence for fine structure at $E_B=31$ Mev, 36 Mev, and 46 Mev corresponding to an excitation of the residual nucleus to 15 Mev, 20 Mev, and 30 Mev. Due to limited statistics the evidence is not conclusive. Proton pairs originating from collisions with s protons are expected to appear around $E_B=36$ Mev. The energy sharing distribution [Fig. 6(C)] is not that expected of collisions with s protons, but that found for p protons. Grouping the events included in this diagram into finer energy intervals does not change this conclusion. It is only for the very lowest-energy proton pairs [Fig. 6(E)] that a change appears to occur toward an s-type shape of the energy sharing distribution. However, it is difficult to estimate how seriously this last distribution is affected by decreased detection efficiency for low-energy pairs. Accidental coincidences make it very difficult to obtain angular distributions with a well defined value of the sharing parameter x^2 for proton pairs under the broad peak and no such distribution is available as yet.

Some of the possible causes of the discrepancy between the observed energy sharing distribution for values of $E_B>26$ Mev and that expected for s shell protons are these:

1. Either the incoming, or one or two of the outgoing protons involved in a collision with a p proton, has a secondary interaction in the target nucleus. Such a secondary interaction can lead to the excitation of the target nucleus without a large change in the angle of the outgoing particles. Cross-section estimates based on the results of Gooding and Pugh[4] and Tyrén and Maris[13] suggest that a sizable fraction of the observed pairs could come from such processes.

2. The thresholds for observing certain reactions other than $C^{12}(p,2p)B^{11}$ are shown by arrows on Fig. 5. In the presumed s proton region, the (p,pd) reaction products seem the most likely contaminants in this experiment. Future experiments will test for this possible contribution. The number of p-d pairs is expected to be much smaller than the number of p-p pairs under the 17-Mev peak, and thus it does not seem likely that the (p,pd) reaction contributes appreciably.

3. The reaction model is too simple, especially when the residual nucleus is excited to a relatively high energy. Further experiments with lighter nuclei (where secondary interactions are less important) and at higher energies (where in addition the calculations are more reliable) are clearly desirable.

Comparing the angular distribution of Fig. 7 for angles larger than 55° to Born approximation calculations for p protons, a value of $\beta^2=9$ Mev is found to be satisfactory. (At smaller angles the Born approximation results are dominated by the minimum and thus not applicable.) Considering the approximate nature of the calculation, the agreement of this value of β^2 with the value of 7.6 Mev calculated from the characteristic nuclear size parameter $a=1.65\times10^{-13}$ cm obtained by electron scattering[14] from carbon is satisfactory.

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This agreement reinforces the previous conclusion that the reaction model used is satisfactory to explain the results obtained, at least for proton pairs emitted such as to leave the residual nucleus in or near its ground state. It must be emphasized that quasi-elastic scattering experiments tend to emphasize those features of the target nucleon wave function which correspond to this nucleon not being close to another nucleon.

VI. ACKNOWLEDGMENTS

We wish to thank above all Inge Sierko who did the difficult job of scanning, Ray Goloskie who served as comparison scanner, Cos Papaliolios who helped take data for both parts of the experiment and worked on the electronic circuits, and Ted Houk who did most of the computations. We also thank Andreas Koehler and the cyclotron crew for their invaluable assistance with the machine.

PHYSICAL REVIEW VOLUME 120, NUMBER 3 NOVEMBER 1, 1960

Relative Ionization of Protons near the End of Their Range*

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(Received June 13, 1960)

The influence of multiple scattering on straggling and consequently on the ionization produced by protons close to the end of their range is derived. The effect due to nuclear reactions is shown to be small at 340 Mev. The observed data at this energy are satisfactorily accounted for.

In range-energy measurements for protons at 340 Mev[1] and 660 Mev[2] the relative ionization in argon produced by the protons after penetration of the absorber foil is measured. This ionization is produced by a combination of stopping power and straggling. Using a pure Gaussian for the straggling function folded with the stopping power [Eq. (2)], the theoretical result is in disagreement with the experimental data of both references 1 and 2.

In this Article it is proposed that most of the disagreement is caused by neglecting the influence of multiple scattering on the straggling.

The effect of protons suffering inelastic nuclear scattering has to be considered. It is assumed that all the inelastic processes can be treated according to the theory discussed by Metropolis *et al.*[3] The elastic Coulomb scattering is included in the multiple scattering theory employed. The question posed is: How many protons appearing in the ion chamber after penetrating thickness t of material have suffered energy losses not connected with conventional electron stopping power and multiple scattering processes? It is important to realize that for $t \geqslant 0.95R_0$ (R_0= mean range of the protons) a relatively small energy loss ΔE due to a nuclear reaction in the stopping material will prevent the proton from appearing in the ion chamber. The limiting energy loss is given by the expression

$$\Delta E_1 = (R_0 - t)\frac{dE}{dt}(E),$$

where E is the energy which the proton has just before undergoing the nuclear reaction. In the experiment of reference 1 with 340-Mev protons in lead, $t \geqslant 116$ g cm^{-2}, $R_0 \sim 122$ g cm^{-2} and for a proton in the middle of the foil, $E \sim 220$ Mev. With $dE/dt \sim 2.2$ Mev g^{-1} cm^2 one obtains $\Delta E_1 \leqslant 13$ Mev.

Using the results of Fig. 15 of reference 3, it is seen that only about 3% of all the protons scattered inelastically will lose between 0 and 10 Mev, and only a fraction of them will be scattered in the forward direction into the ion chamber. With a total inelastic cross section $\sigma \sim 1.7$ barns (approximately independent of energy, Fig. 11 of reference 3), about 50% of the protons will be transmitted without inelastic nuclear interaction. Thus, it is seen that the number of inelastically scattered protons amounts to less than 3% of the unaffected protons. Because of their reduced energy, their energy loss in the ion chamber will be higher, on the average by a factor of about 2. Thus, the contribution by extra protons to the ionization in the detector will be less than 6%. For the present purpose this effect is neglected.

The products of the nuclear interactions could also ionize the gas of the ion chamber, but their ranges will be very small, and the number penetrating into the detector will be small.

The multiple scattering contribution was discussed in

* This work was supported in part by the U. S. Atomic Energy Commission.
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