This agreement reinforces the previous conclusion that the reaction model used is satisfactory to explain the results obtained, at least for proton pairs emitted such as to leave the residual nucleus in or near its ground state. It must be emphasized that quasi-elastic scattering experiments tend to emphasize those features of the target nucleon wave function which correspond to this nucleon not being close to another nucleon.

VI. ACKNOWLEDGMENTS

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Relative Ionization of Protons near the End of Their Range*

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The influence of multiple scattering on straggling and consequently on the ionization produced by protons close to the end of their range is derived. The effect due to nuclear reactions is shown to be small at 340 Mev. The observed data at this energy are satisfactorily accounted for.

IN range-energy measurements for protons at 340 Mev[1] and 660 Mev[2] the relative ionization in argon produced by the protons after penetration of the absorber foil is measured. This ionization is produced by a combination of stopping power and straggling. Using a pure Gaussian for the straggling function folded with the stopping power [Eq. (2)], the theoretical result is in disagreement with the experimental data of both references 1 and 2.

In this Article it is proposed that most of the disagreement is caused by neglecting the influence of multiple scattering on the straggling.

The effect of protons suffering inelastic nuclear scattering has to be considered. It is assumed that all the inelastic processes can be treated according to the theory discussed by Metropolis *et al.*[3] The elastic Coulomb scattering is included in the multiple scattering theory employed. The question posed is: How many protons appearing in the ion chamber after penetrating thickness t of material have suffered energy losses not connected with conventional electron stopping power and multiple scattering processes? It is important to realize that for $t \geqslant 0.95R_0$ (R_0= mean range of the protons) a relatively small energy loss ΔE due to a nuclear reaction in the stopping material will prevent the proton from appearing in the ion chamber. The limiting energy loss is given by the expression

$$\Delta E_1 = (R_0 - t)\frac{dE}{dt}(E),$$

where E is the energy which the proton has just before undergoing the nuclear reaction. In the experiment of reference 1 with 340-Mev protons in lead, $t \geqslant 116$ g cm^{-2}, $R_0 \sim 122$ g cm^{-2} and for a proton in the middle of the foil, $E \sim 220$ Mev. With $dE/dt \sim 2.2$ Mev g^{-1} cm^2 one obtains $\Delta E_1 \leqslant 13$ Mev.

Using the results of Fig. 15 of reference 3, it is seen that only about 3% of all the protons scattered inelastically will lose between 0 and 10 Mev, and only a fraction of them will be scattered in the forward direction into the ion chamber. With a total inelastic cross section $\sigma \sim 1.7$ barns (approximately independent of energy, Fig. 11 of reference 3), about 50% of the protons will be transmitted without inelastic nuclear interaction. Thus, it is seen that the number of inelastically scattered protons amounts to less than 3% of the unaffected protons. Because of their reduced energy, their energy loss in the ion chamber will be higher, on the average by a factor of about 2. Thus, the contribution by extra protons to the ionization in the detector will be less than 6%. For the present purpose this effect is neglected.

The products of the nuclear interactions could also ionize the gas of the ion chamber, but their ranges will be very small, and the number penetrating into the detector will be small.

The multiple scattering contribution was discussed in

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[1] R. Mather and E. Segrè, Phys. Rev. **84**, 191 (1951).

[2] V. P. Zrelov and G. D. Stoletov, J. Exptl. Theoret. Phys. (U.S.S.R.) **36**, 658 (1959) [translation: Soviet Phys-JETP **36(9)**, 461 (1959)].

[3] N. Metropolis, R. Bivins, M. Storm, A. Turkevich, J. M. Miller, and G. Friedlander, Phys. Rev. **110**, 185 (1958).

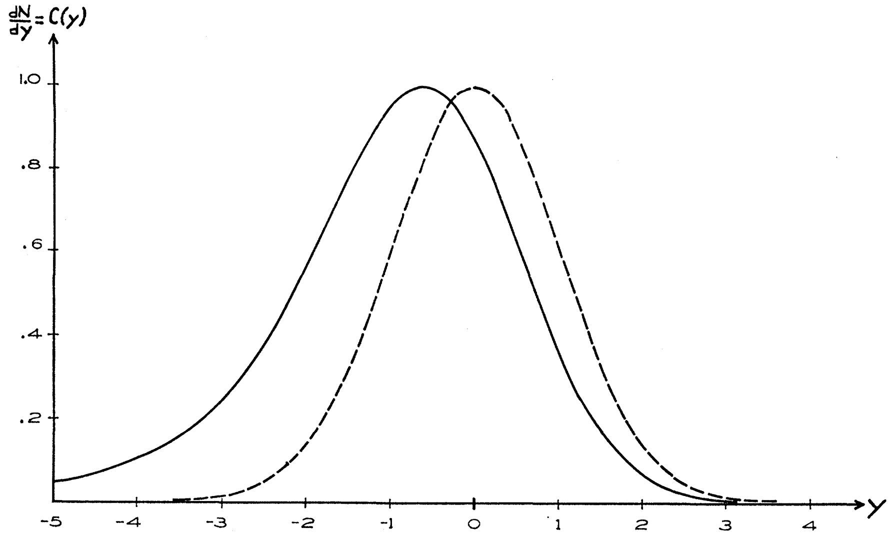

FIG. 1. Straggling distribution for $y=\Delta R/\sigma=(t-R)/\sigma$. (a) Broken line: no multiple scattering (pure Gaussian). (b) Solid line: with multiple scattering, $\xi=1$.

a recent paper[4] in connection with range energy measurements around 20 Mev. It was found that the observed straggling distribution $C(R)$ is obtained by folding the pure Gaussian $g(R)$ of straggling with the multiple scattering distribution $f(R)$. The parameters employed are (a) the width σ of the straggling distribution (called ΔR_s in reference 4), (b) the multiple scattering parameter r (called ΔR_0 in reference 4), and (c) the ratio $\xi=\sigma/r$. The variable used is ΔR, which is the range difference measured from the theoretical mean range R_0. A typical result for $C(y)$ for an experiment with fixed beam energy and variable absorber thickness is given in Fig. 1 with $y=\Delta R/\sigma$ as abscissa, for $\xi=1.0$, together with $g(y)$. The results published in Table I and Fig. 1 of reference 4 are the integrals of curves of the type of Fig. 1 of this paper, converted to an experiment with fixed absorber thickness and variable beam energy.

It is immediately seen that the extended tail of the new distribution curve $C(y)$ will tend to fill up the gap between theory and experiment of references 1 and 2. Results described by Fig. 1 would be obtained in an experiment in which the number of individual protons with zero energy would be detected as a function of the absorber thickness. If on the other hand the ionization $I(\Delta R)$ produced by the protons emerging from the foil is measured (as in the high-energy experiments), it will be the result of folding $C(\Delta R)$, with $(dE/dR)(\Delta R)$, assuming that the energy w for the formation of an ion

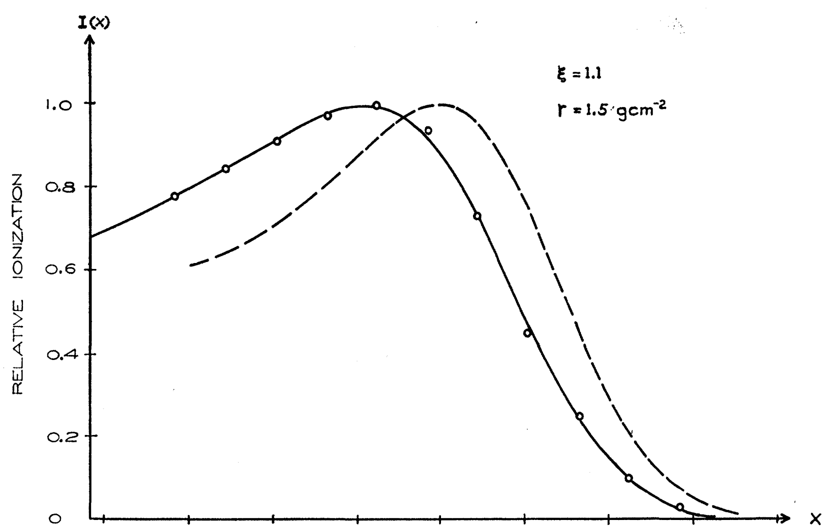

FIG. 2. Relative ionization in 340-Mev range-energy experiment in Pb (reference 1). (a) Broken line: No multiple scattering correction, according to Eq. (2). (b) Solid line: Theoretical curve with multiple scattering, $\xi=1.1$. (c) Experimental points from reference 1: o. The experimental mean range R is obtained by adding $2r/15$ to the value of t at $x=0$, where t is the abscissa of Fig. 1 of reference 1.

[4] H. Bichsel and E. Uehling, Phys. Rev. (to be published); also Bull. Am. Phys. Soc. 4, 217 (1959), and Bull. Am. Phys. Soc. 3, 399 (1958).

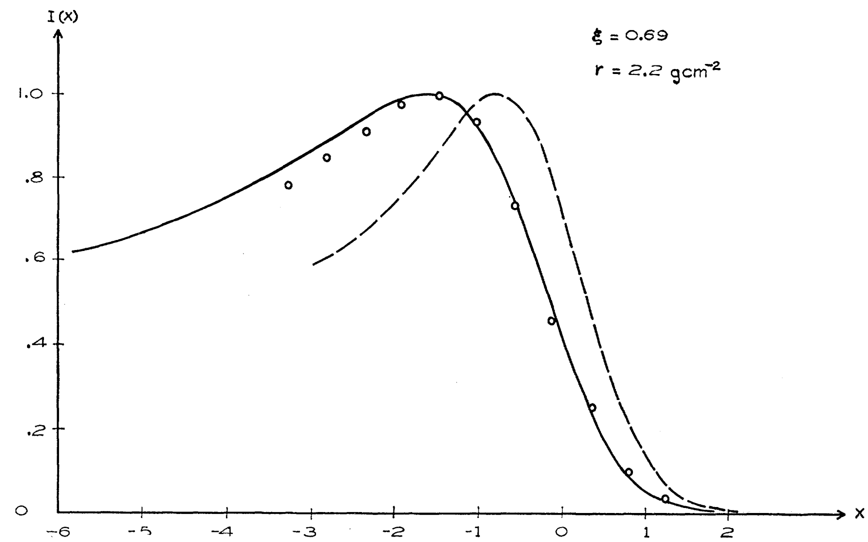

FIG. 3. Similar to Fig. 2, but with $\xi=0.69$. In both figures, r is chosen to give a close fit at 20% and 80%.

pair is independent of R; thus we may write

$$I(x)=\int_0^\infty C(x-z)\frac{dE}{dR}(z)dz,\quad x=\frac{\Delta R}{r},\quad z=\frac{\Delta R'}{r}. \quad (1)$$

Results of a numerical calculation using $\xi=1.10$ and the correct stopping power for argon are given in Fig. 2. The experimental points for lead from reference 1 are plotted for $r=1.50$ g cm^{-2}. The straggling parameter is $\sigma=1.65$ g cm^{-2}. The experimental error for ξ and r is estimated to be about 10%, while σ can probably be determined to an accuracy of $\pm2\%$. In Fig. 3, the theoretical ionization for $\xi=0.69$ and $r=2.24$ g cm^{-2} is compared with the experimental points. In this case σ is 1.55 g cm^{-2}, and is consequently not very sensitive to the value of ξ. The ionization curve for a pure Gaussian,

$$I_0(x)=\int_0^\infty g(x-z)\frac{dE}{dR}(z)dz, \quad (2)$$

is also given, and the displacement between $I(x)$ and $I_0(x)$ gives immediately the multiple scattering correction for the experimental points.

In particular the thickness R corresponding to the theoretical mean range can be found immediately from the curves as the point where the center of the original Gaussian lies. For $\xi=1.1$ this occurs for $I=49\%$. An additional small correction discussed in reference 4 and amounting to $(2/15)r$ cannot be observed experimentally.

The experimental range obtained in this fashion is $R=122.8$ g cm^{-2} in very close agreement with reference 1. The experimentally determined value $r=1.50$ g cm^{-2} found here is also in good agreement with the corresponding theoretical value of reference 1 [Eq. (8)], while the straggling parameter is 10% smaller than the experimental value, but still 20% larger than the theoretical value of reference 1 (presumably caused by the energy spread of the incident beam).

Unfortunately, the experimental points of reference 2 do not cover a sufficient range of t to determine the ξ value for Cu. Also, it would be necessary to consider the effect of the mesons produced at the higher energy of 660 Mev, an effect quite negligible at 340 Mev.

The effect of diffraction scattering[5] has been neglected. For Pb, the protons with wavelength $\lambda\!\!\!^{-}$ will approximately be confined to an angle $\alpha\sim\lambda\!\!\!^{-}/R_n\sim0.03$ rad (with radius R_n of the nucleus), which is small compared to the angle produced by multiple scattering (approximately 0.3 rad). Therefore the correction to the shape of the ionization curve and probably also the correction to the range will be small.

[5] W. H. Barkas, Nuovo cimento 8, 201 (1958), footnote 15.