

(4) we see that the radial coordinate r_∞ of this particle (the suffix denotes that the meeting takes place in the infinite future) is given by

$$r_\infty = R \tan \chi. \quad (19)$$

The acceleration α corresponding to an η as in (18) is found, from (10), to be

$$\alpha = R^{-1} \tan 2\chi, \quad (20)$$

and this allows us to eliminate χ from (19). Thus we easily find

$$r_\infty = \alpha^{-1} \{ (1 + \alpha^2 R^2)^{1/2} - 1 \}. \quad (21)$$

The same result was obtained analytically in paper I, which should be consulted for the connection between r_∞ and the “ α horizon.” If α is infinite, the “accelerating” particle is a photon. This corresponds, by (19) and (20), to $r_\infty = R$. Hence even a light-signal does not overtake all fundamental particles on its line of motion, which is, of course, a well-known fact.

It is also clear from the diagram that there exists a photon which, sent after the accelerating particle, only intercepts it in the infinite future. Its projection on to the (x, y) plane is QQ' , parallel to PP' and tangent to the waist-circle. The coordinates of Q are evidently $(0, R \operatorname{cosec} 2\chi, 0)$, while the coordinates of the point on \mathcal{H} whose projection is Q are $(0, R \operatorname{cosec} 2\chi, R \cot 2\chi)$. By 3 (ii), this corresponds to a time

$$t_{\text{crit}} = R \ln(\operatorname{cosec} 2\chi + \cot 2\chi),$$

which, by (20), is equivalent to

$$t_{\text{crit}} = R \ln[\alpha^{-1} R^{-1} \{ (1 + \alpha^2 R^2)^{1/2} + 1 \}]. \quad (22)$$

The subscript denotes that this is a critical time for photons sent from the origin-particle ($r=0$) to intercept an accelerating particle that was released there from rest at $t=0$. Photons emitted earlier intercept the receding particle, while photons emitted later do not. This result too was obtained analytically in paper I.

Absence of Bound States in a Gravitational Field*

ASHER PERES

Department of Physics, Israel Institute of Technology, Haifa, Israel

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The Klein-Gordon and Dirac equations for a particle in the gravitational field of a point mass are investigated. It is shown that the geometrical properties of the Schwarzschild metric prevent the normalization of any bound-state solutions.

CALLAWAY¹ has recently investigated the general relativistic Klein-Gordon and Dirac equations for an attractive center of mass M and charge e , in the case $M < e$.²

The opposite case, $M > e$, displays wholly different features: Equation (C2) shows that the metric is singular at $r = M \pm (M^2 - e^2)^{1/2}$. We shall take here $e=0$, since this does not essentially differ from $0 < e < M$.

The Klein-Gordon equation (C4) is singular at $r = 2M$. The indicial equation at this point shows that the solutions behave there as $(r - 2M)^{\pm 2iEM}$. Since the

current density is

$$J^k = i(-g)^{1/2} g^{kn} (\bar{\psi} \psi_{,n} - \bar{\psi}_{,n} \psi) / 2m,$$

then it follows from (C1) that $\int J^0 dr d\theta d\phi$ diverges at $r = 2M$.

In the case of the Dirac equation, one has¹

$$J^0 = \bar{\psi}(-g)^{1/2} \gamma^0 \psi \sim (-g)^{1/2} (F^2 + G^2) / r^2 g_{00}.$$

Expansion of (C7) about $r = 2M$ shows that the leading terms are

$$(r - 2M)F' + 2MEG = 0 \quad \text{and} \quad (r - 2M)G' - 2MEF = 0.$$

It follows that F and G behave as $(r - 2M)^{\pm 2iEM}$, so that here also the wave function is not normalizable.

Thus, there are no bound states in the gravitational field of a point mass.

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¹ J. Callaway, Phys. Rev. **112**, 290 (1958), hereafter referred to as C. There is a misprint in Eq. (C7); see D. Brill and J. A. Wheeler, Revs. Modern Phys. **29**, 465 (1957).

² Natural units are used: $c = G = \hbar = 1$.