bound states will of course give a peak of half-width Γ_m at a critical value of the energy $E_{BC}=E_m$. Further study of the hyperspherical harmonics should disclose other ways in which experimental observations can be used in this sort of analysis—various combinations of data including energies, angular correlations, and temporal coincidences or delays, could be examined for 3-body effects.[12]

[12] Compare the study of 3-body events for effects of 2-body forces: G. F. Chew and F. E. Low, Phys. Rev. **113**, 1640 (1959). See also L. Fonda and R. G. Newton, Phys. Rev. **119**, 1394 (1960).

In some problems of chemical kinetics, 3-body reactions occur in a statistical assemblage of colliding particles. Similar events, governed by short-range forces, occur in the 3-body attachment of electrons to atoms or molecules. In cases like these, it should be possible to introduce the angular momentum description of 3-body collisions into a statistical argument. In such a description it is important to look carefully into the relative contributions of pure 3-body processes and events involving a 2-body metastable.

PHYSICAL REVIEW VOLUME 120, NUMBER 3 NOVEMBER 1, 1960

Quenching of Magnetic Moments in Nuclei*

S. D. DRELL AND J. D. WALECKA†
Department of Physics and Institute of Theoretical Physics, Stanford University, Stanford, California
(Received June 20, 1960)

Starting from the premise that with modern dispersion-theoretical techniques one has a reliable method for calculating the anomalous magnetic moment of a nucleon, we have calculated the modification or "quenching" of this moment for a nucleon in nuclear matter. The effect we consider here is due to the fact that nucleons are not allowed by the exclusion principle to recoil into states already occupied by other nucleons in the nucleus. The actual technique we have used in our calculation is to sum all the Feynman diagrams that are included in the dispersion-theory calculation of the single-nucleon moment. We then write the nucleon propagator as a sum over states and remove those states in which the nucleon is inside the Fermi sea. Our result is that the anomalous moment is reduced by $\approx 7\%$.

I. INTRODUCTION

IN this paper we wish to re-examine the question of the quenching of the intrinsic magnetic moments of nucleons in nuclear matter. The idea of quenching the spin-g factor of a nucleon (g_s) in nuclear matter was proposed in 1951, independently, by Bloch,[1] Candler,[2] Miyazawa,[3] and de-Shalit.[4] Their arguments were based on the observation that in almost every case the observed magnetic moments of odd-A nuclei could be explained by a single-particle calculation with the intrinsic nucleon moment lying somewhere between the free-nucleon moment

$$\mu_p=(1+1.79)\text{ nm},\quad \mu_n=-1.91\text{ nm},\qquad (1.1)$$

and a completely quenched moment

$$\mu_p=1\text{ nm},\quad \mu_n=0\text{ nm}.\qquad (1.2)$$

If one plots the magnetic moments of the odd-A nuclei vs the nuclear spin one obtains from the single-particle model two Schmidt lines[5,6] for $l=I\pm\frac{1}{2}$, where l is the orbital angular momentum and I the total angular momentum, or spin, of the odd nucleon considered to be moving in the spherically symmetric potential provided by the even-even core. The experimental moments are found to cluster near these lines but the fit is greatly improved if the value of the intrinsic moment for a nucleon in nuclear matter is taken to lie between values (1.1) and (1.2).

The physical assumption underlying use of unquenched values (1.1) in nuclear matter is that the currents in the meson cloud about a nucleon are not altered by the presence of other nucleons in the nuclear matter. The values (1.2) would apply in the case that the presence of other nucleons at the density of normal nuclear matter completely discouraged a nucleon from developing its normal meson currents. Thereby the moment would be quenched all the way down to the Dirac value which obtains in the absence of all meson-current effects. There have been several attempts to implement this idea with an accurate calculation.[3,7] However, several major obstacles have barred the way:

1. It has not been possible to calculate from meson theory the magnetic moments of free nucleons with any accuracy. Indeed until the dispersion-theory methods of the past two years there has not even existed a systematic approach to a nucleon magnetic

* Supported in part by the U. S. Air Force through the Air Force Office of Scientific Research.
† National Science Foundation Postdoctoral Fellow.
[1] F. Bloch, Phys. Rev. **83**, 839 (1951).
[2] C. Candler, Proc. Phys. Soc. (London) **A64**, 999 (1951).
[3] H. Miyazawa, Progr. Theoret. Phys. (Kyoto) **5**, 801 (1951).
[4] A. de-Shalit, Helv. Phys. Acta **24**, 296 (1951).
[5] T. Schmidt, Z. Physik **106**, 358 (1937).
[6] R. J. Blin-Stoyle, Revs. Modern Phys. **28**, 75 (1956).
[7] F. Villars and V. Weisskopf (unpublished).

moment calculation other than the totally inadequate weak-coupling perturbation expansion.

2. As emphasized especially by Blin-Stoyle, "nuclear moments (particularly the magnetic moments) are very sensitive to certain types of admixture in the nuclear wave function. This means that nuclear moments cannot in general be taken as a good guide to the purity or otherwise of nuclear states. If the deviation in the moment is small there may nevertheless be large admixtures of states which do not appreciably affect the moment. On the other hand, a large deviation may only mean a small admixture of states to which the moment is particularly sensitive."[6] Indeed, as a result of the progress and refinements in our nuclear models which have developed in recent years one no longer expects nuclear moments to be closely correlated with the single-particle Schmidt lines. Even if we confine our attention to the doubly-magic-plus-or-minus-one nuclei it is not clear whether or not the comparison between theory and experiment shows any need for a quenching hypothesis. Thus the observed moment of O^{17} is -1.89 mm, in very close agreement with the Schmidt value of -1.91 mm for an odd-neutron nucleus with $I=l+\frac{1}{2}=\frac{5}{2}$. In contrast, K^{39} which is doubly-magic minus one proton has an observed moment of 0.39 nm and a Schmidt value for $I=l-\frac{1}{2}=\frac{3}{2}$ of 0.12 mm.[6]

For these reasons the idea of the quenching of g_s in nuclear matter has been in eclipse in recent years. Now with the development of improved approximation methods for studying the strong pion couplings we are motivated to reeaxmine this problem. There may be no obvious or significant quenching indicated by experiment. Nevertheless there will be some implied by the recent calculations of free-nucleon moments and we wish to compute this here.

II. QUENCHING MECHANISM AND APPROXIMATIONS

The specific mechanism for quenching which we consider here was discussed by Miyazawa in his original proposal.[3] It is most simply described as the exclusion principle effect. The anomalous magnetic moment of a nucleon may be expressed in terms of integrals over the momentum spectrum of the virtual intermediate mesons contributing to its physical structure. Inside a nucleus those meson states leading a nucleon to recoil into states already occupied by the other nucleons are forbidden by the exclusion principle. Their contribution to the anomalous moment is thus suppressed and this leads to a diminution in its value.

In our calculations we take for simplicity a Fermi gas model of the nucleus. At normal nuclear density the nucleons occupy all states up to an energy $E_f=p_f^2/2M=40$ Mev. We compute the exclusion-principle quenching for a nucleon of momentum p_f. This will be of direct relevance for doubly-magic-plus-or-minus-one nuclei for which the observed magnetic moment is due to the extra-core nucleon, here represented by one at the

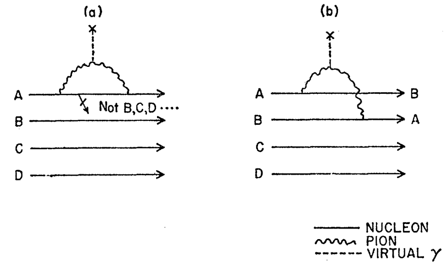

FIG. 1. We illustrate in this figure the sense in which the quenching mechanism we consider is equivalent to an exchange current. In 1(a), we do not allow the intermediate virtual nucleon to recoil into the states $B, C, D \ldots$ because they are already occupied by other nucleons. If, on the other hand, we follow Feynman's prescription and neglect the exclusion principle in intermediate states, we let nucleon A recoil into the filled states in graphs of type 1(a) and in addition include the exchange contributions indicated in 1(b). These also violate the Pauli principle since A recoils into state B (or any of the other states) while B is already filled. However, they enter with a minus sign due to the antisymmetry of the nuclear wave function. The sum of the two effects is exactly the same as excluding the filled virtual states in (1a).

Fermi surface. In this model we have no nuclear force effects on the meson cloud other than that resulting from the fact that there are nucleons held together at the observed density of nuclear matter. This means one may be ignoring possible exchange magnetic moment contributions arising from spin-orbit or charge-exchange forces. In the absence of a complete detailed force theory we have no way of knowing precisely what it is that we are neglecting in this way. In fact our presently considered exclusion-principle effect may be construed equally well as an exchange-current contribution.[3] Following Feynman's prescription we merely ignore the exclusion principle for intermediate states and allow the meson current originating on any one of the nucleons in the nuclear matter to terminate on any other one as well as on the one where it originated. Graphically for a lowest order perturbation diagram we may consider the process as pictured either in Fig. 1(a) or 1(b).

We next outline the approximations applied in the calculation of the magnetic moment of a free nucleon. The spectral representations of the dispersion approach express the moment in terms of an integral over a weight function, W. W is proportional to the amplitude for the magnetic field to produce real particles which then are absorbed by the nucleon; it is a function of the invariant square of the four-momentum m^2 transferred from the field to the nucleon. In practice one includes in W only the amplitude to produce the lightest-mass state connecting with the nucleon; this is the two-pion state. The dispersion integral then extends from the threshold, $m^2=4m_\pi^2$, to produce two pions to $m^2=\infty$. All other states are neglected. This means in particular that in this approximation one calculates only the isotopic vector part of the anomalous magnetic moment $\frac{1}{2}[(\mu_p-1)-\mu_n)=1.85$ nm and makes no attempt to account for the much smaller isotopic scalar part

$\frac{1}{2}[(\mu_p-1)+\mu_n]=-0.06$ nm. Since the momentum transfer to the nucleon is time-like, $m^2>4m_\pi^2$, the two pions may be viewed as scattering the nucleon from a negative-to a positive-energy state—or equivalently, as creating a nucleon-antinucleon pair. The recent calculations of Frazer and Fulco have established that with a resonance interaction between the two intermediate pions one can explain the observed value and structure of the nucleon moment within this approximation.[8] The resonance required for this fit is rather narrow and with a maximum in the region of $3.5m_\pi$ to $4m_\pi$ for the total energy of the two pions in their center-of-mass system. It serves to emphasize contributions from low-momentum pions to the weight function and thereby to provide a more distributed structure. Simultaneously it weighs against the high-momentum contributions. For this reason Frazer and Fulco find it to be a good approximation to describe the absorption of the two pions on the nucleon simply by Born approximation times a form factor taking into account the large scattering phase shift between the two interacting pions. They show that higher order meson-nucleon rescattering corrections contribute only very little in the moment calculation because they become important only at higher momenta and their effect is damped out by the π-π resonance. Schematically we can represent their working approximation for the weight function by Fig. 2.

In the present analysis of quenching we are dealing with the magnetic moment not of one single free nucleon but of a nucleon swimming on the surface of a Fermi sea of nucleons. This makes it impractical to take an excursion into the complex momentum-transfer plane in the canonical manner which yields the dispersion relations, since the electromagnetic interaction depends on the momenta in the Fermi sea as well as on that transferred by the field. We therefore construct a set of Feynman graphs which should be kept to reproduce the accuracy of the dispersion approach and then calculate how much the values of these graphs are altered by the exclusion-principle effect.

This in no way constitutes a defense of the dispersion-theory approximation of keeping only the lighest-mass two-π-meson state in the weight function W. Our result, however, is to be interpreted as an implication of that approximation. Moreover we can appeal to low-energy approximations in the present application since the quenching affects only intermediate nucleons recoiling with low momentum below the surface of the Fermi sea, $E_f=p_f^2/2M=40$ Mev. This will strengthen the accuracy of our method.

III. DISCUSSION OF PROCEDURE

We want now to give a development of our procedure and approximations for calculating the magnetic moment within the framework of Feynman graphs and to compare with the corresponding dispersion approach. It is convenient to begin by defining the electromagnetic vertex of a nucleon. The amplitude for a nucleon of four momentum p^μ, with $p^2=M^2$, to absorb a virtual photon of four momentum k^μ from an external field, emerging with $(p+k)^2=M^2$, is[8a]

$$S_{fi}=-2\pi i\left(\frac{M^2}{E_{p'}E_p}\right)^{\frac{1}{2}}(p'|J_\mu|p)A_{\mathrm{ext}}{}^\mu\delta^4(p'-p-k). \quad (3.1)$$

To lowest order in perturbation theory we write

$$(p+k|J_\mu|p)=e_0\bar{u}(p+k)\gamma_\mu u(p) \quad (3.2)$$

in terms of free Dirac spinors $u(p)$, $\bar{u}(p')=u^\dagger(p')\gamma_0$ and of the unrenormalized bare charge e_0. For the complete vertex we write

$$(p+k|J_\mu|p)=\bar{u}(p+k)\Gamma_\mu(p+k,\,p)u(p). \quad (3.3)$$

The most general form of the vertex operator Γ_μ standing between Dirac spinors may be written

$$\begin{aligned}\bar{u}(p+k)&\Gamma_\mu(p+k,\,p)u(p)\\ &=\bar{u}(p+k)[F_1(k^2)\gamma_\mu+iF_2(k^2)\sigma_{\mu\nu}k^\nu+F_3(k^2)k_\mu]u(p)\\ &\equiv\bar{u}(p+k)[e_0\gamma_\mu+\Lambda_\mu(p+k,\,p)]u(p), \quad (3.4)\end{aligned}$$

where $F_1(0)=e$, the observed physical charge; $F_2(0)=+\lambda e/2M$, the anomalous magnetic moment; and the third term $F_3(k^2)k_\mu$ does not contribute by gauge invariance.[9] Equation (3.4) follows by consideration of Lorentz invariance and by use of the Dirac equation for the spinors $u(p)$: $(\not{p}-M)u(p)=0$, where $\not{p}\equiv p_\mu\gamma^\mu$. Since we are interested here in the anomalous magnetic moment, or $F_2(0)$, we must calculate the general nucleon-electromagnetic vertex through terms linear in k as $k^\mu\to 0$.

In order to project out the magnetic moment term

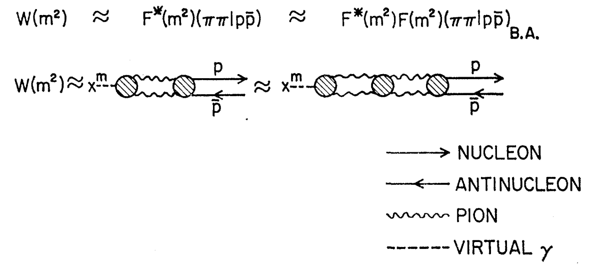

FIG. 2. Working approximation for the dispersion-theoretical calculation of the vector part of the free-nucleon magnetic moment. The subscript B.A. means Born approximation.

[8] W. R. Frazer and J. R. Fulco, Phys. Rev. **117**, 1609 (1960).

[8a] We use the notation of S. Schweber, H. Bethe, and F. de Hoffmann, *Mesons and Fields* (Row, Peterson, and Company, Evanston, Illinois, 1955) Vol. 1.

[9] The appearance of the k_μ term in the operator can be ruled out directly by time-reversal arguments. [F. J. Ernst, R. G. Sachs, and K. C. Wali, Phys. Rev. **119**, 1105 (1960).]

we perform the following readily constructed operation:

$$\tfrac{1}{2}\sum_{\substack{\text{spins}\\E>0}} \bar{u}(p)\left\{\frac{\gamma_\mu}{1+k^2/2M}-\frac{(p+\tfrac{1}{2}k)_\mu}{M(1-k^2/4M^2)}\right\} \times u(p+k)\bar{u}(p+k)\Gamma_\mu(p+k,\,p)u(p) =\frac{k^2}{2M}\left(\frac{2-k^2/2M^2}{1+k^2/4M^2}\right)F_2(k^2), \quad (3.5)$$

and then keep the leading terms as $k^2\to 0$.

The general prescription for calculating the electromagnetic vertex within the framework of Feynman diagrams is as follows. One draws pictures and writes down corresponding expressions for all irreducible graphs contributing to the vertex. In these graphs one then inserts into each nucleon, pion, and photon line the exact propagators, S_F', Δ_F', and D_F', respectively, and, at each vertex, the exact proper vertex parts $e_0\Gamma_\mu$ for the nucleon-photon vertex, e_0V_μ for the pion-photon vertex, and $g_0\Gamma_5$ for the pion-nucleon vertex; e_0 and g_0 are the unrenormalized coupling constants. According to the general renormalization theory arguments these exact quantities are equal to finite parts multiplied by infinite constants, Z_i. The Z_i can always be grouped together in such a way as to renormalize the charges appearing in the theory.[10] This means then that we have effectively inserted into the irreducible graphs the renormalized coupling constants e and g, and the finite parts of the propagators and vertices, S_{Fc}', D_{Fc}', Δ_{Fc}', Γ_{5c}, $V_{\mu c}$ and $\Gamma_{\mu c}$, and are thus left with finite results.

Applying this procedure to the vertex we illustrate in Fig. 3 the first few of the infinite series of graphs contributing to the vertex. Diagram 3(a) represents our basic approximation. In parallel with the dispersion approach we concentrate only on graphs in which the interaction with the electromagnetic field (which is treated in lowest order) is through the meson current which is treated exactly. We find contributions of this type also in the other graphs in Fig. 3 when we expand out the vertex blobs in a perturbation series. For example graph 3(b) includes a process such as in Fig. 4. which corresponds to a meson rescattering correction along the nucleon line. The work of Frazer and Fulco[8] including the π-π scattering showed this to play a minor role in the calculation of the free-nucleon magnetic moment. We shall show by calculation that it is unimportant also in our quenching calculation because the dominant rescattering correction in the 3-3 state of the meson and nucleon cannot operate effectively if the intermediate nucleon must lie within the Fermi sea. Diagrams of the form 3(c) are not included in the Frazer-Fulco analysis of the absorptive amplitude and will also be dropped here along with 3(d) and 3(e) since they correspond to higher-order rescattering corrections.

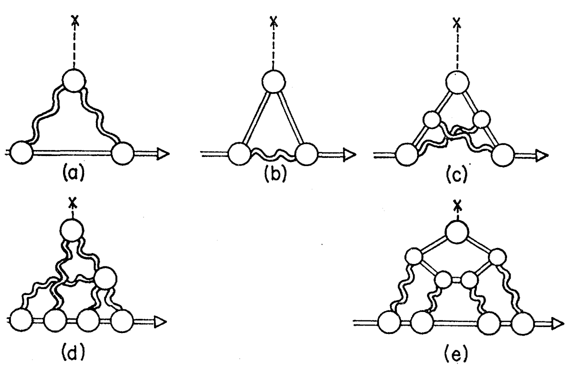

FIG. 3. Irreducible graphs of the nucleon electromagnetic vertex.

[10] J. C. Ward, Phys. Rev. 84, 897 (1951).

Our basis for neglecting these higher graphs is then summarized in these two remarks:

(*a*) They correspond to amplitudes which are shown to contribute negligibly or are entirely neglected in the Frazer-Fulco analysis which is the starting point for us here.

(*b*) Rescattering and higher-order corrections in the meson-nucleon interaction are expected to be unimportant because in the quenching calculation we are interested only in that part of the amplitude for which the nucleon remains in the Fermi sea. Direct calculations of the rescattering in the 3-3 resonant state gives a correction of 15%–20% to our work.

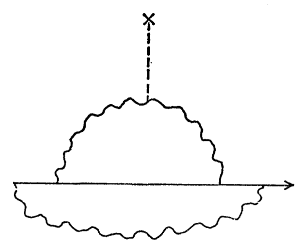

FIG. 4. One graph in 3(b)—a rescattering correction.

In our approximation we have then included all Feynman graphs contained in the final working approximation of the dispersion approach. The difference in the approximations lies in the fact that we here retain the entire Feynman graph rather than just those absorptive parts of the graphs which contribute to diagrams of Fig. 2. However our methods here shall yield information only on the static moment $F_2(0)$ whereas the dispersion analysis also gives the structure, $F_2(k^2)$, for all $k^2\neq 0$.

IV. FORMAL DEVELOPMENT

We concentrate on graph 3(a) where the interaction with the electromagnetic field (which is treated in lowest order) is through the meson current. The general form of all such graphs is indicated in Fig. 5. For present purposes we have replaced the internal nuclear line and the nucleon-pion vertices by a general box; this in no way affects our arguments. The top half of this

graph can be written

$$\frac{-i}{(2\pi)^8}(\delta_{\alpha1}\delta_{\beta2}-\delta_{\alpha2}\delta_{\beta1})\Delta_F'(q^2)e_0V_\mu(q+k,\,q)\times\Delta_F'((q+k)^2), \quad (4.1)$$

where

$$V_\mu(q+k,\,q)=i(2q+k)_\mu+\chi_\mu(q+k,\,q) \quad (4.2)$$

is the sum over all proper pion-electromagnetic vertex parts. The bottom half of the graph has a form similar to the pion-nucleon scattering amplitude. We can use the usual arguments to classify the possible invariants[11] that appear here since our vertex Γ_μ stands between Dirac spinors in Eq. (3.4). However, since the pions are virtual, we will now have two extra scalars in the problem which we take to be

$$q_1^2=m_1^2, \quad q_2^2=m_2^2. \quad (4.3)$$

Therefore, for the lower part of the graph indicated in Fig. 5, we can write the scalar

$$\bar{u}(p_2)T(m_1^2,m_2^2,k^2,W^2)u(p_1), \quad (4.4)$$

where

$$W^2=(q_1+p_1)^2, \quad k^2=(q_2-q_1)^2, \quad (4.5)$$

with

$$T=A+\tfrac{1}{2}(\boldsymbol{q}_1+\boldsymbol{q}_2)B, \quad A=A^+\delta_{\beta\alpha}+A^-\tfrac{1}{2}[\tau_\beta,\tau_\alpha], \quad (4.6)$$

and

$$B=B^+\delta_{\beta\alpha}+B^-\tfrac{1}{2}[\tau_\beta,\tau_\alpha].$$

It must be noted that this box in Fig. 5 is *not* just pion-nucleon scattering off the mass shell since many of the graphs that contribute to pion-nucleon scattering are now contained in the pion vertex $V_\mu(q+k,\,q)$. Our argument here is purely one on the form of the amplitude as limited by covariance and isotopic spin conservation. Joining the two halves of our graph in Fig. 5 and summing over isotopic spins and momenta q gives, with

$$(\delta_{\alpha1}\delta_{\beta2}-\delta_{\alpha2}\delta_{\beta1})\tfrac{1}{2}[\tau_\beta,\tau_\alpha]=-2i\tau_3, \quad (4.7)$$

the result

$$\Lambda_\mu(p+k,\,p)=\frac{-2e_0\tau_3}{(2\pi)^4}\int d^4q\,\Delta_F'(q^2)V_\mu(q+k,\,q)\times\Delta_F'((q+k)^2)T^-(m_1^2,m_2^2,k^2,W^2). \quad (4.8)$$

To approach an evaluation of this vertex we wish to establish first that in evaluating the magnetic moment it is possible to set the momentum transfer k equal to zero everywhere in Eq. (4.8).

The entire k dependence need only be kept in the Dirac spinors. It can then be brought out explicitly for the static magnetic moment in the form $\sigma_{\mu\nu}k^\nu$ through a term p_μ and the Gordon decomposition of the current. Once we establish this result we can draw upon Ward's identity to simplify the calculation greatly.

[11] G. F. Chew, M. L. Goldberger, F. E. Low, and Y. Nambu, Phys. Rev. **106**, 1337 (1957).

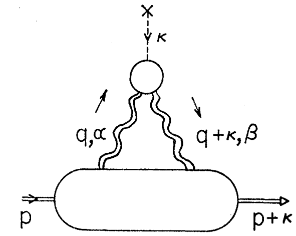

FIG. 5. The general graph where the interaction with the electromagnetic field is through the meson current.

We can readily establish this result by recalling the general form of the operator $\Lambda_\mu(p+k,p)$ which must be a 4×4 matrix which transforms as a four vector. Since $p^2=M^2$ and $(p+k)^2=M^2$ for real initial and final nucleons on the mass shell we can construct only one scalar variable from the vectors p and k for Λ_μ to depend upon. We thus can write, for the general form of Λ_μ,

$$\Lambda_\mu(p+k,\,p)=a_1(k^2)\gamma_\mu+a_2(k^2)k_\mu+a_3(k^2)P_\mu+a_4(k^2)\sigma_{\mu\nu}k^\nu+a_5(k^2)\sigma_{\mu\nu}P^\nu, \quad (4.9)$$

where for convenience we introduce

$$P_\mu=(p+k+p)_\mu=2p_\mu+k_\mu, \quad (4.10)$$

which satisfies the relation $P\cdot k=0$. No terms containing the scalar $\boldsymbol{p}=p_\mu\gamma^\mu$ or $\boldsymbol{k}$ appear here because we anticipate sandwiching Λ_μ between Dirac spinors as in Eqs. (3.4). Then it is possible, using the commutation rules of the Dirac algebra, to reduce such terms to the above form since $(\boldsymbol{p}-m)u(p)=0$, and $\bar{u}(p+k)\boldsymbol{k}u(p)=0$. By straightforward application of the Dirac equation and the γ-matrix algebra we can combine terms of Λ_μ into the form

$$\Lambda_\mu(p+k,\,p)=b_1(k^2)\gamma_\mu+b_2(k^2)k_\mu+b_3(k^2)P_\mu. \quad (4.11)$$

This form follows directly from Eqs. (4.6) and (4.8).

Only the b_3 term in Eq. (4.11) contributes to the anomalous magnetic moment, as we see by comparing with Eq. (3.4). To exhibit the b_3 term in the form of a magnetic moment we recall the Gordon current decomposition,

$$\bar{u}(p+k)P_\mu u(p)=\bar{u}(p+k)(2M\gamma_\mu-i\sigma_{\mu\nu}k^\nu)u(p). \quad (4.12)$$

Comparing with Eq. (3.4) we see the correspondence

$$\begin{aligned}F_1(k^2)&=b_1(k^2)+2Mb_3(k^2),\\ F_2(k^2)&=-b_3(k^2),\\ F_3(k^2)&=b_2(k^2).\end{aligned} \quad (4.13)$$

Thus to evaluate the static moment we can set $k^2=0$ in Eq. (4.8). The procedure then is to first express the general form for the lower half of the Feynman diagram as in Eq. (4.4). All terms of the form $\sigma_{\mu\nu}k^\nu$ are reduced by the Gordon identity (4.12). We can then put $k^2=0$ everywhere and extract the moment term by the projection in Eq. (3.5). In the following we shall always have this sequence of operations in mind upon setting k to zero in Eq. (4.8). With this understanding we have

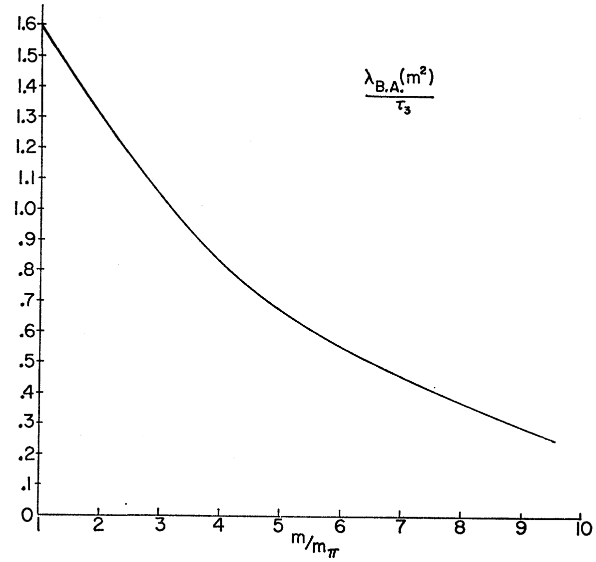

FIG. 6. Born approximation calculation of the anomalous magnetic moment as a function of the intermediate meson mass.

proved our contention that we need only study

$$\Lambda_\mu(p,p)=\frac{-2e_0\tau_3}{(2\pi)^4}\int d^4q\,\Delta_F'(q^2)V_\mu(q,q)\times\Delta_F'(q^2)T^-(q^2,q^2,0,W^2)\quad(4.14)$$

in order to evaluate the static moment.

We may now use Ward's identity[10] to simplify this expression. The product $\Delta_F'(q^2)V_\mu(q,q)\Delta_F'(q^2)$ corresponds to the insertion of a zero-energy photon into the pion propagator and can be simply expressed according to Ward's identity as a derivative of the propagator:

$$\Delta_F'(q^2)V_\mu(q,q)\Delta_F'(q^2)=-i\frac{\partial}{\partial q^\mu}\Delta_F'(q^2).\quad(4.15)$$

Equation (4.14) now reads

$$\Lambda_\mu(p,p)=\frac{-2e_0\tau_3}{(2\pi)^4}\int d^4q\left(-i\frac{\partial}{\partial q^\mu}\Delta_F'(q^2)\right)\times T^-(q^2,q^2,0,W^2).\quad(4.16)$$

We now limit ourselves to the diagrams of type 3(a) as indicated in Fig. 5. $g_0{}^2\Gamma_5(p,p-q)S_F'(p-q)\Gamma_5(p-q,p)$ replaces T^-, with Γ_5 and S_F' the complete nucleon-meson vertex operator and nucleon propagator, respectively.[12] Carrying out the renormalization program we can express the vertex in terms of the renormalized coupling constants and the finite parts of the vertices and propagators. In the approximation of Fig. 5 there results

$$\Lambda_{\mu c}(p,p)=\frac{-2eg^2\tau_3}{(2\pi)^4}\int d^4q\left(-i\frac{\partial}{\partial q^\mu}\Delta_{Fc}'(q^2)\right)\times\Gamma_{5c}(p,p-q)S_{Fc}'(p-q)\Gamma_{5c}(p-q,p).\quad(4.17)$$

Our next step is to introduce a spectral representation for the pion propagator[13]:

$$\Delta_{Fc}'(q^2)=\int_0^\infty\frac{\rho(m^2)dm^2}{q^2-m^2+i\epsilon},\quad(4.18)$$

with a weight function $\rho(m^2)$ that is everywhere nonnegative:

$$\rho(m^2)=\delta(m^2-m_\pi{}^2)+\Theta(m^2-(3m_\pi)^2)\sigma(m^2)\geqslant0.\quad(4.19)$$

The vertex now reads

$$\Lambda_{\mu c}(p,p)=\frac{-2eg^2\tau_3}{(2\pi)^4}\int d^4q\int\rho(m^2)dm^2\frac{2iq_\mu}{(q^2-m^2+i\epsilon)^2}\times\Gamma_{5c}(p,p-q)S_{Fc}'(p-q)\Gamma_{5c}(p-q,p).\quad(4.20)$$

We note in passing that the lowest order perturbation approximation of replacing $\Gamma_{5c}\to\gamma_5$ and

$$S_{Fc}'\to S_F(p-q)=1/(\not p-\not q-M)$$

is simply recognized as an integral over the pion mass spectrum of the lowest order perturbation approximation contribution of the meson current to the nucleon vertex. Projecting out the moment contribution we can write explicitly in this approximation

$$\lambda(m_\pi{}^2)=\int\rho(m^2)\lambda_{\mathrm{B.A.}}(m^2)dm^2=\lambda_{\mathrm{B.A.}}(m_\pi{}^2)+\int_{9m_\pi{}^2}^\infty\sigma(m^2)\lambda_{\mathrm{B.A.}}(m^2)dm^2,\quad(4.21)$$

where $\lambda(m^2)$ means the moment calculated assuming a meson mass m and the subscript B.A. stands for Born approximation in lowest order perturbation theory. Since both σ and $\lambda_{\mathrm{B.A.}}$ are positive functions of m^2 we conclude immediately that

$$\lambda(m_\pi{}^2)>\lambda_{\mathrm{B.A.}}(m_\pi{}^2)=1.6\text{ nm}.\quad(4.22)$$

This gives a pretty fair agreement with the known isotopic vector part of the moment, 1.85 nm, if the higher-mass contributions to the weight function are not too pronounced. The behavior of $\lambda_{\mathrm{B.A.}}(m^2)$ as a function of m^2 helps in this regard since it is a uniformly decreasing function as shown in Fig. 6 and the ratio $\lambda_{\mathrm{B.A.}}(m^2)/\lambda_{\mathrm{B.A.}}(m_\pi{}^2)$ has already fallen to $\sim$0.6 by the threshold of $9m_\pi{}^2$ for additional contributions. However, little more can be said about this added term due to our ignorance of the pion spectral function σ. We mention here that the quenching calculation, since it weights low-

[12] We recall here that we must follow the sequence of operations discussed above Eq. (4.14); that is, we first write

$$g_0{}^2\Gamma_5(p+k,p-q)S_F'(p-q)\Gamma_5(p-q,p)$$

in the form $A^- -\frac{1}{2}(2q+k)B^-$ and then set $k_\mu=0$.

[13] H. Lehmann, Nuovo cimento 11, 342 (1954).

momentum contributions very heavily, will be much less sensitive to σ.

Let us return now to Eq. (4.20) for a more systematic treatment of Γ_{5c} and S_{Fc}'. The renormalized propagator is expressed explicitly in terms of the vacuum expectation value of a time-ordered product of renormalized Heisenberg fields:

$$iS_{Fc}'(t)=\int d^4x\, e^{it\cdot x}(0|T(\psi(x),\bar{\psi}(0))|0). \quad (4.23)$$

Displacing the fields to the origin with

$$O(x)=e^{iP\cdot x}O(0)e^{-iP\cdot x},$$

inserting a complete set of energy momentum eigenstates $P|n)=p_n|n)$, and performing the space-time integral, we obtain

$$S_{Fc}'(t)=(2\pi)^3\sum_n\left[\delta(\mathbf{p}_n-\mathbf{t})\frac{(0|\psi|n)(n|\bar{\psi}|0)}{t_0-E_n+i\epsilon}+\delta(\mathbf{p}_n+\mathbf{t})\frac{(0|\bar{\psi}|n)(n|\psi|0)}{t_0+E_n-i\epsilon}\right]. \quad (4.24)$$

If we limit $|n)$ to one-nucleon states, then, with

$$(n|\bar{\psi}|0)=\frac{1}{(2\pi)^{\frac{3}{2}}}\left(\frac{M}{E_{p_n}}\right)^{\frac{1}{2}}\bar{u}(p_n),$$
$$(0|\psi|n)=\frac{1}{(2\pi)^{\frac{3}{2}}}\left(\frac{M}{E_{p_n}}\right)^{\frac{1}{2}}u(p_n), \quad (4.25)$$

we find

$$S_{Fc}'(t)=\sum_{\text{spins}}\frac{M}{E_t}u_s(t)\frac{1}{t_0-(\mathbf{t}^2+M^2)^{\frac{1}{2}}}\bar{u}_s(t). \quad (4.26)$$

Furthermore, for the quenching calculation we are interested only in the contribution to $S_{Fc}'(t)$ for nuclear states which lie outside of the Fermi sea; i.e., those for which $(\mathbf{t}^2+M^2)^{\frac{1}{2}}>E_F$. Consequently for calculating the difference between the moment of a nucleon which is free and that for one in nuclear matter we are interested in calculating the contribution to (4.20) from

$$\delta S_{Fc}'(t)=-S_{Fc}'(t)\Theta(E_F-(\mathbf{t}^2+M^2)^{\frac{1}{2}}). \quad (4.27)$$

It is here that we make use of the fact that our study is confined to low-energy intermediate nucleons within the Fermi sea. First of all the validity of keeping only the nucleon state in the sum in Eq. (4.24) is strengthened by the fact that very little of the phase space for the higher-mass states corresponds to a nucleon within the Fermi sea. An approximate calculation of the contribution from the one-meson plus nucleon intermediate state, taking into account the resonant 3-3 rescattering, verifies this as will be shown shortly. Secondly, incorporating Eqs. (4.26) and (4.27) into (4.20) and taking its matrix elements between initial and final spinors $\bar{u}(p)\cdots u(p')$, we may evaluate the vertex operator. By invariance arguments it is possible to write

$$g\bar{u}(p)\Gamma_{5c}(p,p')u(p')=g\mathfrak{F}[(p-p')^2]\bar{u}(p)\gamma_5 u(p'). \quad (4.28)$$

g is the renormalized coupling constant,

$$(g^2/4\pi)(m_\pi/2M)^2=0.08,$$

and $\mathfrak{F}$ the form factor for the nucleon-meson vertex. For 4-momentum transfers $(p-p')^2\approx m_\pi^2$ we may approximate $\mathfrak{F}$ to unity since we know from the success of the Chew-Low effective-range theory that $\mathfrak{F}$ has no large variations for momentum transfers of $\approx m_\pi$. In our quenching calculation $(p-p')^2$ must be less than $\approx(4m_\pi)^2$ if the recoil nucleon is to remain within the Fermi sea; most of the contribution will come from $(p-p')^2\approx(2m_\pi)^2$. Therefore we approximate[14] $\mathfrak{F}\approx 1$. If we take Eqs. (4.20), (4.26), (4.27), and (4.28) and anticipate sandwiching the vertex between free-nucleon spinors $\bar{u}(p+k)\cdots u(p)$, we obtain for the modification of the vertex due to quenching

$$\delta\Lambda_\mu(p,p)=\frac{-2eg^2\tau_3}{(2\pi)^4}\int\rho(m^2)dm^2\int\frac{2iq_\mu d^4q}{(q^2-m^2+i\epsilon)^2}\frac{q'\Theta(E_F-[(\mathbf{p}-\mathbf{q})^2+M^2]^{\frac{1}{2}})}{2[(\mathbf{p}-\mathbf{q})^2+M^2]^{\frac{1}{2}}\{(\mathbf{p}^2+M^2)^{\frac{1}{2}}-[(\mathbf{p}-\mathbf{q})^2+M^2]^{\frac{1}{2}}-q_0+i\epsilon\}}, \quad (4.29)$$

with

$$q'=((M^2+\mathbf{p}^2)^{\frac{1}{2}}-[M^2+(\mathbf{p}-\mathbf{q})^2]^{\frac{1}{2}},\mathbf{q}). \quad (4.30)$$

In obtaining this form for the numerator we have used the identity

$$\bar{u}(p+k)[\gamma_5\sum_{\substack{\text{spins}\\E>0}}u(p-q)\bar{u}(p-q)\gamma_5]u(p)=-\frac{1}{2M}\bar{u}(p+k)q'u(p). \quad (4.31)$$

[14] To measure the accuracy of this approximation we note that the difference between the Kroll-Ruderman coupling constant (defined at $p^2=0$) and the Watson-Lepore coupling constant (defined at $p^2=m_\pi^2$) evaluated in second-order perturbation theory is

$$g_{\text{K.R.}}=g_{\text{W.L.}}\left[1-\frac{1}{6\pi}\left(\frac{m_\pi}{2M}\right)^2\frac{g_{\text{W.L.}}^2}{4\pi}\right].$$

We are now ready to project out the magnetic moment term according to Eq. (3.5). This gives

$$\delta F_2(0)=\underset{k\to 0}{L}\frac{ieg^2\tau_3}{(2\pi)^4M^3}\int\rho(m^2)dm^2\int\frac{d^4q}{(q^2-m^2+i\epsilon)^2}\frac{\Theta(E_F-[(\mathbf{p}-\mathbf{q})^2+M^2]^{\frac{1}{2}})}{2[(\mathbf{p}-\mathbf{q})^2+M^2]^{\frac{1}{2}}}$$
$$\times\frac{\left(-3(p\cdot q)(p\cdot q')+M^2q\cdot q'-\dfrac{(k\cdot q)(k\cdot q')}{k^2/M^2}\right)}{\{(\mathbf{p}^2+M^2)^{\frac{1}{2}}-[(\mathbf{p}-\mathbf{q})^2+M^2]^{\frac{1}{2}}-q_0+i\epsilon\}}. \quad (4.32)$$

V. NUMERICAL CALCULATION

Our first step in getting a number is to carry out the q_0 integration in Eq. (4.32). If the contour is closed in the lower half q_0 plane, then one only picks up a double pole coming from the meson propagator. Carrying out the integral gives

$$\delta F_2(0)=\underset{k\to 0}{L}\left(\frac{eg^2\tau_3}{64\pi^3M^3}\right)\int\rho(m^2)dm^2$$
$$\times\int\frac{d^3\mathbf{q}\,\Theta(E_F-[(\mathbf{p}-\mathbf{q})^2+M^2]^{\frac{1}{2}}}{[(\mathbf{p}-\mathbf{q})^2+M^2]^{\frac{1}{2}}}\frac{1}{\omega^2(\omega-\Delta)^2}$$
$$\times[A(2\omega-\Delta)+B\omega^2], \quad (5.1)$$

with

$$\omega=(\mathbf{q}^2+m^2)^{\frac{1}{2}},$$
$$\Delta=(\mathbf{p}^2+M^2)^{\frac{1}{2}}-[(\mathbf{p}-\mathbf{q})^2+M^2]^{\frac{1}{2}}, \quad (5.2)$$
$$A=-M^2\left(\mathbf{q}^2-\frac{(\mathbf{k}\cdot\mathbf{q})^2}{\mathbf{k}^2}\right)+O(1),$$

and

$$B=M\left(\mathbf{q}^2+\mathbf{p}\cdot\mathbf{q}-\frac{(\mathbf{k}\cdot\mathbf{q})(\mathbf{p}\cdot\mathbf{k})}{\mathbf{k}^2}\right)+O\left(\frac{1}{M}\right).$$

A and B have been expanded in terms of momenta-squared over M^2. This expansion is justified since both $\mathbf{p}$ and $\mathbf{q}$ are limited by the Fermi momentum and $\mathbf{k}$ is to go to zero. Nowhere, however, do we expand in powers of ω/M since we must still integrate over the meson mass spectrum. In evaluating (5.1) we set $|\mathbf{p}|=p_F$ corresponding to a nucleon at the surface of the Fermi sea and average over orientations of $\mathbf{k}$. $\mathbf{q}$ is then limited to the region

$$|\mathbf{p}-\mathbf{q}|^2=p_F^2+\mathbf{q}^2-2p_Fq\cos\theta\leqslant p_F^2.$$

The limitation on θ is that

$$O\leqslant\theta\leqslant\cos^{-1}(q/2p_F). \quad (5.3)$$

Using $F_2(0)=+\lambda e/2M$ we find

$$\delta\lambda=\delta\lambda_{\mathrm{B.A.}}(m_\pi^2)+\int_{9m_\pi^2}^{\infty}\sigma(m^2)\delta\lambda_{\mathrm{B.A.}}(m^2)dm^2, \quad (5.4)$$

with

$$\delta\lambda_{\mathrm{B.A.}}(m^2)=\frac{-g^2\tau_3}{32\pi^3M}\int_{|\mathbf{p}_F-\mathbf{q}|^2\leq p_F^2}\frac{q^2dqd\Omega_{\mathbf{q}}}{\omega^4}$$
$$\times\left[\frac{4}{3}q^2-\frac{\omega}{M}\left(q^2+\frac{2}{3}\mathbf{p}\cdot\mathbf{q}\right)\right]. \quad (5.5)$$

Introducing

$$x=q/2p_F,\quad r=m/2p_F, \quad (5.6)$$

and carrying out the angular integrations, we get

$$\delta\lambda_{\mathrm{B.A.}}(m^2)=\frac{-\tau_3}{3\pi}\left(\frac{g^2}{4\pi}\right)\left(\frac{2p_F}{M}\right)\int_0^1\frac{x^4(1-x)dx}{(x^2+r^2)^2}$$
$$\times\left[1-\frac{3p_F}{2M}(x^2+r^2)^{\frac{1}{2}}\left(\frac{1+7x}{6x}\right)\right]. \quad (5.7)$$

For

$$p_F=2m_\pi\quad\text{and}\quad\left(\frac{m_\pi}{2M}\right)^2\frac{g^2}{4\pi}=0.08, \quad (5.8)$$

we find

$$\delta\lambda_{\mathrm{B.A.}}(m\pi^2)=-0.11\tau_3\,\mathrm{nm}. \quad (5.9)$$

In order to evaluate the higher-mass contributions to $\delta\lambda$ in (5.4) we would first have to know something about the presently unknown weighting function $\sigma(m^2)$. However as we see from Fig. 7, $\delta\lambda_{\mathrm{B.A.}}(m^2)$ has fallen to $\approx 1/10$ of its value by the time m^2 is increased from m_π^2 to the threshold $(3m_\pi)^2$ for contributions from the next lightest or three-pion state. We may therefore hope that due to the rapid fall-off of $\delta\lambda_{\mathrm{B.A.}}(m^2)$ with

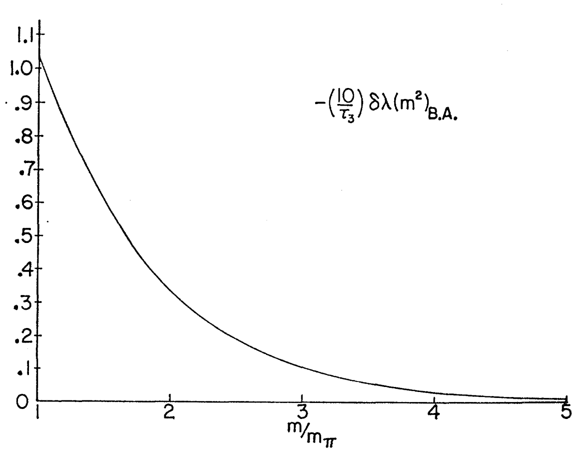

FIG. 7. Born approximation calculation of the quenching of the anomalous magnetic moment as a function of the intermediate meson mass.

increasing mass these additional contributions are small.[15]

So far our result is based on the approximation in (4.26) of neglecting all but the one-nucleon intermediate state in evaluating the nucleon propagator. We expect the major corrections to this to come from one nucleon plus one pion in the resonant 3-3 state. We do not evaluate the one-nucleon-plus-one-pion contribution in (4.24) because this corresponds to Fig. 8 and allows the nucleon and pion to be only in the nonresonant $(\frac{1}{2},\frac{1}{2})$ state. The resonant amplitude arises from crossed graphs of the type in Fig. 4. We evaluate this contribution in the static approximation. From Walecka,[16] the magnetic moment in the static theory can be written

$$\lambda(m_\pi^2)=\frac{g^2\tau_3}{24\pi^3 M}\int\frac{q^2d^3\mathbf{q}}{\omega^4}\left[1+\frac{2}{9}\frac{\omega(2\omega+\omega_R)}{(\omega+\omega_R)^2}\right], \quad (5.10)$$

with $\omega_R\approx 2m_\pi$. The first term is the Born approximation and the second term is due to the resonance. We can obtain an estimate of the ratio of the contribution to the quenching of the resonance term to the Born term by equating the momentum of the intermediate "isobar" to that of the nucleon. This is approximately valid due to the large mass ratio of the nucleon to the pion. We therefore write

$$\delta\lambda(m_\pi^2)=\frac{-g^2\tau_3}{24\pi^3 M}\int\frac{q^2d^3\mathbf{q}}{\omega^4}\Theta[p_F^2-(\mathbf{p}_F-\mathbf{q})^2] \times\left[1+\frac{2}{9}\frac{\omega(2\omega+\omega_R)}{(\omega+\omega_R)^2}\right]. \quad (5.11)$$

The second term increases the integral by less than 20%. Our final value for the quenching is then

$$\delta\lambda(m_\pi^2)\cong -0.13\tau_3\,\mathrm{nm}. \quad (5.12)$$

This means a 7% reduction of the isotopic vector part of the anomalous nucleon moment, 1.85 nm.

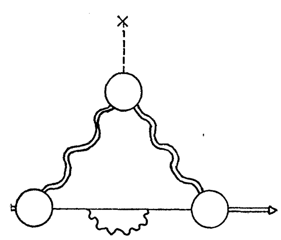

FIG. 8. Graph contributing to the one-meson, one-nucleon term in the nucleon propagator.

[15] Even a Born approximation calculation of the $\pi\rightarrow 3\pi$ amplitude with a ϕ^4 coupling which violates unitarity in $\pi-\pi$ scattering by more than a factor of 10 contributes only an additional 20% to the quenching.

[16] J. D. Walecka, Nuovo cimento 11, 821 (1959).

DISCUSSION OF RESULTS

To conclude we wish to comment on the accuracy and on the experimental significance of our result, (5.12), that the magnetic moment is quenched by 7% in nuclear matter.

The main contributions to $\delta\lambda(m_\pi^2)$ comes from the Born approximation calculation of the contribution of the meson current to the magnetic moment, (5.9). There is, of course, no reason to have confidence in perturbation calculations in pion physics. In this paper, however, we have derived (5.8) and (5.9) by a more general approach which has specific advantages:

(*a*) Our answer emerges in terms of the physical renormalized pion-nucleon coupling constant g which is introduced in (4.28). The form factor there was approximated to unity on the basis of low-energy arguments as used successfully in the effective-range plots of Chew and Low.

(*b*) The accuracy of our result can be defended on the same grounds, as was shown in Sec. III, as the dispersion; theoretic approach to the free-nucleon magnetic moment calculation. This certainly does not put the accuracy beyond question. However, relative to a perturbation approximation, the dispersion-theory approach has the great advantage of giving approximately correct results for the free-nucleon magnetic moment.

We estimated in (5.11) the corrections due to resonance (33) rescattering of the intermediate pion and found it to add an additional quenching contribution of $<20\%$. To calculate the correction from higher masses in (5.4) one needs information on the amplitude for the process[15] $\pi\rightarrow 3\pi$. This amplitude is of great interest in other connections also.

As to the experimental significance of a 7% moment quenching we note that there are many other factors which may alter the magnetic moment of a nucleon in nuclear matter. Among these are the contributions from exchange currents arising in conjunction with charge exchange and velocity-dependent forces. Little can be said about their effect on the magnetic moments in the absence of a detailed nuclear force theory. In the extreme shell model it is possible to estimate the exchange moment arising from the spin-orbit forces phenomenologically introduced to explain the observed level splitting. This effect is shown in Table I and evidently leads to large uncertainties. Another effect is the relativistic correction arising from the Fermi energy of a nucleon in the nuclear matter. This multiplies the Dirac part of the total nucleon moment by a factor M/E. We have included this effect in Table I; it is comparable with the quenching effect for protons. There is also a correction to the single-particle moment when the motion of the center of mass of the system is

TABLE I. Listed are selected odd-proton and odd-neutron nuclei together with indicated moments and corrections. The Schmidt values are given in column I. Modifications of the moment due to a one-body shell model spin orbit coupling for the extra-core nucleon (hole) are given in column II according to calculations of Jensen and Mayer[a] and of Marty[b]; see also Blin-Stoyle.[c] Modifications of the moment due to a relativistic factor M/E, which reduces the Dirac part of the moment only, are given in Column III; for a nucleon at the Fermi surface the correction here is proportional to $[(M/E_F)-1]=-0.04$. The modifications due to quenching as calculated in this paper are given in column IV. The experimental moments are taken from Klinkenberg.[d]

Nucleus	Shell model state	I Schmidt value for moment in nuclear magnetons	II Modifications moment due to spin orbit coupling in shell model a	b	III Relativistic correction to moment	IV Quenching effect	Total moment (I+II+III+IV) a	b	Experimental moment
N^{15}	$p_{1/2}$	-0.26	$+0.17$	$+0.14$	-0.01	$+0.04$	-0.06	-0.09	-0.28
F^{19}	$s_{1/2}$	2.79	-0.17		-0.04	-0.13	2.45		$+2.63$
K^{39}	$d_{3/2}$	$+0.12$	$+0.21$	$+0.46$	-0.05	$+0.08$	$+0.36$	$+0.61$	$+0.39$
Bi^{209}	$h_{9/2}$	$+2.62$	$+0.24$	$+0.30$	-0.16	$+0.10$	$+2.80$	$+2.86$	$+1.08$
O^{17}	$d_{5/2}$	-1.91	0	0	0	$+0.13$	-1.78	-1.78	-1.89
P^{207}	$p_{1/2}$	$+0.64$	0	0	0	-0.04	$+0.60$	$+0.60$	$+0.59$

[a] J. H. D. Jensen and M. G. Mayer, Phys. Rev. **85**, 1040 (1952).
[b] C. Marty, J. phys. radium **15**, 783 (1954).
[c] See reference 6, p. 93.
[d] Z. A. A. Klinkenberg, Revs. Modern Phys. **24**, 63 (1952).

treated correctly. This was studied by Gartenhaus and Schwartz[10] who found the effect to be negligible.

Our conclusion from Table I then is that the quenching effect is neither negligible nor so prominent that it can be separated from other nuclear corrections to the Schmidt lines. In particular it now appears that when quenching is included, the magnetic moment of O^{17} as deduced from the extreme single-particle model no longer agrees so spectacularly with experiment.

[17] S. Gartenhaus and C. Schwartz, Phys. Rev. **108**, 482 (1957).

ACKNOWLEDGMENTS

We wish to thank Professor Amos de-Shalit for a very stimulating letter about the magnetic moments of the potassium isotopes. His analysis, in collaboration with Professor I. Talmi of the Weizmann Institute, indicated that a 15%–20% quenching of the anomalous nucleon moments would lead to good agreement with observed moments and transition rates in K^{39}, K^{40}, K^{41}, and K^{43}. It was this letter that led to the present work.