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Anisotropic Fermion Gas

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An energy term discovered by Kohn and Luttinger is shown to have a simple geometrical meaning.

KOHN and Luttinger[1] have pointed out that the usual linked-cluster expansion for the ground-state energy of a many-fermion system is incomplete if there are deviations from spherical symmetry, e.g., due to lattice forces. By studying a grand canonical ensemble in the limit of zero temperature, they have derived a new energy term of the second order in the interaction. At first sight it seems puzzling that this term involves only that part of the interaction which is *diagonal* in the original occupation number representation: The result should then be derivable even if all off-diagonal parts of the Hamiltonian are omitted. Such an elementary derivation will then reveal the simple meaning of the Kohn-Luttinger term.

To sketch such a derivation, we formally introduce a parameter λ as a factor in the interaction, viz., a "coupling strength" which may be "switched on" adiabatically. If the Fermi surface is nonspherical it will depend on λ. Calling its interior $G(\lambda)$, the ground-state energy (diagonal part) may be written

$$E(\lambda)=\int_{G(\lambda)} d^3k\ \epsilon_{\mathbf{k}}+\tfrac{1}{2}\lambda\int_{G(\lambda)} d^3k\int_{G(\lambda)} d^3k'\ v_{\mathbf{kk}'},$$

where $v_{\mathbf{kk}'}=v_{\mathbf{k}'\mathbf{k}}$. For each value of λ, the Fermi surface is defined as that surface for which $E(\lambda)$ is a minimum:

$$\epsilon_{\mathbf{k}}+\lambda\int_{G(\lambda)} d^3k'\ v_{\mathbf{kk}'}=\mu(\lambda),$$

in conjunction with

$$\int_{G(\lambda)} d^3k=\text{const.}$$

Then, if we vary λ,

$$\frac{\partial E(\lambda)}{\partial\lambda}=\frac{1}{2}\int_{G(\lambda)} d^3k\int_{G(\lambda)} d^3k'\ v_{\mathbf{kk}'},$$

because the other terms cancel, and expressions for $\partial^n E(\lambda)/\partial\lambda^n$ $(n=2, 3, \cdots)$ are easily written down. They are, of course, all zero if G is a sphere and therefore independent of λ. $(\partial^2 E/\partial\lambda^2)_{\lambda=0}$ corresponds precisely to the Kohn-Luttinger term [Eq. (33) in reference 1].

When off-diagonal elements of the interaction are added as perturbations, it is presumably advantageous to start from a ground-state Fermi surface which is correct to the desired order of approximation.[2] The possibility that the surface may not remain "sharp" (superconductor) should not be overlooked.

[1] W. Kohn and J. M. Luttinger, Phys. Rev. **118**, 41 (1960).

[2] A. Klein, Phys. Rev. Letters 4, 601 (1960).