is determined from the experimental spin-orbit coupling constant. Since the shape of the wave function changes very little from element to element, we have determined κ by comparison with SCF wave functions and a from the spin-orbit coupling constant. In the latter case we have used the Thomas-Fermi potential, which is accurate enough for this purpose. This potential is particularly close to SCF potentials near the nucleus, where the main contribution to the spin-orbit coupling originates (see Fig. 6).

No SCF calculations are available for any rare-earth atoms but some have recently been carried out for the Pr^{3+} and Tm^{3+} ions.[9] The difference in shape between the $4f$ wave functions for these ions is very small, and both correspond to a κ value slightly greater than 0.4. Since one would not expect the shape to differ much between the ions and the atoms, this should be a reasonable value also for the atoms. This is in agreement with the value obtained by extrapolation from heavier atoms like W and Hg.

For the wave function (A-1) the following formulas are easily verified (subscript hy indicates hydrogenic value).

$$N^2=\frac{(2a)^{2n+1}}{(2n)!}\frac{1}{C_{2n+1}}=\frac{N_{hy}^2}{C_{2n+1}},$$

$$\langle r^{-m}\rangle=\frac{(2a)^m(2n-m)!}{(2n)!}\frac{C_{2n+1-m}}{C_{2n+1}}=\langle r^{-m}\rangle_{hy}\frac{C_{2n+1-m}}{C_{2n+1}},$$

$$\langle T\rangle=\frac{1}{2}\left[n(n-1)\langle r^{-2}\rangle-\left\langle\frac{1}{R}\frac{d^2R}{dr^2}\right\rangle\right]$$

$$=\frac{1}{2}a^2\left[1-\kappa^2-2\kappa\frac{D_{2n}}{C_{2n+1}}\right],$$

where

$$C_s=\tfrac{1}{4}[e^{-2n\kappa}(1-\kappa)^{-s}+2+e^{2n\kappa}(1+\kappa)^{-s}],$$

and

$$D_s=\tfrac{1}{4}[e^{-2n\kappa}(1-\kappa)^{-s}-e^{2n\kappa}(1+\kappa)^{-s}].$$

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Preformation Factor in Emission of Complex Particles from Nuclear Reactions*

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The reason why complex particles (alphas, tritons, Li^6 nuclei, etc.) are emitted in nuclear reactions as frequently as nucleons (after corrections for Coulomb barrier penetration and energetics), whereas ice crystals are never emitted from evaporating water droplets is investigated. It is shown that the difference is entirely explained by the fact that a nucleus is a highly degenerate system subject to Fermi-Dirac statistics, whereas an evaporating water droplet is a nondegenerate statistical system.

IF one considers an excited compound nucleus to be a conglomeration of neutrons and protons similar to a liquid drop, it is intuitively appealing to assume that neutrons and protons should be emitted most readily in nuclear reactions, and tritons, alpha particles, Li^6 nuclei, etc., should be impeded by a preformation factor, f, relative to the emission of nucleons. (We ignore here other factors affecting emission such as Coulomb barrier penetration factors, energetics, etc.) The problem seems to be analogous to that of an evaporating droplet of water, where the evaporation of a sizeable crystal of ice is certainly very much less probable than the evaporation of water molecules one at a time.

This view indeed prevailed in early treatments of the subject, the best known of which is Bethe's many body theory of alpha decay.[1] However, when it was found experimentally that alpha particles are frequently emitted from nuclear reactions so that f is close to unity, the use of a preformation factor became unfashionable, although it is still occasionally discussed or referred to.[2] Proofs have been offered to show that if complex nuclei are captured with geometric cross sections in experiments where they bombarded nuclei, application of the principle of detailed balance indicates that f must be unity. However, this cannot explain the difference between a decaying compound nucleus and an evaporating water droplet. In the latter case, an ice crystal striking the droplet would certainly be absorbed with the geometric cross section, and detailed balance is essentially an expression of invariance under time reversal which is a classical as well as a quantum mechanical principle.

It is the purpose of this paper to clearly elucidate the difference between the two cases. It will be shown that it is due solely to the fact that a nucleus is a highly

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[1] H. A. Bethe, Revs. Modern Phys. **9**, 69 (1937).

[2] See, for example: J. J. Devaney, Phys. Rev. **91**, 587 (1953); H. A. Toelhoek and P. J. Brussaard, Physics **21**, 449 (1955); G. H. McCormick, H. G. Blosser, B. L. Cohen, and T. H. Handley, J. Inorg. & Nuclear Chem. **2**, 269 (1956).

degenerate system subject to Fermi-Dirac statistics, whereas an evaporating water droplet is a nondegenerate statistical system.

It must be clear from the outset that in either the nuclear or water droplet case, if we consider the decay of a system C into two possible modes, $A+B$ and $A'+B'$, the preformation factor for A, B, A', and B' is just proportional to the number of ways each can be formed; that is, to the density of available states. In the quantum mechanical case, this is a consequence of the well-known theorem that all states have equal *a priori* probability. In the classical case, a state may be defined as a given position and velocity vector attached to each individual particle; any single choice of these, including those choices which correspond to a group of particles with the correct spacings to form an ice crystal and equal velocity vectors directed out of the surface, must have equal *a priori* probability, so that here also the probability is proportional to the density of states.

First, we consider a water droplet with N_0 molecules and we calculate the relative probability for the emission of a single water molecule, p_1, and a crystal of ice containing n molecules, p_n. For simplicity (and also to improve the analogy with the nuclear case) we consider the ice crystal to be at absolute zero temperature, and assume that the kinetic energy, E, of the molecule and the ice crystal are the same. To make the problem solvable, we neglect interactions between molecules in the droplet (effectively treating it as a gas enclosed in the volume of the droplet). It will be clear that none of these simplifying assumptions has an important effect on the final conclusions.

The ratio p_1/p_n is just the ratio of the density of states available to the system after the evaporation has occurred. The density of states of the final system, ρ_f, is the product of the momentum space degeneracy of the outgoing "particle," ρ_p, and the level density of the residual droplet, ρ_R. The former is proportional to $(nmE)^{\frac{3}{2}}$, where m is the mass of each molecule, whence

$$\rho_p(1)/\rho_p(n)=n^{-\frac{3}{2}}. \tag{1}$$

In Maxwell-Boltzmann statistics, the density of states for a gas of N particles of mass m having a total energy U and enclosed in a volume V may readily be shown to be

$$\rho_R(U,N)=\left[\frac{\pi^{\frac{1}{2}}V^{\frac{1}{3}}(2mU)^{\frac{1}{2}}}{h}\right]^{3N}\frac{1}{(\frac{3}{2}N-1)!}\frac{1}{U}. \tag{2}$$

This expression is derived on the assumption that all particles are distinguishable; if they are identical, as in the water droplet case, (2) must be divided by $N!$ In addition, we take the volume per particle, V_0, to be constant, or

$$V=V_0N,$$

whence (2) becomes

$$\rho_R(U,N)=\left[\frac{\pi^{\frac{1}{2}}V_0^{\frac{1}{3}}(2mU)^{\frac{1}{2}}}{h}\right]^{3N}\frac{N^N}{(\frac{3}{2}N-1)!}\frac{1}{N!}\frac{1}{U}, \tag{3}$$

or, in the approximation $N!=N^N$,

$$\rho_R(U,N)=\left[\frac{\pi^{\frac{1}{2}}V_0^{\frac{1}{3}}(\frac{4}{3}mU/N)^{\frac{1}{2}}}{h}\right]^{3N}\frac{3N}{2U}. \tag{4}$$

For a water droplet at 300°K, the quantity in brackets is 10.4; thus, for $n\ll N$, we have

$$\rho_R(U,N)/\rho_R(U,N-n)=(10.4)^{3n}. \tag{5}$$

Combining (1) and (5), the ratio of probabilities for emission of a single molecule and an ice crystal containing n molecules is

$$\frac{p_1}{p_n}=\frac{1}{n^{\frac{3}{2}}}[10.4]^{3(n-1)}, \tag{6}$$

which is very large even for relatively small values of n; for $n=4$, $p_1/p_4=1.8\times10^8$. Thus, an ice crystal is essentially never emitted from a water droplet; the reason is that the density of states of the residual droplet increases very rapidly with the number of molecules even under the constraints that the total energy and the volume per molecule remain constant.

The difference between this classical case and the nuclear problem is apparent from (4) and (6); the bracketed term in (4) for the nuclear case is near unity, indicating the degenerate state of the system;[2a] hence, alpha emission is not much reduced compared to nucleon emission. A more correct estimate for the nuclear case may readily be obtained by using Fermi statistics in calculating the density of states of the residual nucleus. Expressions for level densities of nuclei have been given by Bethe,[1] Blatt and Weisskopf,[3] Lang and LeCouteur,[4] and others. The Lang-LeCouteur formula is

$$\rho_R(U,N)\propto\frac{1}{N^2U^2}\exp\left[2\left(\frac{NU}{11\text{ Mev}}\right)^{\frac{1}{2}}\right], \tag{7}$$

whence, neglecting the variation in the coefficient and differentiating,

$$\frac{\rho_R(U,N)}{P_R(U,N-n)}\simeq\exp\left[n\left(\frac{U}{N\times11\text{ Mev}}\right)^{\frac{1}{2}}\right], \tag{8}$$

where we have assumed $n\ll N$.

The ratio p_1/p_4 may be calculated following the procedure used above for the classical case except that (8) is used in place of (5). For $N=100$ and $U=10$ Mev

$$\frac{p_1}{P_4}=2\times4^{-\frac{3}{2}}\times e^{3/10.5}=0.33,$$

where the first factor of 2 was inserted to take account of the spin degeneracy of nucleons. The Bethe formula[1]

[2a] *Note added in proof.* It is interesting to point out that the bracketed term is essentially d/λ where d is the average distance between particles, and λ is their wavelength. The well-known condition for a system to be degenerate is $d/\lambda\approx1$.

[3] J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley & Sons, New York, 1952).

[4] J. M. B. Lang and K. J. LeCouteur, Proc. Phys. Soc. (London) **A67**, 586 (1954).

yields $p_1/p_4=0.45$, and the Blatt-Weisskopf level densities[3] give $p_1/p_4=0.40$. Thus, the emission of alpha particles is not impeded, so that the preformation factor, f, is near unity (note that $f=p_4/p_1$). At higher excitation energies, f decreases; for example, with $U=100$ Mev and $N=100$, $p_1/p_4=0.63$. It is thus apparent that the reason why alphas are frequently emitted in nuclear reactions although ice crystals never evaporate from water droplets is that nuclei are highly degenerate Fermi systems, whereas a water droplet is a nondegenerate system. The preformation factor is strictly an intuitive concept based on our experience with classical systems, and has no meaning in the nuclear case. The confusion may well serve as a warning against the time honored custom of visualizing a compound nucleus as a classical evaporating liquid drop.

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jj Coupling Model in Odd-Odd Nuclei*

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An analysis has been made, using the *jj* coupling model, of the spins of 76 low-lying levels of odd-odd nuclei with $20<A<120$. Levels have been excluded from the analysis if any ambiguity exists in the assignment of a configuration. Excluding particle-hole configurations we find Nordheim's "strong" rule obeyed in the 22 cases in which it is applicable. Nordheim's "weak" rule is replaced by a rule predicting a ground state of the highest or lowest allowable spin. This revised rule is obeyed in 38 out of 41 cases. This competition between two levels of greatly differing spin results in the frequent occurrence of isomerism. For particle-hole configurations the state of the highest spin minus one is the ground state in 6 out of 13 cases. This agreement with experiment is obtained using proton and neutron configurations which, in 66 of the 76 cases, are found as ground states in the neighboring odd-even nuclei. The three revised coupling rules are predicted by the calculations of Schwartz, in which the residual proton-neutron interaction has a delta-function radial dependence, if the singlet-to-triplet strength is taken as 0.6, independent of mass number. This value is the same as that required to fit the free two-nucleon data. Exceptions to the empirical coupling rules and this theory will be discussed.

I. INTRODUCTION

A STUDY of the low-lying levels in odd-odd nuclei can provide useful information on the nature of the effective interaction between protons and neutrons in nuclear matter. This problem can be conveniently discussed by extending the odd-group model, as normally applied to odd-even nuclei. In the odd-group model the spin and magnetic properties of the nucleus are assumed to be determined by the properties of the odd group of particles. In the extension of this representation to odd-odd nuclei it is assumed that the wave function is a simple vector-coupled product of the wave functions of the two odd groups. A further simplification which permits the use of *jj* coupling, is obtained if it is assumed that the residual interactions are weak compared to the spin-orbit force.[1]

Under these assumptions the levels arising from a given proton and neutron configuration can take on all integral spins between the sum and the difference of the spins of the two odd groups. The degeneracy of these levels is removed by the residual proton-neutron interaction. Furthermore, the low-lying levels in an odd-odd nucleus should result from combinations of the lowest configurations in the adjacent odd-proton and odd-neutron nuclei. If it is then possible to consider only those odd-odd nuclei where the low-lying levels result from a single proton-neutron configuration, a study of the level ordering should provide information about the residual proton-neutron interaction.

The rules governing the coupling of the proton and neutron angular momenta have been studied both empirically and theoretically. In 1950, Nordheim[2] proposed two coupling rules which, with the data available at the time, provided a satisfactory description of the spins of the majority of odd-odd nuclei. However, later empirical studies[3] showed that there were frequent violations of the so-called "weak" rule. These studies,

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[1] For a general discussion of the *jj* coupling model and its application to odd-odd nuclei, see the review article of J. P. Elliott and A. M. Lane, *Handbuch der Physik*, edited by S. Flügge (Springer-Verlag, Berlin, 1957) Vol. 39.

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[3] K. Way, D. N. Kundu, C. L. McGinnis, and R. van Lieshout, *Annual Review of Nuclear Science* (Annual Reviews, Inc., Palo Alto, California, 1956) Vol. 6, p. 129; C. A. Mallman, *Proceedings of the Second United Nations International Conference on the Peaceful Uses of Atomic Energy, Geneva, 1958* (United Nations, Geneva, 1958) Vol. 14, p. 68; C. J. Gallagher, Jr and S. A. Moszkowski, Phys. Rev. **111**, 1282 (1958).