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Single and Multiple Stripping of α Particles*

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Assuming the stripping process to take place mainly at the nuclear surface, expressions are derived for the differential cross sections for the processes in which one, two, or three nucleons are stripped from an incident α particle by the target nucleus. Comparison between the derived formulas and experimental data was carried out for the (He^3,p) and (α,p) reactions and shows fairly good agreement.

INTRODUCTION

RECENT experimental work with projectiles heavier than deuterons seems to indicate that stripping mechanism plays an important role in the rearrangement reactions caused by these projectiles. The recent experimental work on (α,t),[1] (He^3,p),[2] (α,d),[3] (α,p),[4] and (α,n)[5] are examples of heavy projectile reactions in which the angular distribution of the outgoing particles in each case shows strong similarity to the patterns produced by the usual theories of deuteron stripping.[6]

The theory of the two-nucleon stripping process[7,8] was discussed previously and fairly good agreement with experiment was obtained, which may suggest that reactions such as (He^3,p) and (α,d) could contain large contributions from the stripping mechanism.

In the present work the stripping of one, two, or three nucleons from an α particle is considered. The first Born approximation method is used for the calculation of the stripping differential cross section, together with the surface stripping assumption introduced first by Bhatia *et al.*[6] Assuming the stripping process to take place at the nuclear surface, the radial coordinates of the captured particles may be put equal to the nuclear radius in the integrand of the stripping matrix element.[9] This assumption is justified by the agreement between experimental data and the theoretical formulas for the deuteron stripping,[6] and also for the stripping of He^3 and α particles as will be shown later, in the present work.

For the purposes of simplification, the Coulomb effects of the interacting particles will be neglected, and also the nuclear interaction between the outgoing particles and the residual nucleus will not be taken into account. The derived formulas will hence be considered applicable when the energies of the incident particles are well above the Coulomb barrier.

The (α,t) reaction is considered in Sec. I and an expression is derived for the differential cross section. In Sec. II, the two-nucleon stripping from an α particle is derived by making the surface stripping assumption and the derived expression is compared with the corresponding formula derived before.[8] This may show the actual effect of the interaction potential on the form of the differential cross section. In Sec. III, an expression is derived for the differential cross section for the process (α,p) in which three nucleons are stripped from the incident α particle, using the surface stripping assumption. Section IV contains a discussion of the derived formulas and comparison with experimental data.

In Appendix I, a general expression is given for the differential cross section of the stripping process of an incident α particle, in terms of the spatial scattering amplitude, taking into account the spins of the interacting particles.

In Appendix II, a discussion is included concerning the previously derived expression for the two-nucleon stripping process.[8]

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9 In the work of Bhatia *et al.* on the stripping of deuterons, this replacement was done only in the factors of the integrand which represent the incident wave. This partial replacement is also used in Sec. I of the present work, while complete replacement is adopted in Secs. II and III.

II. (α,t) REACTION

The differential cross section for the $i(\alpha,t)f$ stripping reaction, according to the first Born approximation, may be written as

$$d\sigma=\frac{M_t^*M_\alpha^*}{4\pi^2\hbar^4}\frac{k_t}{k_\alpha}\langle|I|^2\rangle, \quad (1)$$

where

$$M_t^*=[M_tM_f/(M_t+M_f)],$$
$$M_\alpha^*=[M_\alpha M_i/(M_\alpha+M_i)],$$
$$k_\alpha=(2M_\alpha^*E_\alpha)^{\frac{1}{2}}/\hbar,$$

and

$$k_t=(2M_t^*E_t)^{\frac{1}{2}}/\hbar;$$

E_α and E_t are the energies of the incident and outgoing particles. The quantity $\langle|I|^2\rangle$ is formed from the matrix element

$$I=\int\psi_f^*(\mathbf{r}_p,\xi)\exp[-i\mathbf{k}_t\cdot\mathbf{r}_{t'}]\varphi^*(r_t)V(\mathbf{r}_p,\xi)\psi_i(\xi)\times\exp[i\mathbf{k}_\alpha\cdot\mathbf{r}_\alpha]\varphi(r_\alpha)d\xi d\tau, \quad (2)$$

by taking the square of its amplitude, then averaging over spin projections in the initial state and summing over spin projections in the final state, i.e.,

$$\langle|I|^2\rangle=\frac{1}{[2J(i)+1]}\sum_{\mu(i)\mu(f)\mu(t)}|I|^2.$$

In Eq. (2) $\mathbf{r}_{t'}=\mathbf{r}_t-(M_p/M_f)\mathbf{r}_p$; $\varphi(r_t)$ and $\varphi(r_\alpha)$ represent the internal wave functions of the triton and α particle, respectively; ξ represents the collective coordinates of the target nucleus i; and

$$d\tau=d\mathbf{r}_{n(1)}d\mathbf{r}_{p(1)}d\mathbf{r}_{n(2)}d\mathbf{r}_p$$

[See Fig. 1(a).]

The process of capture of the proton p from the incident α particle will be assumed to take place close to the surface of the target nucleus, whose radius is denoted by R. This assumption implies that the major contribution to the integral in Eq. (2) comes from a small range of r_p close to R, and so in the factors of the integrand which represent the incident wave, one may replace $\mathbf{r}_p=(r_p,\theta_{p'},\varphi_p)$ by $\mathbf{R}=(R,\theta_p,\varphi_p)$ and obtain

$$I=\int\exp[-i\mathbf{k}_t\cdot\mathbf{r}_{t'}]\varphi^*(r_t)\varphi(r_tR)\times\exp[i\mathbf{k}_\alpha\cdot(\mathbf{R}+3\mathbf{r}_t)/4]\times\left\{\int\psi_f^*(\mathbf{r}_p)V(\mathbf{r}_p,\xi)\psi_i(\xi)r_p^2dr_pd\xi\right\}\times d\mathbf{r}_{n(1)}d\mathbf{r}_{n(2)}d\mathbf{r}_{p(1)}. \quad (3)$$

The expression in brackets in (3) is a function of the angular coordinates of p and may be expanded as follows:

$$\sum_{l(p)m(p)}A_{l(p)m(p)}Y_{l(p)}{}^{m(p)}(\Omega_p)^*,$$

where

$$A_{l(p)m(p)}=\int\psi_f^*(\mathbf{r}_p,\xi)V(\mathbf{r}_p,\xi)\psi_i(\xi)Y_{l(p)}{}^{m(p)}(\Omega_p)d\mathbf{r}_pd\xi. \quad (4)$$

Using this expansion, Eq. (3) becomes

$$I=\sum_{l(p)m(p)}A_{l(p)m(p)}\int\exp[-i\mathbf{k}_t\cdot\mathbf{r}_{t'}]\varphi(t)^*\varphi(r_tR)\times\exp[i\mathbf{k}_\alpha\cdot(3\mathbf{r}_t+\mathbf{R})/4]Y_{l(p)}{}^m(\Omega_p)^*d\Omega_p\times d\mathbf{r}_{p(1)}d\mathbf{r}_{n(1)}d\mathbf{r}_{n(2)}. \quad (5)$$

Introducing the coordinate $\boldsymbol{\varrho}$ defined by $\mathbf{r}_t=\mathbf{R}+\boldsymbol{\varrho}$, the exponent of the integrand of (5) becomes

$$X=i\mathbf{k}_\alpha\cdot(3\mathbf{r}_t+\mathbf{R})/4-i\mathbf{k}_t\cdot[\mathbf{r}_t-(M_p/M_f)\mathbf{R}]=i\mathbf{Q}\cdot\mathbf{R}-i\mathbf{K}\cdot\boldsymbol{\varrho}, \quad (6)$$

where $\mathbf{Q}=\mathbf{k}_\alpha-(M_i/M_f)\mathbf{k}_t$, $\mathbf{K}=\mathbf{k}_t-\frac{3}{4}\mathbf{k}_\alpha$.

The Gaussian form will now be assumed for the

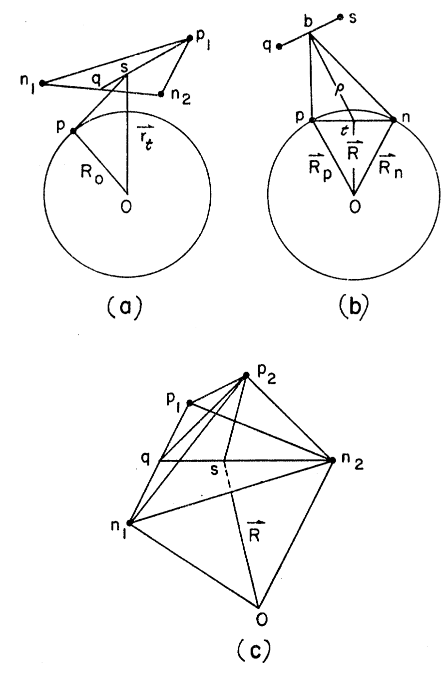

FIG. 1. The stripping of α particles by target nuclei: Figure 1(a) shows the single-particle stripping, where the particle p is captured, and the H^3 particle is going out. In 1(b) the deuteron qs is going out while the nucleons p and n are captured. Similarly 1(c) shows the three-particle stripping, where $p(2)$ is the only outgoing particle. The symbols n_1, n_2, p_1, p_2 in the figure appear in the text as $n(1)$, $n(2)$, $p(1)$, $p(2)$, respectively.

internal wave functions of the α and t particles, i.e.,

$$\varphi(r_t)=N_t^{\frac{1}{2}}\exp[-\gamma_t^2\sum_{i\neq j}r_{ij}^2]$$
$$=N_t^{\frac{1}{2}}\exp[-\gamma_t^2(2u^2+\tfrac{3}{2}v^2)], \quad (7)$$

where $u=r_{qp(1)}$, $v=r_{n(1)n(2)}$ [see Fig. 1(a)], and the normalization constant $N_t=(24\sqrt{3}/\pi^3)\gamma_t^6$. Also

$$\varphi(r_\alpha)=N_\alpha^{\frac{1}{2}}\exp[-\gamma_\alpha^2(8u^2/3+2v^2+3\rho^2)], \quad (8)$$

where the normalization constant $N_\alpha=(1024\sqrt{2}/\pi^{9/2})\gamma_\alpha$.[9]

Using Eqs. (6), (7), and (8), the integral in Eq. (5) is divided into four separate integrals over $d\Omega_p$, $d\boldsymbol{\varrho}$, $d\mathbf{u}$, and $d\mathbf{v}$. Carrying out these integrations with

$$\int e^{i\mathbf{Q}\cdot\mathbf{R}}Y_{l(p)}{}^{m(p)}(\Omega_p)^*d\Omega_p$$
$$=i^{l(p)}[4\pi(2l(p)+1)]^{\frac{1}{2}}j_{l(p)}(QR)\delta_{m(p),0},$$

$$\int\exp[-i\mathbf{K}\cdot\boldsymbol{\varrho}-3\gamma_\alpha^2\rho^2]d\boldsymbol{\varrho}=\frac{\pi^{\frac{3}{2}}}{3\sqrt{3}\gamma_\alpha^2}\exp\left[-\frac{K^2}{12\gamma_\alpha^2}\right],$$

one gets

$$I=T\exp\left[-\frac{K^2}{12\gamma_\alpha^2}\right]\sum_{l(p)}A_{l(p)}i^{l(p)}$$
$$\times[2l(p)+1]^{\frac{1}{2}}j_{l(p)}(QR), \quad (9)$$

where the constant factor $T=7.67\pi^{5/4}\gamma_t^3\gamma_\alpha^{3/2}/(\gamma_\alpha^2+\frac{3}{4}\gamma_t^2)^3$.

The values of γ_α and γ_t may be taken as $1/\gamma_\alpha=1/\gamma_t=4.5\times10^{-13}$ cm. as this is found to give reasonable values for the binding energies of the α particle and the triton.[10] In Eq. (13) the spherical Bessel function $j_{l(p)}(QR)$ is related to the standard notation[11] $F_{l(p)}(QR)$ used in the literature by

$$j_{l(p)}(QR)=F_{l(p)}(QR)/QR.$$

From Eqs. (A7) and (13) one gets

$$d\sigma=\frac{M_t^*M_\alpha^*}{4\pi^2\hbar^4}\frac{k_t}{k_\alpha}\frac{[2J(f)+1]T^2}{2[2J(i)+1]}$$
$$\times\exp\left[-\frac{K^2}{6\gamma_\alpha^2}\right]\sum_{l(p)}A_{l(p)}{}^2j_{l(p)}{}^2(QR),$$
$$=\frac{M_t^*M_\alpha^*}{4\pi^2\hbar^4}\frac{k_t}{k_\alpha}\frac{[2J(f)+1]T^2}{2[2J(i)+1]}$$
$$\times\exp\left[-\frac{K^2}{6\gamma_\alpha^2}\right]\sum_{l(p)}A_{l(p)}{}^2F_{l(p)}{}^2(QR)/Q^2R^2, \quad (10)$$

for the differential cross section for the (α,t) stripping reaction. A formula similar to (10) was used by Holmgren *et al.*[12] for the interpretation of their data on $C^{13}(He^3,\alpha)C^{12}$. Equation (15) is also similar to that of deuteron stripping, which was found to represent Yntema's results[1] on the (α,t) reaction fairly well. Equation (4) for the factor A implies the following selection rule:

$$\mathbf{J}(f)=\mathbf{J}(i)+\mathbf{l}(p)+\tfrac{1}{2},$$

together with the parity conservation rule.

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III. TWO-NUCLEON STRIPPING

In this section the process in which two nucleons are captured from the incident α particle will be considered. The (α,d) stripping reaction was studied before,[8] and it will be seen that the present result, derived on the basis of the surface stripping assumption (or approximation), will be quite comparable with the previous one. The matrix element may be written in this case as [see Fig. 1(b)].

$$I=\int d\mathbf{r}_pd\mathbf{r}_nd\mathbf{r}_d\;\varphi(r_d)$$
$$\times\exp[-i\mathbf{k}_d\cdot\mathbf{r}_{d'}]\psi_f^*(\xi\mathbf{r}_p\mathbf{r}_n)(V_n+V_p)$$
$$\times\exp[i\mathbf{k}_\alpha\cdot\mathbf{r}_\alpha]\varphi(r_\alpha)\psi_i(\xi)d\xi, \quad (11)$$

where $\mathbf{r}_{d'}=\mathbf{r}_d-[(M_n+M_p)/M_f]\mathbf{R}$, where

$$\mathbf{R}=\tfrac{1}{2}(\mathbf{r}_n+\mathbf{r}_p).$$

Introducing now the surface stripping approximation, one may replace $\mathbf{r}_p\equiv(r_p,\theta_p,\varphi_p)$ and $\mathbf{r}_n\equiv(r_n,\theta_n,\varphi_n)$ by $\mathbf{R}_p\equiv(R_0,\theta_p,\varphi_p)$ and $\mathbf{R}_n\equiv(R_0,\theta_n,\varphi_n)$ in the integrand of (11), which may then be written as

$$I=\int d\mathbf{r}_pd\mathbf{r}_nd\mathbf{r}_d\exp[-i\mathbf{k}_d\cdot\mathbf{r}_{d'}]\varphi(r_d)\varphi(r_\alpha)$$
$$\times\exp[i\mathbf{k}_\alpha\cdot(\mathbf{R}_p+\mathbf{R}_n+2\mathbf{r}_d)/4]$$
$$\times\left\{\int\psi_f^*(\xi\mathbf{R}_p\mathbf{R}_n)(V_n+V_p)\psi_i(\xi)d\xi\right\}. \quad (12)$$

The expression in the brackets of (12) is a function of the angular coordinates and may be expanded as $\sum_{LM}A_Lf_{l(m)l(p)}(R_0)Y_L{}^M(\Omega)^*$, where Ω is the angular coordinates of the center of mass of the captured particles p and n;

$$Y_L{}^M(\Omega)=\sum_{m(p)m(n)}\begin{pmatrix}l(p) & l(n) & L\\ m(p) & m(n) & M\end{pmatrix}$$
$$Y_{l(p)}{}^{m(p)}(\theta_p\varphi_p)Y_{l(n)}{}^{m(n)}(\theta_n\varphi_n),$$

where the bracket symbol on the right-hand side denotes the Wigner vector-addition coefficient.[13]

[12] Holmgren *et al.*, reference 2.
[13] G. Breit, reference 6, p. 115.

The internal wave functions of both the α particle and deuteron will also be approximated here by the Gaussian form. Hence

$$\varphi(r_\alpha)=N_\alpha^{\frac{1}{2}}\exp[-\gamma^2\sum_{i<j} r_{ij}^2],$$

where r_{ij} is the distance between the two particles i and j, and summation extends over i and j from 1 to 4 with the condition that $i<j$. Adopting the notation [see Fig. 1(b)] $u=r_{pn}$, $v=r_{qs}$, and $\rho=r_{bt}$, one gets

$$\sum_{i<j}^{4} r_{ij}^2=2u^2+2v^2+4\rho^2. \tag{13}$$

The exponent in the integrand of Eq. (12) may be written as

$$i\mathbf{k}_\alpha\cdot(\mathbf{R}_p+\mathbf{R}_n+2\mathbf{r}_d)/4-i\mathbf{k}_d\cdot\mathbf{r}_{d'}=i\mathbf{Q}\cdot\mathbf{R}-i\mathbf{K}\cdot\boldsymbol{\rho}, \tag{14}$$

where $\mathbf{Q}=\mathbf{k}_\alpha-(M_i/M_f)\mathbf{k}_d$ and $\mathbf{K}=\mathbf{k}_d-\frac{1}{2}\mathbf{k}_\alpha$. To carry out the integration in (12), using (13) and (14) one may effect a coordinate transformation from $\mathbf{r}_p\cdot\mathbf{r}_n$ and $\mathbf{r}_d$ (i.e., r_q and r_s) to $\boldsymbol{\rho}$, $\mathbf{R}$, $\mathbf{u}$, and $\mathbf{v}$. The integral (12) becomes[14]

$$\begin{aligned}I=&\textstyle\sum_{LM} A_L f_{l(n)l(p)}(R_0)\displaystyle\int d\boldsymbol{\rho}d\mathbf{u}d\mathbf{v}d\mathbf{R}\\ &\times\exp[i\mathbf{Q}\cdot\mathbf{R}-i\mathbf{K}\cdot\boldsymbol{\rho}-\gamma_\alpha^2(2u^2+2v^2+4\rho^2)\\ &\qquad-\gamma_d^2v^2]Y_L^M(\Omega)^*\\ =&T\exp[-K^2/16\gamma_\alpha^2]\\ &\times\textstyle\sum_L A_L i^L(2L+1)^{\frac{1}{2}}F_L(QR_0)/QR_0. \end{aligned}\tag{15}$$

Squaring (15) and employing Eq. (A7) one gets for the differential cross section for the (α,d) stripping reaction

$$\begin{aligned}d\sigma=&\frac{M_d^*M_\alpha^*}{4\pi^2\hbar^4}\frac{k_d}{k_\alpha}\frac{[2J(f)+1]T^2}{3[2J(i)+1]}\\ &\times\exp[-K^2/8\gamma_\alpha^2]\sum_L A_L^2F_L^2(QR_0)/Q^2R_0^2,\end{aligned}\tag{16}$$

where

$$T=\frac{16R_0^3(\pi\gamma_\alpha^3\gamma_d^3)^{\frac{1}{2}}}{3(1+\gamma_d^2/2\gamma_\alpha^2)^{\frac{3}{2}}}f_{l(n)l(p)}(R_0).$$

A similar procedure for (He^3,p) using the surface reaction approximation leads to the following ex-

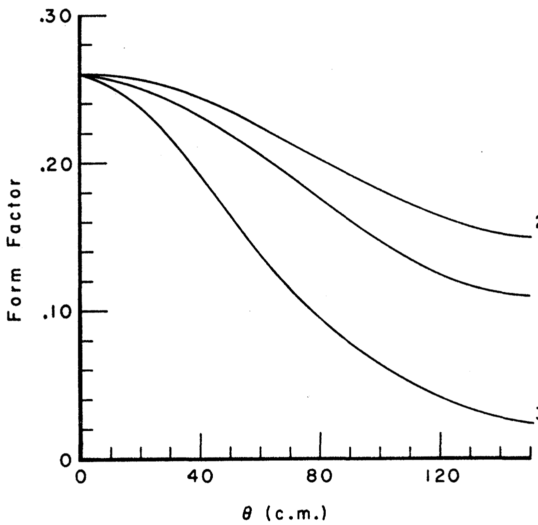

FIG. 2. Comparison between the form factors of the two-nucleon stripping formulas, (1) $[2\exp(-K^2/32\gamma^2)+\exp(-K^2/8\gamma^2)]^2$, (2) $\exp(-K^2/16\gamma^2)$, and (3) $\exp(-K^2/4\gamma^2)$ based on the surface reaction assumption.

pression for the differential cross section:

$$\begin{aligned}d\sigma=&\frac{M_p^*M_h^*}{4\pi^2\hbar^4}\frac{k_p}{k_h}\frac{[2J(f)+1]T^2}{2[2J(i)+1]}\\ &\times\exp\left[-\frac{K^2}{4\gamma_h^2}\right]\sum_L A_L^2F_L^2(QR_0)/Q^2R_0^2,\end{aligned}\tag{17}$$

where

$$T=(0.83\pi^2R_0^3/\gamma_h^3)f_{l(n)l(p)}(R_0),$$

$$\mathbf{Q}=\mathbf{k}_h-(M_i/M_f)\mathbf{k}_p,\quad \mathbf{K}=\mathbf{k}_p-\tfrac{1}{3}\mathbf{k}_h.$$

Equations (16) and (17) are similar to the corresponding expressions derived in reference 8, except for a slight difference in the value of the form factor. This difference may be largely due to the special form of the interaction potential (the delta-function form) introduced in the earlier work. To get an estimate of this difference, Eq. (17) may be compared with the corresponding previous expressions[8]

$$\begin{aligned}d\sigma\simeq&\{2\exp[-K^2/32\gamma_h^2]+\exp[-K^2/8\gamma_h^2]\}^2\\ &\times\textstyle\sum_L |A_L|^2F_L^2(QR_0)/Q^2R_0^2,\end{aligned}\tag{18}$$

or simply (see Appendix II below)

$$d\sigma\simeq\exp[-K^2/16\gamma_h^2]\textstyle\sum_L |A_L|^2F_L^2(QR_0)/Q^2R_0^2. \tag{19}$$

Figure 2 shows a plot for the form factors of Eqs. (17), (18), and (19) for the reaction $Be^9(He^3,p)B^{11}$ for the first excited state protons and for $E_h=4.5$ Mev (See Fig. 4). While a slight difference is observed between the form factors of Eqs. (18) and (19), the form factor of Eq. (17) decreases more rapidly at large (backward) angles. Taking into account the fact that the stripping contributions are mostly peaked in the forward angles

[14] In deriving the result (15), the spherical Bessel function $j_L(QR)\equiv F_L(QR)/QR$, being a slowly varying function of R, is taken outside the integral over R at the value $R=R_0$ (see reference 17). For more accurate calculations one may replace $F_L(QR_0)/QR_0$ in Eq. (15) by $(3/QR_0^3)\int_0^{R_0}F_L(QR)RdR$. The spherical Bessel function $F_L(QR_0)/QR_0$, for a given L, could be expressed in terms of trigonometric functions and the integral be easily evaluated. This applies also to similar formulae in Secs. II, III, and reference 8. Adopting this approximation, the multiple stripping formulas would look similar to those of lump stripping discussed in Appendix III.

(due to the behavior of the function $F_L(QR_0)/QR_0$ especially for small values of L) the difference might be relatively small.

The form factors for the (α,d) reaction corresponding to (18) and (19) are[15] $[4.6\exp(-K^2/48\gamma_\alpha^2)+\exp(-K^2/16\gamma_\alpha^2)]^2$ and $\exp[-K^2/24\gamma_\alpha^2]$.

The allowed values of $\mathbf{L}$ may be deduced from the matrix element giving the value of the expansion coefficient A_L, i.e.,

$$\mathbf{J}(f)=\mathbf{J}(i)+\mathbf{L}+\tfrac{1}{2}+\tfrac{1}{2},$$

together with the conservation of parity rule which requires $|\mathbf{L}|$ to be even or odd according as the parity of the final state f is the same or different from that of the initial state i.

III. THREE-NUCLEON STRIPPING

For definiteness the (α,p) reaction will be considered where three nucleons are captured by the target nucleus.[16] The matrix element for such a reaction, adopting the first Born approximation, runs as [see Fig. 1(c)]

$$\begin{aligned}I=\int & d\mathbf{r}_{p(1)}d\mathbf{r}_{p(2)}d\mathbf{r}_{n(1)}d\mathbf{r}_{n(2)}d\xi\\ &\times\exp[-i\mathbf{k}_p\cdot\mathbf{r}_{p'}]\psi_f^*(\xi\mathbf{r}_{p(1)}\mathbf{r}_{n(1)}\mathbf{r}_{n(2)})\\ &\times(V_{n(1)}+V_{n(2)}+V_{p(1)})\exp[i\mathbf{k}_\alpha\cdot\mathbf{r}_\alpha]\varphi(r_\alpha)\psi_i(\xi).\end{aligned}\quad(20)$$

The point s in Fig. 1(c) represents the center of mass of the three particles $p(1)$, $n(1)$, and $n(2)$ captured by the target nucleus. The internal wave function of the α particle may be written as

$$\varphi(r_\alpha)=N_\alpha^{\frac{1}{2}}\exp[-\gamma_\alpha^2(8u^2/3+2v^2+3\rho^2)],$$

where $u=r_{qn(2)}$, $v=r_{p(1)n(1)}$, $\rho=r_{sp(2)}$, and N_α is a normalization constant whose value is given before [See Eq. (8)]. The exponent of the integrand in (20) is

$$\begin{aligned}x&\equiv-i\mathbf{k}_p\cdot\mathbf{r}_{p'}+i\mathbf{k}_\alpha\cdot(\mathbf{r}_{p(2)}+3\mathbf{R})/4\\ &=-i\mathbf{k}_p\cdot\left(\mathbf{r}_{p(2)}-\frac{M_{p(1)}+M_{n(1)}+M_{n(2)}}{M_f}\mathbf{R}\right)\\ &\qquad\qquad+i\mathbf{k}_\alpha\cdot(\mathbf{r}_{p(2)}+3\mathbf{R})/4\\ &=i\mathbf{Q}\cdot\mathbf{R}-i\mathbf{K}\cdot\boldsymbol{\rho},\end{aligned}\quad(21)$$

where $\mathbf{Q}=\mathbf{k}_\alpha-(M_i/M_f)\mathbf{k}_p$, $\mathbf{K}=\mathbf{k}_p-\frac{1}{4}\mathbf{k}_\alpha$, and $\mathbf{R}$ is the radius vector defining the center of mass "s" of the three captured particles.

According to the surface reaction assumption the coordinates $\mathbf{r}_{n(1)}$, $\mathbf{r}_{p(1)}$, and $\mathbf{r}_{n(2)}$ may be replaced by $\mathbf{R}_{n(1)}$, $\mathbf{R}_{p(1)}$, and $\mathbf{R}_{n(2)}$, and one has, for the integral over ξ in (20),

$$\begin{aligned}\int\psi_f^*(\xi\mathbf{R}_{p1},\mathbf{R}_{n1},\mathbf{R}_{n2})(V_{n1}+V_{n2}+V_{p1})\psi_i(\xi)d\xi\\ =\sum_{LM}A_Lf_{l[n(1)]l[p(1)]l[n(2)]}(R_0)Y_L^M(\Omega)^*,\end{aligned}\quad(22)$$

where Ω in (22) represents the angular coordinates of the vector $\mathbf{R}$. Changing now the coordinates in (20) from $\mathbf{r}_{p(1)}$, $\mathbf{r}_{p(2)}$, $\mathbf{r}_{n(1)}$ and $\mathbf{r}_{n(2)}$ to $\mathbf{u}$, $\mathbf{v}$, $\boldsymbol{\rho}$, and $\mathbf{R}$, and using (22) one gets[14]

$$\begin{aligned}I=T&\exp[-K^2/12\gamma_\alpha^2]\\ &\times\textstyle\sum_L A_Li^L(2L+1)^{\frac{1}{2}}F_L(QR_0)/QR_0,\end{aligned}\quad(23)$$

where

$$T=[64(2\pi)^{\frac{1}{2}}/3]R^3f_{l[n(1)]l[p(1)]l[n(2)]}(R_0).$$

Squaring the expression (23) and using (A7) one gets for the differential cross section

$$\begin{aligned}d\sigma=&\frac{M_p^*M_\alpha^*}{4\pi^2\hbar^4}\frac{k_p}{k_\alpha}\frac{[2J(f)+1]T^2}{2[2J(i)+1]}\\ &\times\exp[-K^2/6\gamma_\alpha^2]\textstyle\sum_L A_L^2F_l^2(QR_0)/Q^2R_0^2.\end{aligned}\quad(24)$$

From the value of the expansion factor A, as given by Eq. (22), one gets the following selection rule

$$\mathbf{J}(f)=\mathbf{J}(i)+\mathbf{L}+\tfrac{1}{2}+\tfrac{1}{2}+\tfrac{1}{2},\quad(25)$$

together with conservation of parity rule which gives another restriction to the allowed values of $\mathbf{L}$.

IV. DISCUSSION

Comparing Eqs. (10), (16), and (24) one observes the similarity between the expressions for the differential cross section for the three different stripping processes for the α particle. When a single nucleon is stripped from the projectile and is being captured into a specified nuclear level with a given value l of the orbital angular momentum, then one generally gets for the differential cross section a single term,

$$d\sigma\cong\exp[-K^2/6\gamma_\alpha^2]F_l^2(QR_0)/Q^2R_0^2.\quad(26)$$

On the other hand, when two nucleons are stripped from the projectile and are captured into two levels with orbital angular momenta $l(1)$ and $l(2)$, then one has, according to (16),

$$d\sigma\cong\exp[-K^2/8\gamma_\alpha^2]\textstyle\sum_L A_L^2F_L^2(QR_0)/Q^2R_0^2,\quad(27)$$

where now a summation over L is implied over the values from $l(1)-l(2)$ to $l(1)+l(2)$. The coefficient A_L in (27) gives the probability amplitude that the final nucleus is composed of a core with angular momentum J_i and a proton-neutron system with total angular momentum L. All terms with different L have to be added, unless forbidden by parity conservation or by other selection rules as those implied by the collective properties of some nuclei.[6,8]

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[15] In calculating the factor 4.6, account is taken of the fact, pointed out in footnote 7 in reference 8, that the ratio between the effective interaction between p and d to that between p and n is 1.5.

[16] J. Sawicki, Nuclear Phys. 6, 575 (1958); G. R. Satchler, Ann. Phys. 3, 275 (1958).

Similarly, when three nucleons are stripped from a given projectile and captured into the individual levels $l(1)$, $l(2)$ and $l(3)$, one gets Eq. (24) which contains the same sum shown in (27). The orbital angular momentum quantum number L can take all values implied by the relation $\mathbf{L}=\mathbf{l}(1)+\mathbf{l}(2)+\mathbf{l}(3)$.

Thus when fitting is achieved by some values of L, one may perhaps be able to determine the values of the orbital angular momenta of the single-particle $l(i)$'s. On the other hand, one may also be able to determine the parity of the final state when the parity of the initial state and the even or odd character of L (obtained by fitting the experimental data) are known.

Considering the formulas given in Sec. II, one may notice that if the experimental data are peaked in the forward direction (i.e., at small forward angles), then the difference between the formula derived here and that derived before[8] (see Appendix II) is not large. If, on the other hand, experimental data show sizeable contribution at intermediate angles, in addition to the forward angle contribution, then although fitting could be obtained, the values of the factor A_L for the large angles may be different in both cases. Fitting and also the values of the quantum numbers L will not, in general, be affected by this difference.

In Appendix III the differential cross sections for (α,d) and (α,p) stripping processes are given for the cases when the captured particles are complex, i.e., a deuteron and a triton, respectively. Apart from the possible change in the corresponding values of the constant γ, these formulas have the same form as those derived above. Hence one may deduce that in the present approximate treatment, using the first Born approximation method, both mechanisms give rise to similar results.

However, in the above formulas, one has to sum over

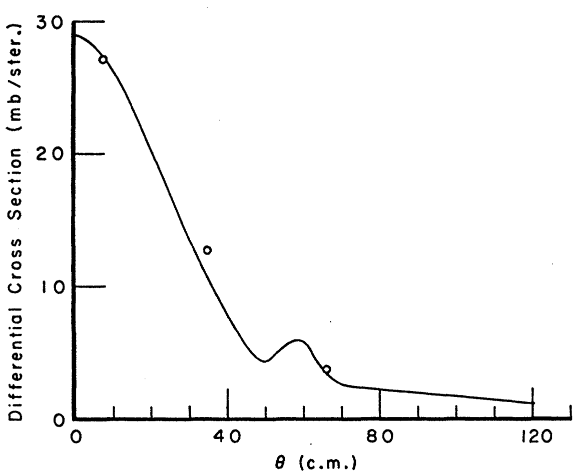

FIG. 3. The differential cross section of the second excited state protons in the center-of-mass system as a function of angle at the energy 3.75 Mev for $C^{12}(He^3,p)N^{14}$ [R. S. Johnston *et al.* Phys. Rev. **109**, 884 (1958)]. The solid curve is derived from Eq. (17) as $d\sigma=[\exp(-K^2/4\gamma^2)](54.95F_0^2+57.0F_2^2)/Q^2R_0^2$.

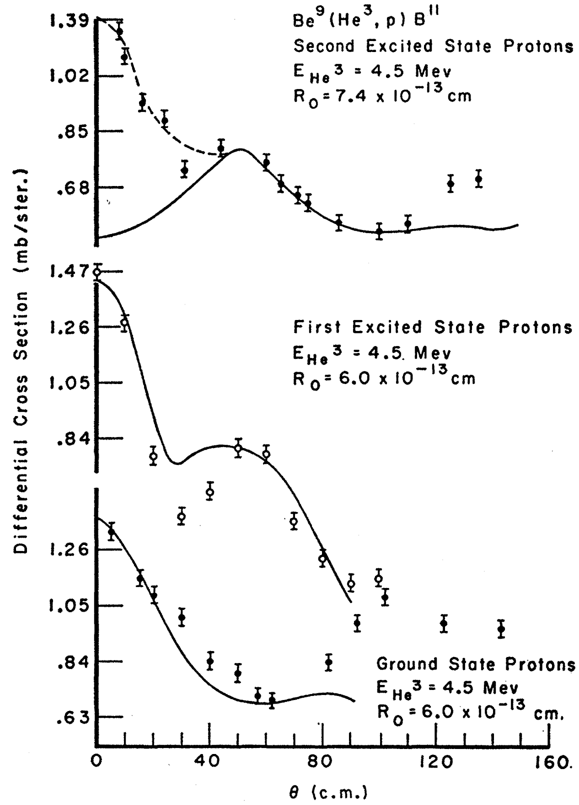

FIG. 4. The center-of-mass differential cross sections for the ground, first and second excited state proton groups from the $Be^9(He^3,p)B^{11}$ reaction as a function of center-of-mass angle at 4.5 Mev incident He^3 energy. The experimental points are those of E. A. Wolicki *et al.*, Phys. Rev. **116**, 1585 (1959). The solid lines represent the theoretical formulas (29), (30), and (31) given in the text.

all possible values of L, while in the latter picture one value of L is only required. Moreover, for the (α,p) reaction the selection rule in the latter, complex particle capture, is

$$\mathbf{J}(f)=\mathbf{J}(i)+\mathbf{L}+\tfrac{1}{2}, \tag{28}$$

as compared with the selection rule (25) given above.

The selection rule (28) is the same as that implied by the direct knockout process as developed by Austern, Butler, and McManus,[17] which was used by Herrmann and Pieper[4] for the interpretation of their data on the (α,p) reactions. Satisfactory fit was not obtained for all cases considered. Since the selection rule (28) allows only some of the values implied by the rule (25), it is clear that better fitting could be obtained when using the latter. Moreover, the cases for which better fitting is obtained by (25) may favor the mechanism of three-nucleon stripping over the H^3 capture from the incident α particle.

Figures 3 and 4 show a comparison between the theoretical formula (17) and some of the recent experi-

[17] N. Austern, S. T. Butler, and H. McManus, Phys. Rev. **92**, 350 (1953).

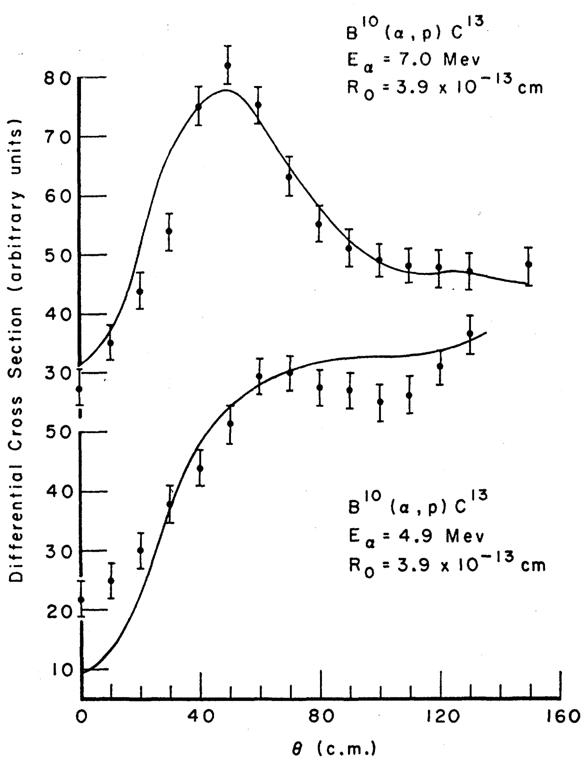

FIG. 5. Angular distribution in the c.m. system of protons from the ground-state transition in $B^{10}(\alpha,p)C^{13}$ at alpha-particle energies of 4.9 and 7.0 Mev, respectively. The experimental points are those of P. von Herrmann and G. F. Pieper [Phys. Rev. **105**, 1556 (1957)]. The solid lines are representations of Eqs. (33) and (34) in the text.

mental observations.[2] For the reaction $C^{12}(He^3,p)N^{14}$ shown in Fig. 3, the selection rule may be written as $1^+=0^++L+\frac{1}{2}+\frac{1}{2}$, which suggests that L should be 0 or 2. The solid line in Fig. 3 represents $d\sigma=\exp(-K^2/4\gamma^2)(54.95F_0^2+57.0F_2^2)/Q^2R_0^2$ where R_0 was taken as 6.0×10^{-13} cm.

Figure 4 shows similar calculations for the reaction $Be^9(He^3,p)B^{11}$, for the ground, first, and second excited state protons. The selection rules and allowed values of L for these states run respectively as follows[18]:

$$\frac{3^-}{2}=\frac{1^-}{2}+L+\frac{1}{2}+\frac{1}{2},\quad \text{therefore}\quad L=0 \text{ or } 2$$
(ground state);

$$\frac{1^-}{2}=\frac{1^-}{2}+L+\frac{1}{2}+\frac{1}{2},\quad \text{therefore}\quad L=0 \text{ or } 2$$
(first excited state);

$$\frac{5^-}{2}=\frac{1^-}{2}+L+\frac{1}{2}+\frac{1}{2},\quad \text{therefore}\quad L=2 \text{ or } 4$$
(second excited state).

[18] F. Ajzenberg-Selove and T. Lauritsen, Nuclear Phys. **11**, 1 (1959).

Fitting the experimental data for the first two levels was achieved by the following values:

$$d\sigma=0.69+[\exp(-K^2/4\gamma^2)][3.68F_0^2(QR_0)/Q^2R_0^2], \quad (29)$$

where $R_0=6.0\times10^{-13}$ cm and 0.69 represents the isotropic contribution due to the compound nucleus formation; and

$$d\sigma=[\exp(-K^2/4\gamma^2)][5.61F_0^2(QR_0)+57.88F_2^2(QR_0)]/Q^2R_0^2, \quad (30)$$

with $R_0=6.0\times10^{-13}$ cm.

For the second excited state protons, the solid line in Fig. 4 represents the relation

$$d\sigma=0.54+\exp(-K^2/4\gamma^2)[10.17F_2^2(QR_0)/Q^2R_0^2], \quad (31)$$

with $R_0=7.4\times10^{-13}$ cm. The value 0.54 represents also the contribution due to compound nucleus formation. The dotted line together with the solid line for angles greater than 50°, for the second excited state, represent

$$d\sigma=0.54+[\exp(-K^2/4\gamma^2)][1.74F_0^2(QR_0)+10.17F_2^2(QR_0)]/Q^2R_0^2, \quad (32)$$

which shows better fitting than Eq. (31) alone. This better fitting with $L=0$ and 2 suggests that the spin of the second excited state of B^{11} may be $\frac{3}{2}^-$ rather than $\frac{5}{2}^-$.

Figure 5 shows the angular distribution in the center-of-mass system of protons from the ground-state transition in $B^{10}(\alpha,p)C^{13}$ at alpha-particle energy (lab. system) of 4.9 and 7.0 Mev, respectively. The experi-

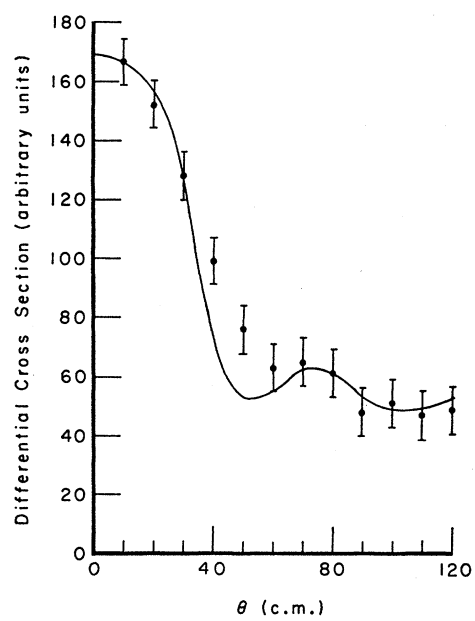

FIG. 6. Angular distribution in the c.m. system of protons from the ground-state transition in $P^{31}(\alpha,p)S^{34}$ at alpha-particle energies (lab. system) of 7.0 Mev. The experimental points are those of P. von Herrmann and G. F. Pieper [Phys. Rev. **105**, 1556 (1957)]. The solid line is a representation of the theoretical formula (35) given in the text.

mental points are those of von Herrmann and Pieper.[4] The selection rule for this case is

$$\frac{1^-}{2}=3^+ + \mathbf{L}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2},$$

suggesting that L could be 1, 3, or 5. Fitting of the experimental data for the 4.9 and 7.0 Mev is given, respectively, by

$$d\sigma=[\exp(-K^2/6\gamma^2)](673.3F_1^2+6047F_3^2+28640F_5^2)/Q^2R_0^2, \quad (33)$$

and

$$d\sigma=[\exp(-K^2/6\gamma^2)][949F_1^2+4026F_3^2+20670F_5^2)/Q^2R_0^2. \quad (34)$$

The value of R_0 was taken as 3.9×10^{-13} cm in both cases.

Similarly one sees in Fig. 6 the angular distribution in the center-of-mass system of protons from the ground-state transition[4] in $P^{31}(\alpha,p)S^{34}$ at alpha-particle energy 7.0 Mev (lab. system). The selection rule

$$\frac{1^+}{2}=0^+ + \mathbf{L}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2},$$

suggests that $L=0$ or 2. Fitting, which is represented by the solid line in Fig. 6, was obtained by

$$d\sigma=48+[\exp(-K^2/6\gamma^2)]\times(1000F_0^2+2270F_2^2)/Q^2R_0^2, \quad (35)$$

where $R_0=7.6\times10^{-13}$ cm. The value 48 on the right-hand side represents the isotropic contribution arising from the compound nucleus mechanism.

From this comparison one observes a general agreement between the formulas derived above and the experimental data considered.

ACKNOWLEDGMENTS

The author would like to express his sincere thanks to Professor G. Breit for his constructive criticism and helpful discussions of this work and to him and the theoretical physics staff at Yale University for their hospitality.

APPENDIX I

It is assumed that in a given nuclear reaction the α particle is decomposed into two particles c and a, where the first particle (which may be one or more nucleons) is captured by the target nucleus i and the second, a, is the outgoing particle. Assuming φ to represent a spin wave function, one may write the spin wave function of the α particle as

$$\varphi(\alpha)=\sum_{\mu(c)\mu(a)}\begin{pmatrix} c & a & 0 \\ \mu(c) & \mu(a) & 0\end{pmatrix}\varphi_c^{\mu(c)}(c)\varphi_a^{\mu(a)}(a)$$

$$=\sum_{\mu(a)}\frac{(-)^{a+\mu(a)}}{(2a+1)^{\frac{1}{2}}}\varphi_a^{-\mu(a)}(c)\varphi_a^{\mu(a)}(a). \quad (A1)$$

Let $\varphi_{J(i)}^{\mu(i)}$ and $\varphi_{J(f)}^{\mu(f)}$ be the spin wave functions of the initial and final nuclei and $\mathbf{L}$ be the orbital angular momentum vector of the captured particle c; then in the usual approximation for such problems, the complete wave function of the residual nucleus may be written as

$$\varphi_{J(f)}^{\mu(i)}(\boldsymbol{\xi}\mathbf{c})=\sum_{Ls,\mu(i)\mu(c),\mu(s)M}\begin{pmatrix} J(i) & c & s \\ \mu(i) & \mu(c) & \mu(s)\end{pmatrix}\begin{pmatrix} s & L & J(f) \\ \mu(s) & M & \mu(f)\end{pmatrix}\varphi_{fLs}(\xi c)Y_L^M(\Omega)\varphi_{J(i)}^{\mu(i)}(\xi)\varphi_{c'}^{\mu(c')}(c), \quad (A2)$$

where $\varphi_{fLs}(\xi c)$ is the radial wave function of the final nucleus. The matrix element for the (α,a) reaction may be written as

$$M=\int\varphi_{j(f)}^{\mu(f)}(\boldsymbol{\xi}\mathbf{c})^*\exp[-i\mathbf{k}_a\cdot\mathbf{r}_{a'}]\varphi(a)^*\varphi_{a'}^{\mu(a')*}V(\boldsymbol{\xi}\mathbf{c})\varphi_{j(i')}^{\mu(i')}(\boldsymbol{\xi})\varphi(\alpha)\exp[i\mathbf{k}_\alpha\cdot\mathbf{r}_\alpha]d\boldsymbol{\tau}, \quad (A3)$$

where $\varphi(a)$ and $\varphi(\alpha)$ are the internal wave functions of the outgoing (a) and incident (α) particle. Substituting from (A1) and (A2) into (A3) one gets

$$M=\sum_{LsM\mu(s)}\frac{(-)^{a'+\mu(a')}}{(2a'+1)^{\frac{1}{2}}}\begin{pmatrix} j(i') & a' & s \\ \mu(i') & -\mu(a') & \mu(s)\end{pmatrix}\begin{pmatrix} s & L & j(f) \\ \mu(s) & M & \mu(f)\end{pmatrix}$$

$$\times\int\varphi_{fLs}(\boldsymbol{\xi}\mathbf{c})^*\exp[-i\mathbf{k}_a\cdot\mathbf{r}_{a'}]Y_L^M(\Omega)^*\varphi(a)^*V(\boldsymbol{\xi}\mathbf{c})\varphi_{j(i')}^{\mu(i')}(\boldsymbol{\xi})\varphi(a)\exp[i\mathbf{k}_\alpha\cdot\mathbf{r}_\alpha]d\boldsymbol{\tau}$$

$$=\sum_{LsM\mu(s)}\frac{(-)^{a'+\mu(a')}}{(2a'+1)^{\frac{1}{2}}}\begin{pmatrix} J(i') & a' & s \\ \mu(i') & -\mu(a') & \mu(s)\end{pmatrix}\begin{pmatrix} s & L & J(f) \\ \mu(s) & M & \mu(f)\end{pmatrix}I(fLsM). \quad (A4)$$

The differential cross section for the stripping process (α,a) may now be written as

$$d\sigma=\frac{1}{4\pi^2\hbar^4}M_a{}^*M_\alpha{}^*\frac{k_a}{k_\alpha}\langle|I|^2\rangle; \qquad \text{(A5)}$$

M^*, k represent the reduced mass and wave propagation vector of the corresponding particle, and $\langle|I|^2\rangle$ represents the stripping amplitude averaged over spin projections in the initial state and summed over spin projections in the final state, i.e.,

$$\langle|I|^2\rangle=\frac{1}{[2J(i)+1]}\sum_{\mu(i')\mu(a')\mu(f)}|M|^2. \qquad \text{(A6)}$$

From (A4) and (A6) one gets

$$\langle|I|^2\rangle=\frac{2J(f)+1}{[2J(i)+1](2a'+1)}\sum_{LsM}\frac{1}{2L+1}I^2(fLsM). \qquad \text{(A7)}$$

$I(fLsM)$ is the amplitude calculated when spin wave functions are not taken into account. One observes from this consideration that in the stripping of α particles, the spin wave functions simply introduce a constant factor and thus do not affect the angular distribution.

APPENDIX II

In this Appendix some remarks will be mentioned about the scattering amplitude given before[8] for the two-nucleon stripping reaction. The notation and argument of Gerjuoy[18] and reference 8 will be adopted. The total Hamiltonian for the (α,d) reaction may be written as

$$H=T_d+T_p+T_n+V_d+V_p+V_n+V_{np}+V_{dp}+V_{dn}, \qquad \text{(A8)}$$

and the complete wave function Ψ satisfies

$$(H-E)\Psi=0. \qquad \text{(A9)}$$

A solution of (A9) was written in the form

$$\Psi=\psi_0-G(V_d+V_{nd}+V_{pd}+V_{np})\Psi, \qquad \text{(A10)}$$

where $(T_d+T_n+T_p+V_n+V_p-E)\Psi_0=0$, and G is the outgoing Green's function

$$G(\mathbf{r}_d,\mathbf{r}_d';\mathbf{r}_n,\mathbf{r}_n';\mathbf{r}_p,\mathbf{r}_p')=\textstyle\sum_\lambda g(E-\lambda)\varphi(\mathbf{r}_n,\mathbf{r}_p,\lambda)\varphi(\mathbf{r}_n',\mathbf{r}_p',)\lambda^*. \qquad \text{(A11)}$$

The wave functions $\varphi(\lambda)$ form the complete set of eigenfunctions of the neutrons and protons in the field of the initial nucleus,

$$(T_n+T_p+V_n+V_p-\lambda)\varphi(\mathbf{r}_p,\mathbf{r}_n,\lambda)=0. \qquad \text{(A12)}$$

The choice of the solution (A10) was done so that one may use the independent-particle wave function representation for $\varphi(\mathbf{r}_p,\mathbf{r}_n,\lambda)$.

However, one may consider the following solution in which the interaction V_{np} between the captured proton and neutron is taken into account:

$$\Psi=\psi_0-G(V_d+V_{nd}+V_{pd})\Psi, \qquad \text{(A13)}$$

where

$$(T_d+T_n+T_p+V_n+V_p+V_{np}-E)\psi_0=0. \qquad \text{(A14)}$$

The function $\varphi(\mathbf{r}_p,\mathbf{r}_n,\lambda)$ of (A12) will be a solution of

$$(T_n+T_p+V_n+V_p+V_{np}-\lambda)\varphi'(\mathbf{r}_p,\mathbf{r}_n,\lambda)=0. \qquad \text{(A15)}$$

Following the same procedure used in reference 8, one gets the following expression for the stripping amplitude, according to the first Born approximation method:

$$I=-\frac{1}{4\pi}\frac{2M_d}{\hbar^2}\int d\mathbf{r}_d d\mathbf{r}_p d\mathbf{r}_n \exp[-i\mathbf{k}_d\cdot\mathbf{r}_{d'}]\times\varphi'(\mathbf{r}_p,\mathbf{r}_n,\lambda_f)^*(V_{nd}+V_{pd})\psi_\alpha. \qquad \text{(A16)}$$

This gives for the differential cross section, if one uses as an approximation (in this procedure) the independent-particle representation for $\varphi'(\mathbf{r}_p,\mathbf{r}_n,\lambda_f)^*$,

$$d\sigma=\frac{M_d{}^*M_\alpha{}^*}{4\pi^2\hbar^4}\frac{k_d}{k_\alpha}\frac{2J(f)+1}{3[2J(i)+1]}(2.4V_0R_0)^2\gamma^3\times\exp[-K^2/24\gamma^2]\sum_L A_L{}^2F_L{}^2(QR_0)/Q^2R_0{}^2, \qquad \text{(A17)}$$

with a similar expression for (He^3,p) except that the form factor becomes

$$\exp[-K^2/16\gamma_h{}^2]. \qquad \text{(A18)}$$

Comparison between (A18) and the earlier derived form factor $[2\exp(-K^2/32\gamma^2)+\exp(-K^2/8\gamma^2)]^2$ is shown in Fig. 2. Small difference is observed even at large angles so that one may use the simpler results (A17) and (A18) for the comparison with the experimental data.

ACKNOWLEDGMENTS

The writer is grateful to Professor G. Breit for discussions concerning the material in this appendix.

APPENDIX III

It might be of interest to consider the possibility that the α particle in the process of stripping is dissociated into two deuterons or a triton and a proton with the former particle in each process being captured by the target nucleus while the latter escapes. This process would be similar to those from deuteron stripping. For the α, d process the matrix element in the first Born approximation method may be written as

$$I=\int d\mathbf{r}_d d\mathbf{R} d\xi \exp[-i\mathbf{k}_d\cdot\mathbf{r}_{d'}]\psi_f(\xi\mathbf{R})^*V(\mathbf{R}\xi)\varphi(\mathbf{r}_\alpha)\psi_i(\xi)\times\exp[i\mathbf{k}_\alpha\cdot(\mathbf{R}+\mathbf{r}_d)/2],$$

where $\mathbf{r}_d$ and $\mathbf{R}$ refer to the coordinates of the outgoing and captured deuterons. Assuming the Gaussian form for the internal wave function of the α particle and using the relation (13) above, one may write $\varphi(r_\alpha) = c\exp[-4\gamma^2\rho^2]$, where the binding energy constant γ may have now a different value from that considered above. c is a constant factor. Following the same method used in Sec. I above, one gets for the (α,d) differential cross section

$$d\sigma \cong \exp[-K^2/8\gamma^2] \sum_L A_L{}^2 F_L{}^2(QR_0)/Q^2R_0{}^2.$$

This has the same form as Eq. (16) above.

Carrying out the same calculation for the (α,p) and using the following form for the internal wave function $\varphi(r_\alpha) = c\exp[-3\gamma^2\rho^2]$, see Fig. 1(c), one gets for the differential cross section the value

$$d\sigma \cong \exp[-K^2/6\gamma^2] \sum_L A_L{}^2 F_L{}^2(QR_0)/Q^2R_0{}^2, \quad \text{(A19)}$$

which is also of the same form as Eq. (24) above. The value of γ may be different in both cases. The selection rule, associated with (A19), for the allowed values of L will be

$$\mathbf{J}(f) = \mathbf{J}(i) + \mathbf{L} + \tfrac{1}{2},$$

instead of Eq. (25) above.

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Angular Distributions of the $Be^9(d,n)B^{10}$ Neutrons

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The angular distributions of neutrons to the ground state and to the first excited state of B^{10} in the reaction $Be^9(d,n)B^{10}$ have been studied at incident deuteron energies of 1.41 Mev, 1.88 Mev, and 2.35 Mev. A proton recoil spectrometer utilizing a xenon gas scintillation trigger was developed to study this problem. This spectrometer operated with a resolution of the order of 7% at the neutron energies involved in this reaction.

The analysis of the data in terms of a direct interaction mechanism indicates that the results are consistent with an interpretation which indicates that the majority of the neutrons emitted at bombarding energies below the Coulomb barrier originate in the Be^9 target nucleus.

I. INTRODUCTION

WHEN considering the general physical problem of nuclear stripping, the Be^9 nucleus is unique. Deuteron stripping has in general been thought of as a consequence of the low binding energy of the proton relative to the neutron; a low binding energy implying an extended wave function and thus a large average separation of the nucleons.

The Be^9 nucleus not only has a lower binding energy for the last neutron, but in addition this particular neutron is in a P state. Thus the average separation of the last neutron from the Be^9 core is even greater than the average extension of the deuteron. Thus one[1] should expect that the outer neutron would be stripped off quite readily.

In spite of the fact that the reaction is of fundamental interest, only a limited number of experiments[2,3] have been performed to study these neutrons. The major experimental difficulty has been that the $Be^9(d,n)B^{10}$ reaction results in neutron groups of relatively low energy and groups which are relatively close in energy. When one considers this fact along with the presence of a high background of gamma rays, it is apparent that high-resolution neutron spectroscopy associated with low background rates is required.

The previous work[2,3] was done utilizing photographic plate techniques, and in one case a time-of-flight method was used to measure the first excited state group.[3] This earlier work performed at low energies indicated the possibility of stripping from the Be^9 nucleus.

The results to be reported here were extended to higher energies with better statistics. In addition to the angular distributions, the total cross sections for the ground and first excited state transitions were measured. The detector used was a recoil proton telescope employing a gas scintillator as the trigger. This instrument will be described.

II. EXPERIMENTAL ARRANGEMENT

The neutron spectrometer used in these experiments was of the same type as developed by other experimenters,[4] with the exception of the trigger mechanism. The conventional telescope employs a trigger which

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[2] F. Ajzenberg, Phys. Rev. **88**, 298 (1952); J. S. Pruitt, C. D. Swartz, and S. S. Hanna, Phys. Rev. **92**, 1456 (1953); L. L. Green, J. P. Scanlon, and J. C. Willmott, Proc. Phys. Soc. (London) **A68**, 386 (1955); A. I. Shpetny, J. Exptl. Theoret. Phys. (U.S.S.R.) **32**, 423 (1957) [translation: Soviet Phys. JETP **5**, 357 (1957)].
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