Creation of the Heavy Elements by Neutron Capture on a Fast Time Scale*

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The mathematical problem concerning the creation of the heavy elements by neutron capture on a fast time scale involves the solution of more than a thousand simultaneous differential equations. This set of equations is solved by replacing them with a mathematical equivalent—a single partial differential equation which turns out to assume the form of the equation of continuity. The stream lines of the analogous hydrodynamical problem are determined, from which the problem of nucleogenesis may be solved once the initial condition is given. By assuming a neutron capture mean lifetime of about 0.1 sec for all nonmagic-number nuclides and by making use of the fact that the neutron magic-number nuclides have smaller neutron-capture cross sections, the position, width, and height of the abundance peaks at Br, Xe, and Pt may be determined; these agree reasonably well with the experimental results.

1. INTRODUCTION

It has been suggested that the heavy elements in the universe are formed from the lighter ones by successive neutron capture and beta-decay processes either "In the Beginning" of the universe[1] or at a later stage in the stellar interior.[2] The cosmological abundance curve of the elements (see Fig. 1) based on recent data by Suess and Urey[3] shows pronounced peaks at Br, Xe, and Pt in addition to peaks at Y, La, and Bi. The latter group of peaks, because of their coincidence with the magic numbers of neutrons (50 for Y, 82 for La, and 126 for Bi) may be explained by the neutron-capture process on a slow time scale. In this process, the mean life of a nucleus against neutron capture being larger than the beta-decay lifetime, the nucleus undergoing successive transmutations will remain close to the beta-stability line. Magic-number nuclides on the beta-stability line will be formed in large abundance because of their smaller neutron-capture cross sections.[4] An explanation for the other group of peaks has been suggested by Coryell.[5] According to him, the groups of elements around Br, Xe, and Pt are considered as the beta-decay products of groups of unstable elements with 50, 82, and 126 neutrons, respectively, which are formed in large abundance, because of their small capture cross sections due to the magic numbers, in a neutron capture process on a fast time scale. The capture mean life must be shorter than the ordinary beta-decay lifetimes so that beta-unstable isotopes far away from the beta-stability line may be formed. In order to account for the peaks at Br, Xe, and Pt, the unstable nuclides formed on the neutron-rich side of the stability line must be, respectively, 5, 7, and 9.5 beta steps away from the stability line.

In this paper we shall formulate and solve the general mathematical problem of element synthesis by neutron capture on a fast time scale as originally outlined by Coryell, i.e., a rapid series of neutron-capture processes interrupted by beta-decay processes.[6] The mathematical formulation of the slow capture process, as given by Alpher and Herman,[1] is represented by a set of simultaneous differential equations, one for each mass number A. About two hundred equations are required. Solution by computer is simplified by approximating this set of equations by a set of 27 equations. Also, the variation of the neutron-capture cross section at magic numbers is ignored. It is evident that, in fast capture processes, one equation for one mass number is not adequate. Each member of the isobaric chain must be considered individually as the capture paths are now no longer limited to the immediate neighborhood of the

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[6] In the meantime the Coryell idea has been developed elaborately by E. M. Burbidge, G. R. Burbidge, W. A. Fowler, and F. Hoyle, Revs. Modern Phys. **29**, 547 (1957); and by A. G. W. Cameron, Chalk River Laboratory Report CRL-41, 1957 (unpublished), and subsequent reports. On the basis that element synthesis takes place in stellar interior, Burbidge, Burbidge, Fowler, and Hoyle included the effect of the (γ,n) processes which becomes appreciable in stellar interior when temperature rises to 10^9 °K, the temperature necessary for the neutrons to be produced by nuclear reactions in stellar interior. On the other hand, such a high temperature may give rise to other types of reactions which may lead to difficulties. B. Pontecorvo, J. Exptl. Theoret. Phys. (U.S.S.R.) **36**, 1615 (1959) [translation: Soviet Phys.-JETP **36(9)**, 1148 (1959)], has considered the energy loss of stars by neutrino pair emission and found its rate to be larger than that of radiation loss when temperature exceeds 10^8 °K. The author is indebted to Dr. Cameron for bringing his attention to this work.

beta-stability line. One differential equation being necessary for each nuclide (N,Z), the total number of equations may well be over a thousand. Furthermore, if the magic numbers have an important part to play, the variation of capture cross section at magic numbers may not be ignored. In spite of the increased complexity, the mathematical system may be solved by approximating the numerous equations by a single partial differential equation. To simplify our problem we shall assume that a strong neutron flux is made available for the light elements to capture for a short duration. The change of abundances due to neutron capture and beta-decay processes will be investigated. We shall not concern ourselves with the origin of the neutron flux, nor its intensity, duration, and time variation. However, in the fast capture process, the time intervals between successive capture and beta-decay processes are so short that we may reasonably approximate the neutron flux to be a constant. In the following (Sec. 2) we shall concern ourselves with the problem of determining the final abundance distribution once the initial abundance distribution of light elements and the intensity and duration of the neutron flux are specified.

2. GENERAL MATHEMATICAL FORMULATION AND ITS SOLUTION

Let the relative abundance of a nuclide (N,Z) at time t be $n(N,Z,t)$, the neutron-capture cross section of the nuclide (N,Z) at the energy corresponding to that of the neutron flux be $\sigma(N,Z)$ and the beta-decay constant of the nuclide (N,Z) be $\lambda(N,Z)$. Assuming a constant neutron flux φ we obtain the following equation for the time rate of change of the abundance of the nuclide (N,Z):

$$\begin{aligned} dn(N,Z,t)/dt &= \varphi n(N-1,\, Z,\, t)\sigma(N-1,\, Z) - \varphi n(N,Z)\sigma(N,Z) \\ &\quad + \lambda(N+1,\, Z-1)n(N+1,\, Z-1,\, t) \\ &\quad - \lambda(N,Z)n(N,Z). \qquad (1) \end{aligned}$$

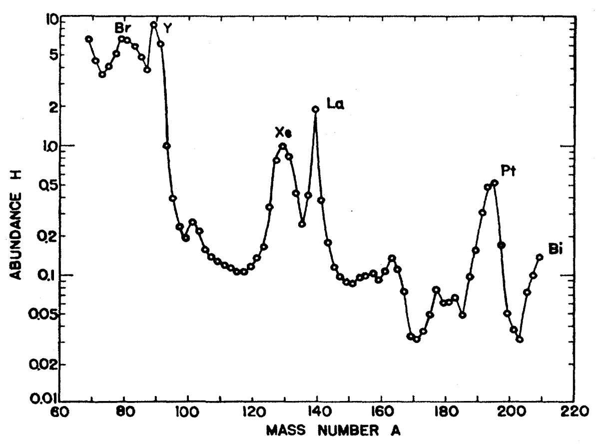

FIG. 1. Abundance of elements ($A>70$) plotted as a function of the mass number A. This graph shows odd-A nuclides only. The abundance curve of even-A nuclides shows the same peaks. The ordinate is so normalized that Si has an abundance of 10^6.

There will be as many simultaneous differential equations as the number of nuclides to be considered.

We make the approximation that $n(N,Z,t)$ is to be a continuous function of the indices N and Z. This assumption is reasonable as the abundance curve actually is continuous in the heavy element region. [$\lambda(N,Z)$ and $\sigma(N,Z)$ are assumed to be continuous functions of N and Z except at magic numbers.] By this assumption we may approximate the more than one-thousand simultaneous differential equations by one single partial differential equation:

$$\begin{aligned} \frac{\partial n(N,Z,t)}{\partial t} &= -\frac{\partial}{\partial N}[\varphi\sigma(N,Z)n(N,Z,t)] \\ &\quad + \frac{\partial}{\partial N}[\lambda(N,Z)n(N,Z,t)] \\ &\quad - \frac{\partial}{\partial Z}[\lambda(N,Z)n(N,Z,t)], \qquad (2) \end{aligned}$$

where $n(N,Z,t)$ is now considered as a function of three variables N, Z, and t. This equation may be rearranged as follows:

$$\begin{aligned} \frac{\partial n(N,Z,t)}{\partial t} &+ \frac{\partial}{\partial N}\{[\varphi\sigma(N,Z)-\lambda(N,Z)]n(N,Z,t)\} \\ &+ \frac{\partial}{\partial Z}\{\lambda(N,Z)n(N,Z,t)\} = 0. \qquad (3) \end{aligned}$$

The form of this equation is identical with that of the equation of continuity for a compressible fluid flowing in the (N,Z) space with a density function $n(N,Z,t)$ and a velocity function the components of which are:

$$\begin{aligned} v_N &= \varphi\sigma(N,Z) - \lambda(N,Z), \qquad (4) \\ v_Z &= \lambda(N,Z). \end{aligned}$$

Therefore, the mathematical problem is identical with that of the flow of a fluid in a two-dimensional space and may be solved by a hydrodynamical analogy.

This analogy has a simple physical interpretation. First $[\varphi\sigma(N,Z)]^{-1}$ is the mean life against neutron capture of a nuclide (N,Z). After capturing one neutron the position of the nucleus moves one unit in the positive N direction in the (N,Z) space. Therefore, the mean velocity of a nucleus in the $+N$ direction due to neutron capture is $1/[\varphi\sigma(N,Z)]^{-1}$, i.e., $\varphi\sigma(N,Z)$. Similarly, the beta decay gives rise to a velocity of the nucleus with a magnitude $\sqrt{2}/[\lambda(N,Z)]^{-1}$, i.e., $\sqrt{2}\lambda(N,Z)$, in a direction making an angle 135° with the $+N$ direction. The vector sum of these two velocities is just the velocity function given by Eq. (4).

The hydrodynamical problem may be solved by determining the stream lines. Equation (4) shows that the stream lines are time independent, being dependent

only on the nuclear properties, $\sigma(N,Z)$ and $\lambda(N,Z)$, and the neutron flux φ. (In the more complicated situation where the neutron flux is not constant in time, the problem may be solved by a set of stream lines changing with time.) Once the initial distribution $n(N,Z,t_0)$ is given, the distribution at any later time $n(N,Z,t)$ may be determined by tracing through the fluid particles along the stream lines. Analytically the solution may be obtained as follows. The differential equation for the stream lines may be found from Eq. (4) to be:

$$dZ/dN=\lambda(N,Z)/[\varphi\sigma(N,Z)-\lambda(N,Z)]. \tag{5}$$

If σ and λ can be expressed by analytic functions, this differential equation may be solved to obtain a series of curves representing the stream lines. Let the equation of the curves be:

$$Z=\mathfrak{z}(N,C), \tag{6}$$

where C is an integration constant, fixed for one specified stream line. Equation (6) may be transformed as follows to express N as a function of Z:

$$N=\mathfrak{N}(Z,C). \tag{7}$$

Equation (4) may now be integrated as follows:

$$\begin{aligned} t-t_0&=\int_{N_0}^{N_t}\frac{dN}{\varphi\sigma(N,\mathfrak{z}(N,C))-\lambda(N,\mathfrak{z}(N,C))},\\ t-t_0&=\int_{\mathfrak{z}(N_0,C)}^{Z_t}\frac{dZ}{\lambda(\mathfrak{N}(Z,C),Z)}. \end{aligned} \tag{8}$$

Equations (8) are the equations of transformation from the point $[N_0,\mathfrak{z}(N_0,C)]$ to the point (N_t,Z_t), i.e., from the coordinates of a particle at t_0 to that at t, the particle being moving along a stream line identified by a given value of C. Equations (8) enable us to transform any point on a given stream line. In order to transform any other initial point (N_0,Z_0) we must find a new stream line which passes through this point. This new stream line may be identified by C' satisfying the following equation:

$$Z_0=\mathfrak{z}(N_0,C'). \tag{9}$$

If we now replace the value C in Eqs. (8) by C' we have the transformation equations for the point (N_0,Z_0). Actually Eqs. (8) may be regarded as the equations of transformation from variables (N_0,C) to variables (N_t,Z_t). Thus the transformation of any initial point (N_0,Z_0) to its later position (N_t,Z_t) may be considered as being made of two successive transformations: first from (N_0,Z_0) to (N_0,C) by Eq. (9), then from (N_0,C) to (N_t,Z_t) by Eqs. (8). If the initial abundance distribution at t_0 is represented by a distribution function $n_0(N_0,Z_0)$ then the abundance distribution at any later time t may be represented by:

$$\begin{aligned} n_t(N_t,Z_t)&=n_0(N_0,\mathfrak{z}(N_0,C))\frac{\partial(N_0,Z_0)}{\partial(N_0,C)}\\ &=n_0(N_0(N_t,Z_t),\mathfrak{z}[N_0(N_t,Z_t),C(N_t,Z_t)])\\ &\quad\times\frac{\partial}{\partial C}\mathfrak{z}[N_0(N_t,Z_t),C(N_t,Z_t)]\frac{\partial(N_0,C)}{\partial(N_t,Z_t)}. \end{aligned} \tag{10}$$

Equation (10) is thus the general solution of the fast capture problem. The abundance distribution at any time t is expressed in terms of the initial distribution n_0, the nuclear properties and the neutron flux, the latter two being contained in the equation of the stream lines.

3. AN APPROXIMATE SOLUTION

The general mathematical theory is applicable if the functions $\sigma(N,Z)$ and $\lambda(N,Z)$ can be expressed by analytic functions. While we may reasonably approximate $\lambda(N,Z)$ by an analytic expression by ignoring the fluctuation due to the different orders of forbidden transition, there is no simple analytic approximation for the discontinuous change of $\sigma(N,Z)$ at magic numbers. As we expect the magic-number effect to be important, a new approach to the solution of the problem will have to be made. Actually the peculiar situation of $\sigma(N,Z)$ at the magic numbers enables us to devise special method for the solution of the problem.

We start by a qualitative consideration of the stream lines in the (N,Z) space. The velocity vector is the sum of two vectors: $\varphi\sigma(N,Z)$ in the $+N$ direction and $\sqrt{2}\lambda(N,Z)$ in the direction making an angle 135° with the $+N$ direction. φ is constant and $\sigma(N,Z)$ is nearly constant for $A>100$ except for magic-number nuclides. We shall assume that $\sigma(N,Z)$ is constant for nonmagic-number nuclides and that for magic-number nuclides its value $\sigma^m(N,Z)$ is smaller[4] by a factor of 50. $\lambda(N,Z)$ is assumed to be proportional to the fifth power of the beta-decay energy E_β according to the Fermi theory. When a stream line originates from a point on the beta-stability line, the value of $\lambda(N,Z)$ at that point is zero and the velocity vector equals $\varphi\sigma(N,Z)$ along the $+N$ direction. Thus, stream lines coming out of the stability valley are at first parallel to the N axis. As they depart from the stability valley, $\lambda(N,Z)$ increases; the velocity vector becomes smaller and turns gradually towards $+Z$ axis. Because of the fifth-power dependence of λ on E_β, this tendency will be accelerated and the direction of the stream line will soon turn parallel to the beta-stability line. If the beta-stability valley has a constant parabolic curvature, the stream line will remain parallel to the valley. This we assume for simplicity. This tendency continues until the line reaches a nuclide with magic-number neutrons. It may be shown that all stream lines starting at different positions along the beta-stability line will eventually

converge into a main artery parallel to the beta-stability line. These qualitative results are represented schematically in Fig. 2.

When the main artery reaches the magic-number line, the sudden drop of the value of $\sigma(N,Z)$ cuts down the velocity component along the $+N$ axis. However, to the right of the magic-number line $\sigma(N,Z)$ is still large. The result is that the main artery will turn into a direction parallel to the $+Z$ axis. If the value of $\sigma^m(N,Z)$ is actually zero, the main artery will proceed until it reaches the beta-stability line. Actually $\sigma^m(N,Z)$ is smaller by a factor[4] of 50. Therefore, a few stream lines will spill over to the right of the magic-number line, the branching fraction being $\varphi\sigma^m(N,Z)/\lambda(N,Z)$ (see Fig. 2). As the main artery moves closer to the beta-stability line, $\lambda(N,Z)$ decreases and the branching fraction increases. When $\lambda(N,Z)$ has decreased by a factor of 50, the branching of the stream lines to the right of the magic-number line becomes appreciable and soon afterwards all stream lines will be found to come over to the right of the magic-number line. From then on they will proceed in a similar manner as they first started.

When the main artery proceeds from P to Q in Fig. 2, the magnitude of the velocity vector decreases rapidly, because now $\varphi\sigma$ is very small and λ decreases rapidly when approaching the beta-stability line. If the fluid flows constantly, a decrease in velocity means an increase in density and a bunching effect may result. After reaching a maximum at M, the density decreases because of the branching of stream lines to the right of the magic-number line. If the neutron flux is turned off suddenly, the nuclei congested between P and Q will follow beta-decay paths to reach beta-stability line. The points P, M, Q will reach the points P', M', Q'. A well-defined abundance peak will result with its maximum M' a few mass units away from the stable magic-number nuclide. The observed abundance peaks at Br, Xe, and Pt may thus be qualitatively explained.[5]

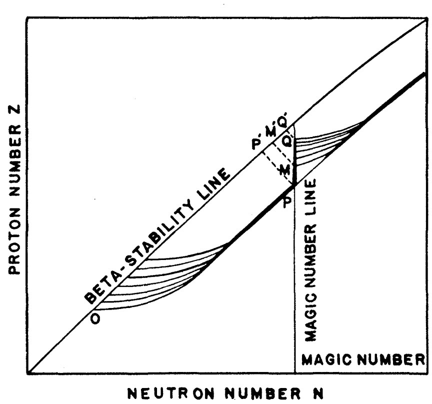

FIG. 2. Schematic diagram of the stream lines representing the paths of neutron capture and beta decay in the neutron-capture process on a fast time scale.

To obtain quantitative results we concern ourselves with the steady-state flow problem. At a later time the neutron flux will be assumed to be turned off suddenly. The problem is to find the abundance distribution of the nuclei undergoing transmutation as a function of the mass number A after they have reached the beta-stability line. This hypothetical problem actually may not be too far from reality. As all stream lines converge into the main artery, an initially constant distribution along the beta-stability line will change over, at a later time, to a nearly constant flow along the main artery. Also, constant flow with a sudden turnoff may be considered as a statistical representation of a random distribution of neutron flux duration. Without further detailed information concerning the actual condition of nucleogenesis,[7] we limit ourselves to this simplest approximation. In doing so we have averaged out the initial conditions of the abundance distribution of the light elements and of the neutron-flux duration. The only condition left undetermined is the neutron flux intensity φ which is taken to be the adjustable parameter of this theory; for best fit we shall postulate that the flux intensity is such that $\varphi\sigma(N,Z)$ is about 8 sec^{-1}, or the mean life against neutron capture is about 0.12 sec. As all stream lines converge into the main artery we need only consider the latter. Considering the bunching and branching of the main artery at the magic-number line we derive the following formulas for the abundance H as a function of mass number A in a region where a peak is expected to appear:

$$H(A)=\text{constant}\times[1/\varphi\sigma(N,Z)] \text{ from 0 to } P',$$

$$H(A)=\text{constant}\times\frac{1}{\lambda(N,Z)}\exp\left(-\int_{Z(P)}^{Z}\frac{\sigma^m(N,Z)}{\sigma(N,Z)}\times\frac{\varphi\sigma(N,Z)-\lambda(N,Z)}{\lambda(N,Z)}dZ\right) \text{ from } P' \text{ to } Q'. \quad (11)$$

In the second expression, N is held constant at a magic number. The general trend of the abundance curve is as follows. The abundance is constant from 0 to P' as $\varphi\sigma(N,Z)$ is constant. At point P' the abundance suddenly changes from $1/\varphi\sigma(N,Z)$ to $1/\lambda(N,Z)$ where $\lambda(N,Z)$ is to be calculated at the point P. From P' to M' the abundance increases because of the factor $1/\lambda(N,Z)$. After reaching the maximum value at M', the abundance will decrease from M' to Q' on account of the exponential factor, which expresses the effect of

[7] The current view[6] seems to be that heavy element synthesis starts from the iron peak (Fe^{56}). Right after the neutron flux is turned on, the hydrodynamical problem is certainly not one of steady state. However, the steady-state solution may still be regarded as a statistical average over a random distribution of neutron flux duration. If the duration is extremely long, many cycles of spontaneous fission may have happened when the synthesized nuclide reaches a mass number greater than 265 and the hydrodynamical state may become steady.

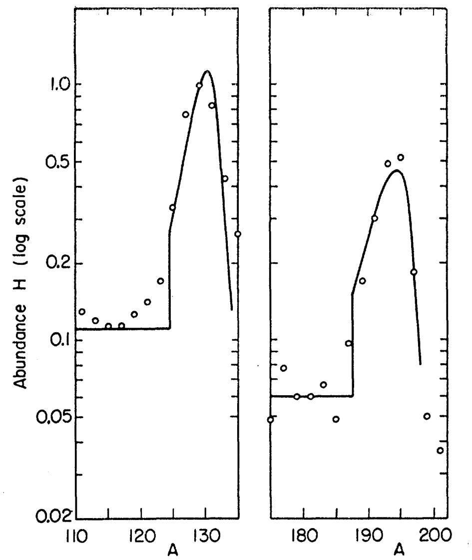

FIG. 3. Theoretical curves representing the abundance peaks calculated on the basis of $\varphi\sigma=8\ \mathrm{sec}^{-1}$ are compared with the experimental results of Fig. 1.

branching. The numerical values assumed for calculation are:

$$\varphi\sigma(N,Z)=8\ \mathrm{sec}^{-1}, \tag{12}$$

$$\sigma(N,Z)/\sigma^m(N,Z)=50. \tag{13}$$

For the beta-decay constant λ we use

$$\lambda=e^{-11.4}E_\beta{}^5, \tag{14}$$

where E_β is the total beta-decay energy in Mev. The value 11.4 is estimated from a graph by Way and Wigner.[8] The value of E_β may be obtained, as a first approximation, from an atomic mass formula by Fermi,[9] i.e.,

$$M(A,Z)=M_A+B_A(Z-Z_A)^2+\delta_A, \tag{15}$$

where

$$Z_A=A/(2+0.015A^{\frac{2}{3}}), \tag{16}$$

$$B_A=0.0419/Z_A,\ \text{in Mev.} \tag{17}$$

For convenience, we neglect the even-odd effect (δ_A) in calculating the beta-decay energy. Thus

$$E_\beta=2B_A(Z_A-Z-\tfrac{1}{2}). \tag{18}$$

The numerical values given by Eqs. (12–18) (among which the value of $\varphi\sigma$ is the only adjustable parameter) enable us to calculate the abundance curve for the Xe peak and the Pt peak. The results are shown in Fig. 3, the relative abundance being normalized to the pre-peak yields. The position, width and height of the peaks agree well with the experimental results. The abundance peak at Br which corresponds to mass number around 80 cannot be treated in a similar manner; for the assumption of constant $\sigma(N,Z)$ is not valid for mass number below 100. It may be mentioned that the break of the calculated curve is not real; it appears as a result of the mathematical approximations employed. The lower right part of the calculated curve cannot be compared with the experimental data since the data in this region are complicated by the neutron-capture process on a slow time scale. The calculated result is rather insensitive to the change of the adjustible parameter $\varphi\sigma$.

4. DISCUSSION

On the basis of the general solution, Eq. (10), a qualitative analysis of the fast capture process may be made. If the stream-line pattern is not singular, the Jacobian of the transformation is not fast-varying and the future abundance distribution will be largely determined by the initial abundance distribution $n_0(N_0,Z_0)$. On the other hand, if the stream-line pattern is singular, exhibiting strong tendency of crowding and bunching, the Jacobian of the transformation is a rapidly varying function and the future status will be essentially determined by the Jacobian instead of by the initial abundance distribution. The effect of the sudden change of the neutron-capture cross section at the magic-number line is to cause crowding among the stream lines and bunching along them. The places where the lines are densest and the velocity smallest are the places of high density, more or less independent of the initial conditions. Therefore, the association of the abundance peaks with the magic numbers of the nucleus seems quite natural and reasonable.

The assumption that $\varphi\sigma(N,Z)$ is constant is, of course, one made for convenience. For very neutron-rich nuclides, the neutron binding energy is small and the beta-decay energy is large. However, in the peak regions considered here the binding energies of the unstable nuclides first produced are not too small. For nuclides of greater chain lengths the binding energy may be very small and capture cross section will be small, too. Therefore, there seems to exist a natural limit for the distance between the beta-stability line and the main artery stream-line no matter how large the neutron flux may be. It is quite possible that the widths of the peaks are set by this natural limit. The emission of delayed neutrons is not considered here, the effect of which may be represented by an effective neutron flux varying with (N,Z), i.e., φ will be dependent on (N,Z). The general feature of the results remains the same.

To handle the discontinuity due to the sudden change of $\sigma(N,Z)$, an analytic method may be conceived in which the two sides of the magic-number line are solved analytically and the two solutions are joined together over the line of discontinuity. The continuity condition

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to join the two solutions turns out to be just the branching effect we considered in Sec. 3. If we omit the minor stream lines, the result is exactly the one obtained in Sec. 3. The condition under which the minor lines may be omitted is $\sigma/\sigma^m \gg 1$ which is satisfied in our case. This consideration thus justifies the approximate method we used in Sec. 3.

5. ACKNOWLEDGMENT

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Mott-Scattering Analysis of Longitudinal Polarization of Electrons from Co^{60}†*

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Mott scattering has been used to analyze the degree of longitudinal polarization of beta particles emitted from radioactive nuclei. The reliability of this method and the influences of the various systematic errors associated with this method on the accuracy of the measurement have been investigated in detail and are discussed. On the basis of a linear extrapolation of the inverse of the Mott asymmetry to zero scatterer thickness, the polarization of 194-kev electrons from Co^{60} was found to be $-(0.994\pm0.057)v/c$ with all known corrections applied. The effects of atomic screening and finite nuclear size have not been included. Using the quoted value for the polarization measured in the pure Gamow-Teller transition in Co^{60} yields $C_A'=(0.7 \text{ to } 1.45)C_A$.

I. INTRODUCTION

The discovery of the violation of space inversion symmetry in beta decay led to the prediction that electrons were emitted from unoriented nuclei with their spins polarized parallel to their direction of flight.[1–5] The study of the magnitude and direction of the longitudinal polarization of beta particles has been of considerable interest during the past three years as an aid in the investigation of the beta-decay interaction. In addition to providing further unambiguous evidence for parity nonconservation in beta decay, these experiments furnish a measurement of the relative magnitudes of the even and odd beta-coupling constants. The latter quantity is of particular interest in establishing a necessary condition for the validity of the two-component neutrino theory.[6–8] The accuracy attainable with polarization measurements compares very favorably with the other experimental methods that are capable of extracting the same information: (1) the now classical experiments of Wu *et al.*[2] on the beta asymmetry from polarized nuclei, and (2) the measurement of the β-γ (circular polarization) correlation from unpolarized nuclei.[9] Another useful feature of this technique is its applicability to nuclei not suitable for the other two methods.

In this paper the results of a Mott scattering analysis of the longitudinal polarization of beta rays from the allowed Gamow-Teller transition in Co^{60} are presented, with particular emphasis on the systematic errors associated with this technique.

The longitudinal polarization for beta particles in an allowed transition can be written in the following manner[4,10]:

$$P=\frac{Gv/c}{1+bm/E}, \qquad (1)$$

where

$$\begin{aligned}\xi G=|M_{\mathrm{F}}|^2\Big[&\pm 2\,\mathrm{Re}(C_SC_S'^*-C_VC_V'^*)\\&+\frac{\alpha Z}{p/m}2\,\mathrm{Im}(C_SC_V'+C_S'C_V^*)\Big]\\+|M_{\mathrm{GT}}|^2\Big[&\pm 2\,\mathrm{Re}(C_TC_T'^*-C_AC_A'^*)\\&+\frac{\alpha Z}{p/m}2\,\mathrm{Im}(C_TC_A'^*+C_T'C_A^*)\Big],\end{aligned} \qquad (2)$$

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* Submitted by D. P. Malone in partial fulfillment of the requirements for the degree of Doctor of Philosophy to the faculty of the Graduate School of Yale University.

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