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Depolarization Due to Target Motion*

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The projection upon both of the lab scattering planes of a spin component along the center-of-mass angular momentum direction is computed for arbitrary target motion. The resulting depolarization of Λ hyperons produced by 1080-Mev/c pions on nuclear matter and on deuterons is presented as a function of production angle. At this energy, the polarization, if constant for all production angles, would be reduced to 85% and 95% of the original value in heavy nuclei with respect to the $\pi\times\Lambda$ and $K\times\Lambda$ planes, respectively. In deuterium, the reductions would be to 98.4% and 99.5% of the initial polarization.

I. INTRODUCTION

WHENEVER the target in a scattering or two-body production process is in motion the angular momentum in the center-of-mass system is in general no longer along the normal to either laboratory scattering plane. The result is a reduction of any initial polarization one of the particles may have had with respect to the c.m. angular momentum. If one knew the target momentum at collision, it would be possible in principle to determine the new direction of spin of the outcoming particle of interest. However, this information is usually not available so that we must average over the target momentum spectrum and accept certain depolarizations with respect to the lab scattering planes.

We calculate the projection of the initial polarization along the normals to two possible lab scattering planes. In the process $1+2\rightarrow 3+4$, where 1 is the incident particle and 3 is the resultant particle polarized along the angular momentum in the c.m. system, we define $\mathbf{1}\times\mathbf{3}$ and $\mathbf{4}\times\mathbf{3}$ as the unit vector normals to the lab scattering planes. If there is no depolarization, $\mathbf{1}\times\mathbf{3}$ and $\mathbf{4}\times\mathbf{3}$ will be collinear with each other and with the true direction of the angular momentum.

The problem is solved relativistically throughout and with no approximations. Various approximations will usually be admissible in evaluating the rather complicated results, however. Wigner's[1] method of transforming spins is used throughout. This method consists essentially of determining the spin components in and normal to the plane containing the necessary Lorentz transformations. Spin components normal to this plane are conserved. The rotation of the component lying in the plane will be given below, following that in Wigner's paper.

The magnitude of the effect is presented as a function of c.m. production angle for 1080-Mev/c pions producing $\Lambda+K^0$ on deuterium and on heavy nuclei. Really the initial polarization should be folded in as a function of energy and c.m. production angle but since this information isn't available yet the projection of a constant spin vector is given.

II. KINEMATIC SOLUTION

Figure 1 displays the kinematic situation. Particles 1 and 2 collide in the plane of the paper with angle u between them. The resultant center of mass has momentum $\mathbf{P}_0=\mathbf{P}_1+\mathbf{P}_2$ with magnitude

$$P_0=\Gamma BM_0=[P_1^2+P_2^2+2P_1P_2\cos u]^{\frac{1}{2}}, \tag{1}$$

where

$$\begin{aligned} &M_0=[M_1^2+M_2^2+2M_1M_2D]^{\frac{1}{2}} \text{ is the total c.m. energy,}\\ &D=\gamma_1\gamma_2(1-\beta_1\beta_2\cos u),\\ &\gamma_1=E_1/M_1,\quad \beta_1=P_1/E_1,\quad \text{etc.,}\\ &B=P_0/(E_1+E_2),\\ &\Gamma=(E_1+E_2)/M_0. \end{aligned} \tag{2}$$

The coordinate system is chosen so that the $1-2$ collision plane is the $x-z$ plane. This choice means that we do not have to average over the azimuthal angle of $\mathbf{P}_2$ about $\mathbf{P}_1$.

The angle w between $\mathbf{P}_1$ and $\mathbf{P}_0$ in the lab system satisfies the condition

$$P_0\cos w=P_1+P_2\cos u. \tag{3}$$

The angle Δ between the c.m. momentum of particle 1, $\mathbf{P}_1^*$, and the c.m. motion can be found from the Lorentz transformation of the longitudinal component of $\mathbf{P}_1$.

$$P_1^*\cos\Delta=\Gamma(P_1\cos w-BE_1), \tag{4}$$

where

$$P_1^*=\frac{M_0}{2}\left[1-2\left(\frac{M_1^2+M_2^2}{M_0^2}\right)+\left(\frac{M_1^2-M_2^2}{M_0^2}\right)^2\right]^{\frac{1}{2}}. \tag{5}$$

After substituting Γ, B, P_1^*, and $\cos w$ into Eq. (4) and reducing, we find

$$\cos\Delta=\frac{\gamma_1(M_1D+M_2)-\gamma_2(M_2D+M_1)}{P_0[D^2-1]^{\frac{1}{2}}}. \tag{6}$$

In the center-of-mass system, the only direction with physical meaning, the direction along which particles

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[1] E. P. Wigner, Revs. Modern Phys. **29**, 255 (1957). See Appendix I.

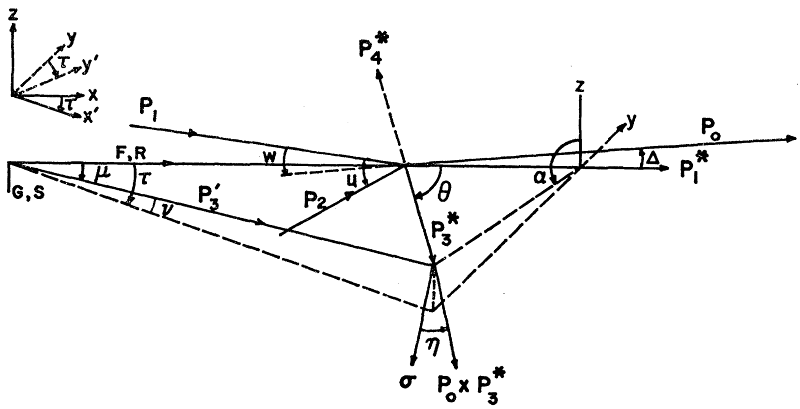


FIG. 1. Kinematic diagram of collision of two moving particles yielding two outgoing particles.

1 and 2 approach each other, $\mathbf{P}_1{}^*$, is chosen as the x axis of our coordinate system.

Particles 3 and 4 are emitted at angles θ and $\pi-\theta$, respectively, with respect to $\mathbf{P}_1{}^*$. The plane containing $\mathbf{P}_1{}^*$ and $\mathbf{P}_3{}^*$ is rotated about $\mathbf{P}_1{}^*$ by a uniformly distributed azimuthal angle α. The unit vector $\boldsymbol{\sigma}$ represents the spin and points in the direction $\mathbf{P}_1{}^*\times\mathbf{P}_3{}^*$, the c.m. angular momentum direction. Let us represent Lorentz transformations by the velocity difference between the two frames. Then particle 3 is first transformed from rest to the c.m. system by $\beta_3{}^*$ and to the lab by B. If we rotate the spin about $\mathbf{P}_3{}^*$ by angle η, until it is parallel to $\mathbf{P}_0\times\mathbf{P}_3{}^*$, before transforming by $\beta_3{}^*$ and B, it will be normal to the plane containing $\beta_3{}^*$ and B and will point in the direction $\mathbf{P}_4\times\mathbf{P}_3$ in the lab system. Thus $\cos\eta$ is the fraction of the original polarization that appears along $\mathbf{4}\times\mathbf{3}$, or $\cos\eta=\boldsymbol{\sigma}\cdot\mathbf{4}\times\mathbf{3}$:

$$\cos\eta=\frac{\mathbf{P}_0\times\mathbf{P}_3{}^*\cdot\mathbf{P}_1{}^*\times\mathbf{P}_3{}^*}{|\mathbf{P}_0\times\mathbf{P}_3{}^*|\,|\mathbf{P}_1{}^*\times\mathbf{P}_3{}^*|}=\frac{\cos\Delta-l\cos\theta}{\sin\theta[1-l^2]^{\frac{1}{2}}},\tag{7}$$

where

$$l=\cos\Delta\,\cos\theta+\sin\Delta\,\sin\theta\,\cos\alpha.\tag{8}$$

As it should, $\cos\eta=1$ when $\mathbf{P}_1$, $\mathbf{P}_2$, $\mathbf{P}_3$, and $\mathbf{P}_4$ all lie in a plane, a condition indicated by $\alpha=0$ or π. Since $\cos\eta$ contains only $\cos\alpha$, it is necessary to average only over $0\leq\alpha\leq\pi$. If $\sin\Delta\ll\cos\Delta$, $\cos\eta$ will be symmetric about $\cos\theta=0$.

The depolarization with respect to $\mathbf{1}\times\mathbf{3}$ is much more involved. The plan of attack is the following. We first express the Lorentz transformation by B to the lab system as a product of two transformations, the first along x by R and the second along the z axis by S. After the R transformation, we rotate the coordinate system about z until the momentum of particle 3 lies in the $x'-z$ plane. At this stage momenta are primed. Since the R transformation is along the line $\mathbf{P}_1{}^*$, the spin is still along $\mathbf{P}_1{}'\times\mathbf{P}_3{}'$. Then the spin components in and normal to the $x'-z$ plane are found. In the transformation by S to the lab system, the component normal to the $x'-z$ plane is conserved while the effect on the component in the $x'-z$ plane is computed by the technique published by Wigner.[1] Finally the projection of the spin upon $\mathbf{1}\times\mathbf{3}$ in the lab is expressed.

To express the Lorentz transformation by B along the c.m. direction as a product of R followed by S, we require that the matrix products of the R and S transformations equal the product of a rotation about y until the x axis points along the c.m. direction of motion, a Lorentz transformation by velocity B along the new x axis, and a rotation again about the y axis by an angle b yet to be determined. While angle b is of no interest to us here it is a convenient link in calculating R and S, given B and Δ.

$$\begin{bmatrix} F & 0 & 0 & FR \\ 0 & 1 & 0 & 0 \\ GSFR & 0 & G & GSF \\ GFR & 0 & GS & GF \end{bmatrix}=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & G & GS \\ 0 & 0 & GS & G \end{bmatrix}\begin{bmatrix} F & 0 & 0 & FR \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ FR & 0 & 0 & F \end{bmatrix}$$

$$=\begin{bmatrix} \cos b & 0 & -\sin b & 0 \\ 0 & 1 & 0 & 0 \\ \sin b & 0 & \cos b & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \Gamma & 0 & 0 & \Gamma B \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \Gamma B & 0 & 0 & \Gamma \end{bmatrix}\begin{bmatrix} \cos\Delta & 0 & \sin\Delta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\Delta & 0 & \cos\Delta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$=\begin{bmatrix} \Gamma\cos b\cos\Delta+\sin b\sin\Delta & 0 & \Gamma\cos b\sin\Delta-\sin b\cos\Delta & \Gamma B\cos b \\ 0 & 1 & 0 & 0 \\ \Gamma\sin b\cos\Delta-\cos b\sin\Delta & 0 & \Gamma\sin b\sin\Delta+\cos b\cos\Delta & \Gamma B\sin b \\ \Gamma B\cos\Delta & 0 & \Gamma B\sin\Delta & \Gamma \end{bmatrix},\tag{9}$$

where

$$F=[1-R^2]^{-\frac{1}{2}} \quad \text{and} \quad G=[1-S^2]^{-\frac{1}{2}}. \tag{10}$$

The convenient way to satisfy this equality is to equate first the $1-3$ terms.

$$\tan b=\Gamma \sin\Delta/\cos\Delta. \tag{11}$$

Further study shows that

$$R=B\cos\Delta, \quad S=B\sin\Delta/[1-B^2\cos^2\Delta]^{\frac{1}{2}}, \quad G=\Gamma[1-B^2\cos^2\Delta]^{\frac{1}{2}}. \tag{12}$$

After the transformation by R, we rotate the coordinate system about the z axis by angle τ until the momentum of particle 3 lies in the $x'-z$ plane or $P_{3y'}'=0$.

$$\begin{bmatrix} P_{3x'}' \\ P_{3y'}' \\ P_{3z}' \\ E_3' \end{bmatrix} = \begin{bmatrix} \cos\tau & -\sin\tau & 0 & 0 \\ \sin\tau & \cos\tau & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} F & 0 & 0 & FR \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ FR & 0 & 0 & F \end{bmatrix} \begin{bmatrix} P_3^*\cos\theta \\ -P_3^*\sin\theta\sin\alpha \\ P_3^*\sin\theta\cos\alpha \\ E_3^* \end{bmatrix}, \tag{13}$$

$$P_{3y'}'=0=F(P_3^*\cos\theta+RE_3^*)\sin\tau - (P_3^*\sin\theta\sin\alpha)\cos\tau, \tag{14}$$

or

$$\sin\tau=\frac{P_3^*\sin\theta\sin\alpha}{[F^2(P_3^*\cos\theta+RE_3^*)^2+(P_3^*\sin\theta\sin\alpha)^2]^{\frac{1}{2}}}. \tag{15}$$

The component of spin normal to the plane containing R and S, $\sigma_\perp$, is then the normalized y' component of $\mathbf{P}_1'\times\mathbf{P}_3'$.

$$\sigma_\perp=\frac{P_{1z}'P_{3x'}'-P_{1x'}'P_{3z}'}{P_1'(P_3'\sin\mu)}=\frac{-P_{1x'}'P_{3z}'}{P_1'(P_3^*\sin\theta)} =\frac{-P_1'P_{3z}'\cos\tau}{P_1'(P_3^*\sin\theta)}, \tag{16}$$

$$P_{3z}'=P_3^*\sin\theta\cos\alpha, \tag{17}$$

so

$$\sigma_\perp=-\cos\tau\cos\alpha \quad \text{and} \quad \sigma_{\text{II}}=[1-\sigma_\perp^2]^{\frac{1}{2}}. \tag{18}$$

The angle ν is needed in determining the effect of the two Lorentz transformations, β_3' and S, on the component of spin which lies in the $x'-z$ plane which contains both β_3' and S.

$$\sin\nu=\frac{P_3^*\sin\theta\cos\alpha}{[(P_3^*\sin\theta)^2+F^2(P_3^*\cos\theta+RE_3^*)^2]^{\frac{1}{2}}} =\frac{P_3^*\sin\theta\cos\alpha}{P_3'}. \tag{19}$$

We now apply Wigner's technique to determine the angle in the $x'-z$ plane between σ_{II} and the velocity of particle 3 after the transformation to the lab system by S. This angle leads directly to the projection of σ_{II} along the component of $\mathbf{P}_1\times\mathbf{P}_3$ in the $x'-z$ plane. We will need to compute only the y' component of $\mathbf{P}_1\times\mathbf{P}_3$ in the lab system to complete the scalar product of $\boldsymbol{\sigma}$ and 1×3.

The state of particle 3 in the lab system is developed in Fig. 2 with a series of diagrams to be read from right to left as indicated by the arrows. A double line indicates an imparted velocity and the location of the $X-Z$ coordinate system simply indicates the Lorentz system in which particle 3 is being described. The state of particle 3 in the lab in terms of the matrices, Eq. (20) (see Fig. 2) under the diagrams is developed by reading again from right to left: The state of a particle with spin along the Z axis and velocity β_3' along the X axis is multiplied on the left by a coordinate system rotation through the angle ν and then by the Lorentz transformation S along Z to give the following state with superscripts and subscripts suppressed temporarily:

$$\begin{pmatrix} \gamma\cos\nu & \sin\nu & \gamma\beta\cos\nu \\ -G(\gamma\sin\nu+S\gamma\beta) & G\cos\nu & -G(\gamma\beta\sin\nu+S\gamma) \\ G(S\gamma\sin\nu+\gamma\beta) & -GS\cos\nu & G(\gamma+S\gamma\beta\sin\nu) \end{pmatrix}. \tag{21}$$

We can now ask by what angle ϵ the spin should have been rotated before β_3' and S in order that the spin and velocity be parallel in the lab. To find the angle ϵ, the above state must be multiplied on the right by a rotation through angle ϵ or

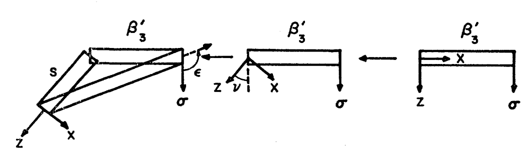
$$\begin{pmatrix} \cos\epsilon & \sin\epsilon & 0 \\ -\sin\epsilon & \cos\epsilon & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{22}$$

To satisfy the form of a state with spin and velocity parallel,[1] the $3-1$ term of the product must be zero,

$$G(S\gamma\sin\nu+\gamma\beta)\cos\epsilon+GS\sin\epsilon\cos\nu=0, \tag{23}$$

or

$$\tan\epsilon=\frac{-\gamma(S\sin\nu+\beta)}{S\cos\nu}=\frac{-\gamma(\sin\nu+\beta/S)}{\cos\nu}. \tag{23}$$



$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & G & -GS \\ 0 & -GS & G \end{pmatrix} \begin{pmatrix} \cos\nu & \sin\nu & 0 \\ -\sin\nu & \cos\nu & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \gamma_3' & 0 & \gamma_3'\beta_3' \\ 0 & 1 & 0 \\ \gamma_3'\beta_3' & 0 & \gamma_3' \end{pmatrix} \tag{20}$$

FIG. 2. [Equation (20).] Kinematic development of state of particle with spin in the lab.

The projection of $\sigma_{\parallel}$ along the normal to β_3 in the $x'-z$ plane is then given by the $\sin\epsilon$ times the magnitude of $\sigma_{\parallel}$ where with superscripts and subscripts restored,

$$\sin\epsilon=\frac{(1+S/\beta_3'\sin\nu)}{[1+(S/\beta_3')^2-S^2\cos^2\nu+2(S/\beta_3')\sin\nu]^{\frac{1}{2}}}. \quad (24)$$

We next find the magnitude of the y' component of $\mathbf{1}\times\mathbf{3}$.

$$(1\times3)_{y'}=\frac{P_{1z}P_{3x'}-P_{1x'}P_{3z}}{|P_1\times P_3|}. \quad (25)$$

The necessary momentum components are found by multiplying (13) by the Lorentz transformation S along the z axis. It is convenient to find all the momentum components and then the direction cosines.

$$\begin{aligned}
P_{1x'}&=F(P_1^*+RE_1^*)\cos\tau,\\
P_{1y'}&=F(P_1^*+RE_1^*)\sin\tau,\\
P_{1z}&=GSF(E_1^*+RP_1^*),\\
P_{3x'}&=F(P_3^*\cos\theta+RE_3^*)\cos\tau\\
&\qquad+(P_3^*\sin\theta\sin\alpha)\sin\tau,\\
P_{3y'}&=0,\\
P_{3z}&=G(P_3^*\sin\theta\cos\alpha)+GSF(E_3^*+RP_3^*\cos\theta).
\end{aligned} \quad (26)$$

Finally, the projection along $\mathbf{1}\times\mathbf{3}$, or $\boldsymbol{\sigma}\cdot\mathbf{1}\times\mathbf{3}$, becomes

$$\boldsymbol{\sigma}\cdot\mathbf{1}\times\mathbf{3}=\sigma_{\perp}(1\times3)_{y'}+\sigma_{\parallel}\sin\epsilon[1-(1\times3)_{y'}{}^2]^{\frac{1}{2}}. \quad (27)$$

This function also need only be averaged over $0\leqslant\alpha\leqslant\pi$.

III. NUMERICAL EXAMPLE

While the above solution is completely general, the original problem arose in attempting to understand the polarization of Λ hyperons produced by π mesons bombarding heavy nuclei and deuterium. Since scattering on nucleons other than particle 2 by particles 1, 3, and 4 in the production nucleus will lead to further depolarization, only an upper limit to $\boldsymbol{\sigma}\cdot\mathbf{1}\times\mathbf{3}$ and $\boldsymbol{\sigma}\cdot\mathbf{4}\times\mathbf{3}$ could be found from an evaluation of the above effect. As well, the initial polarization at all energies and c.m. angles is not known. Yet, it seemed worthwhile to know the magnitude of the effect for some representative cases.

We chose 1080-Mev/c (950-Mev) π mesons producing Λ plus K^0. If the depolarization due to target motion were not serious at this relatively low energy, it would be less serious at higher energies. The Michigan[2] group has actually observed many Λ hyperons produced by 950-Mev π^- on xenon. Also a very large fraction of the hyperons produced at this energy must be Λ's because the $\pi^-+p\rightarrow\Lambda+K^0$ cross section has a peak at 910 Mev.

The left ordinate, projection, in Fig. 3 gives $\boldsymbol{\sigma}\cdot\mathbf{1}\times\mathbf{3}$ and $\boldsymbol{\sigma}\cdot\mathbf{4}\times\mathbf{3}$ averaged over angle α, angle u, and P_2 in heavy nuclei as a function of $\cos\theta^*$, the c.m. production angle of the Λ. In the previous section this angle was simply θ. To exemplify the order of integration, $\langle\boldsymbol{\sigma}\cdot\mathbf{1}\times\mathbf{3}\rangle_{\text{av}}$ is written as

$$\langle\boldsymbol{\sigma}\cdot\mathbf{1}\times\mathbf{3}\rangle_{\text{av}}=\int_0^{(P_2)_{\max}}w(P_2)dP_2\int_{-1}^{s_{\max}}\frac{d\rho}{d(\cos\theta^*)}ds\int_0^{\pi}\frac{(\boldsymbol{\sigma}\cdot\mathbf{1}\times\mathbf{3})d\alpha}{\pi}\bigg/\int_0^{(P_2)_{\max}}w(P_2)dP_2\int_{-1}^{s_{\max}}\frac{d\rho}{d(\cos\theta^*)}ds, \quad (28)$$

where $d\rho/d(\cos\theta^*)$ is the differential cross section for the reaction $\pi^-+p\rightarrow\Lambda+K^0$ and $s=\cos u$. The values of $s_{\max}$ are those at which $M_0=M_\Lambda+M_{K^0}$ for given P_2. We employed an IBM 650 to evaluate the first integral on the right in the numerator. In Fig. 3, the right ordinate gives in arbitrary units the values of $(1/\rho)(d\rho/d(\cos\theta^*))$ for collisions with c.m. energy greater than that corresponding to a 970-Mev/c π on a nucleon at rest. For lesser c.m. energies, an isotropic angular distribution was allowed. These angular distributions are reasonable fits to the available data[3,4] for the range of energies covered by this calculation. The angular distributions have little effect on the presented results because most of the production occurs with the same distribution. Figure 4 gives the values of ρ employed.[3,4] Both Fermi and Gaussian momentum distributions gave projections within 0.01 (the computation accuracy) of each other so only one set of curves appears in Fig. 3. The Fermi distribution had $(P_2)_{\max}$

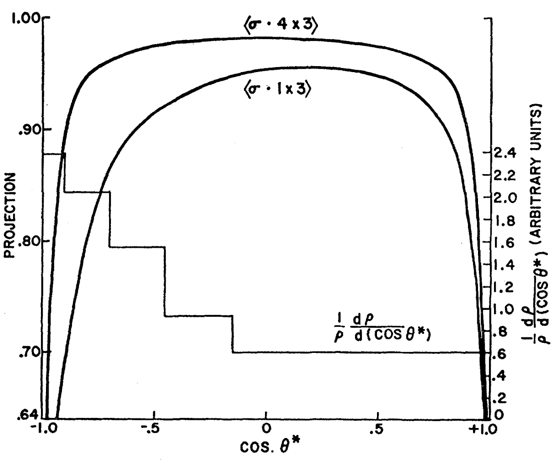


FIG. 3. Average value of spin projection, left ordinate, along the $\mathbf{1}\times\mathbf{3}$ ($\boldsymbol{\pi}\times\boldsymbol{\Lambda}$) and $\mathbf{4}\times\mathbf{3}$ ($\mathbf{K}\times\boldsymbol{\Lambda}$) lab planes as a function of the cosine of the c.m. angle for 1080-Mev/c pions producing $\Lambda+K^0$ on heavy nuclei. The right ordinate gives the angular distribution used for all $\Lambda+K^0$ production with c.m. energy greater than the amount present in the collision of a 970-Mev/c pion with a nucleon at rest.

[2] J. C. Vander Velde (private communication).

[3] F. S. Crawford (private communication).

[4] *Proceedings of 1958 Annual International Conference on High-Energy Physics at CERN* (CERN Scientific Information Service, Geneva, 1958), Session 5.

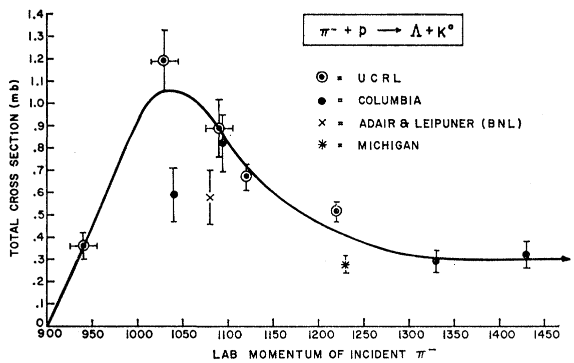


FIG. 4. The total cross section used in our numerical examples represented by the full line. For the experimental points, see references 3 and 4.

equal to 232 Mev/c. In the Gaussian distribution,

$$w(P_2)dP_2 = P_2^2 \exp[-P_2^2/P_0^2]dP_2, \tag{29}$$

$$P_0^2/2M_2 = 14 \text{ Mev}. \tag{30}$$

Figure 5, gives $\langle \boldsymbol{\sigma}\cdot \mathbf{1}\times\mathbf{3}\rangle_{\text{av}}$ and $\langle \boldsymbol{\sigma}\cdot \mathbf{4}\times\mathbf{3}\rangle_{\text{av}}$ for 950-Mev pions producing $\Lambda+K^0$ on deuterium. The depolarization due to target motion is clearly negligible except at extreme forward and backward c.m. angles. In Fig. 5, the right ordinate gives $\sin\theta_3$ and $\langle\sin\theta_3\rangle_{\text{av}}$. The former is the sine of the emission angle of the Λ with respect to the π beam direction for the target at rest; the latter is the same function averaged over α, u, and P_2. The two curves differ only for large emission angles, an effect partly due to averaging $\sin\theta_3$ rather than the angle θ_3, a time-saving approximation on our relatively slow computer. The total cross section in Fig. 4 was also employed in the deuteron calculation. No shadow correction, which would reduce the high-momentum collision weights, was applied to the deuteron calculation. Therefore, the presented result should be an upper limit to the depolarization due to target motion.

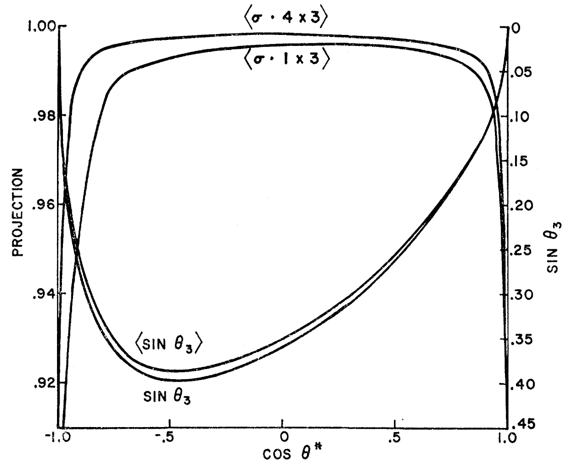


FIG. 5. Average value of spin projection, left ordinate, along the $\mathbf{1}\times\mathbf{3}$ ($\boldsymbol{\pi}\times\boldsymbol{\Lambda}$) and $\mathbf{4}\times\mathbf{3}$ ($\mathbf{K}\times\boldsymbol{\Lambda}$) lab planes as a function of the cosine of the c.m. angle for 1080-Mev/c pions producing $\Lambda+K^0$ on deuterium. The right ordinate gives the sine of the Λ emission angle; one averaged over the deuteron momentum distribution, and the other for a target at rest.

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