Lepton Decays of Hyperons*

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An expression is given for the lepton energy spectrum and asymmetry for the lepton decay modes of hyperons in terms of 6 form factors. A weak interaction having the form of the usual lepton current coupled to an unspecified strong current is assumed, and the momentum transfer dependence of the form factors is neglected. Some applications to the decays $\Lambda\rightarrow p+e+\bar{\nu}$ and $\Lambda\rightarrow p+\mu+\bar{\nu}$ are given.

I. INTRODUCTION

At the present time little is known about the interaction responsible for the weak decays of hyperons. The most natural assumption would be an extension of the universal V-A theory[1] which has been successful in describing weak decays not involving strange particles. If, however, an interaction of this form is assumed with the "universal" coupling constant, and if the effects of strong interactions are ignored, then the resulting calculated rate for the decay $\Lambda\rightarrow p+e+\bar{\nu}$ is an order of magnitude larger than the experimental rate.[2] This shows either that the universal V-A theory does not apply to Λ β decay or that renormalization effects are very large.

It is the purpose of this paper to determine what one can hope to learn about the weak interactions of hyperons from experimental studies of their lepton decay modes. The lepton, rather than pion, modes are chosen because the structure of the weak interaction is almst completely masked by strong interactions in the pion modes. Although at present experimental information concerning the lepton modes is extremely limited,[2] it is likely that sufficient data eventually will be accumulated.

A study of these decays has been made previously by Shekhter,[3] who limits himself to direct V and A interactions (form factors f_1 and g_1). In the present paper, contributions from different form factors (induced couplings) are considered; these may well be important because renormalization effects appear to be large. Some consequences of these other factors have previously been treated by Albright.[4]

II. MATRIX ELEMENT AND FORM FACTORS

In this calculation an interaction of the form[5]

$$GJ^{\mu}(x)L_{\mu}(x)+\text{H.c.} \tag{1}$$

is assumed. As in the V-A theory, the lepton current, $L_{\mu}(x)$, is taken to be

$$L^{\mu}(x)=\bar{\psi}_e(x)\gamma^{\mu}\frac{(1+\gamma_5)}{\sqrt{2}}\psi_{\nu}(x)+\bar{\psi}_{\mu}(x)\gamma^{\mu}\frac{(1+\gamma_5)}{\sqrt{2}}\psi_{\nu}(x). \tag{2}$$

There are arguments, based upon K-meson lepton decay modes, for assuming this form in strangeness-nonconserving decays.[6] $J^{\mu}(x)$ is a current constructed from the fields of strongly interacting particles, possibly, but not necessarily, including hyperons. With this interaction the matrix element for the transition $B\rightarrow b+l+\bar{\nu}$ is, to lowest order in the weak interaction,

$$G(2\pi)^4\delta^4(p_B-p_b-p_e-p_{\nu})\times\langle b|J^{\mu}(0)|B\rangle \times\left(\frac{m_l}{E_{\nu}}\frac{m_{\nu}}{E_{\nu}}\right)^{\frac{1}{2}}\bar{u}_l\gamma_{\mu}\frac{(1+\gamma_5)}{\sqrt{2}}v_{\nu}, \tag{3}$$

where B and b are baryons and l is a μ^- or e^-. (The calculation is essentially identical for $B'\rightarrow b'+\bar{l}+\nu$.) Electromagnetic corrections have been neglected. The mass of the neutrino will be allowed to go to zero later.

As is well known,[7] because of its transformation properties, $\langle b|J^{\mu}(0)|B\rangle$ can always be expressed in terms of six dimensionless form factors:

$$\langle b|J^{\mu}(0)|B\rangle=\left(\frac{m_bm_B}{E_bE_B}\right)^{\frac{1}{2}} \times\bar{u}_b\left(f_1\gamma^{\mu}+\frac{f_2}{m_B}\sigma^{\mu\nu}k_{\nu}+\frac{f_3}{m_B}k^{\mu}+g_1\gamma^{\mu}\gamma_5 +\frac{g_2}{m_B}\sigma^{\mu\nu}\gamma_5k_{\nu}+\frac{g_3}{m_B}\gamma_5k^{\mu}\right)u_B, \tag{4}$$

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1 R. P. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1958).

2 J. Leitner *et al.*, Phys. Rev. Letters **3**, 186 (1959); F. S. Crawford *et al.*, Phys. Rev. Letters **1**, 377 (1958); J. Orear *et al.*, Phys. Rev. Letters **1**, 380 (1958).

3 V. M. Shekhter, J. Exptl. Theoret. Phys. (U.S.S.R.) **35**, 458 (1958) [translation: Soviet Phys.-JETP **8**, 316 (1959)]; J. Exptl. Theoret. Phys. (U.S.S.R.) **36**, 1299 (1959) [translation: Soviet Phys.-JETP **9**, 920 (1959)]. *Note added in proof*. After submitting this paper, it came to the author's attention that a paper containing similar results had been published by V. P. Belov, B. S. Mingalev, and V. M. Shekhter, J. Exptl. Theoret. Phys. (U.S.S.R.) **38**, 541 (1960) [translation: Soviet Phys.—JETP **11**, 392 (1960)].

4 C. H. Albright, Phys. Rev. **115**, 750 (1959).

5 The notation of S. S. Schweber, H. A. Bethe, and F. de Hoffmann, *Meson and Fields* (Row, Peterson and Company, New York, 1956), is used, except that here $\gamma_5=-i\gamma^0\gamma^1\gamma^2\gamma^3$ and $a\cdot b$ represents a four-vector product.

6 See, for example, R. Dalitz, Revs. Modern Phys. **31**, 823 (1959).

7 M. Goldberger and S. Treiman, Phys. Rev. **111**, 354 (1958).

where $\sigma^{\mu\nu}=(\gamma^\mu\gamma^\nu-\gamma^\nu\gamma^\mu)/2$, $k=p_B-p_b=p_e+p_\nu$, and $f_1, \ldots, g_3$ are the form factors, which, in general, depend upon k^2. An alternative expression, more convenient for calculation in some cases, is

$$\langle b|J^\mu(0)|B\rangle=\left(\frac{m_bm_B}{E_bE_B}\right)^{\frac{1}{2}}\bar{u}_b\left(F_1\gamma^\mu+\frac{F_2}{m_B}p_B{}^\mu+\frac{F_3}{m_B}k^\mu +G_1\gamma^\mu\gamma_5+\frac{G_2}{m_B}\gamma_5p_B{}^\mu+\frac{G_3}{m_B}\gamma_5k^\mu\right)u_B. \quad (5)$$

These expressions are equivalent when

$$\begin{aligned}&F_1=f_1+(1+m_b/m_B)f_2,\quad G_1=g_1-(1-m_b/m_B)g_2,\\&F_2=-2f_2,\quad G_2=-2g_2,\\&F_3=f_2+f_3,\quad G_3=g_2+g_3.\end{aligned}\quad (6)$$

If time-reversal invariance is assumed then all the above form factors may be chosen to be real numbers.

Since γ_5u satisfies the Dirac equation with negative mass, one finds that the matrix element is invariant under each of the two transformations:

$$m_B\rightarrow -m_B\begin{cases}F_1\leftrightarrow G_1,\quad F_2\leftrightarrow -G_2,\quad F_3\leftrightarrow -G_3\\ f_1\leftrightarrow g_1,\quad f_2\leftrightarrow -g_2,\quad f_3\leftrightarrow -g_3,\end{cases}\quad (7)$$

$$m_b\rightarrow -m_b\begin{cases}F_1\leftrightarrow G_1,\quad F_2\leftrightarrow -G_2,\quad F_3\leftrightarrow -G_3\\ f_1\leftrightarrow g_1,\quad f_2\leftrightarrow -g_2,\quad f_3\leftrightarrow -g_3.\end{cases}\quad (8)$$

Similar considerations show that the matrix element must be independent of the sign of m_l. Starting with the matrix element (3) a fairly long but straightforward calculation shows that the decay rate from a B particle, at rest with polarization $\mathbf{P}$, into $dE_ld\Omega_ld\Omega_\nu$ is

$$\omega(E_l,\Omega_l,\Omega_\nu;\mathbf{P})dE_ld\Omega_ld\Omega_\nu =\frac{|G|^2}{2(2\pi)^5}\left[C_1+C_2\mathbf{p}_l\cdot\mathbf{P}+C_3\mathbf{p}_\nu\cdot\mathbf{P}+C_4\mathbf{P}\cdot(\mathbf{p}_l\times\mathbf{p}_\nu)\right] \times\frac{|p_l|\,|p_\nu|}{m_B-E_l+|p_l|\cos\theta_{l\nu}}dE_ld\Omega_ld\Omega_\nu. \quad (9)$$

The C_i are written most compactly in terms of the F_i's and G_i's, using the symmetry (8).

$$\begin{aligned}C_1=&|F_1|^22[E_\nu(p_b.p_l)+E_l(p_b.p_\nu)-m_b(p_\nu.p_l)]\\&+|F_2|^2(E_b+m_b)[2E_lE_\nu-(p_l.p_\nu)]\\&+2\mathrm{Re}F_1F_2{}^*[E_\nu(p_b.p_l)+E_l(p_b.p_\nu)-E_b(p_\nu.p_l)\\&+m_b\{2E_lE_\nu-(p_l.p_\nu)\}]\\&+2\mathrm{Re}F_1G_1{}^*[E_\nu(p_b.p_l)-E_l(p_b.p_\nu)]\\&+\mathrm{Re}F_1F_3{}^*(m_l{}^2/m_B)[(p_b.p_\nu)+m_bE_\nu]\\&+2\,\mathrm{Re}F_2F_3{}^*(m_l{}^2/m_B)(E_b+m_b)E_\nu\\&+|F_3|^2(m_l/m_B)^2(E_b+m_b)(p_\nu.p_\nu)\\&+\text{symmetric.}\end{aligned}\quad (10)$$

$$\begin{aligned}C_2=&|F_1|^22[m_bE_\nu-(p_b.p_\nu)]+2\,\mathrm{Re}F_1F_2{}^*[E_bE_\nu\\&-(p_b.p_\nu)]+2\,\mathrm{Re}F_1G_1{}^*(p_b.p_\nu)-\mathrm{Re}F_2G_2{}^*\\&\times[2E_lE_\nu-(p_l.p_\nu)]-2\,\mathrm{Re}F_1G_2{}^*[2E_lE_\nu-(p_l.p_\nu)\\&+(E_b-m_b)E_\nu]-2\mathrm{Re}F_1G_3{}^*(m_l{}^2/m_B)E_\nu\\&-2\,\mathrm{Re}F_2G_3{}^*(m_l{}^2/m_B)E_\nu-\mathrm{Re}F_3G_3{}^*(m_l/m_B)^2\\&\times(p_l.p_\nu)+\text{symmetric.}\end{aligned}\quad (11)$$

$$\begin{aligned}C_3=&-|F_1|^22[m_bE_l-(p_b.p_l)]-2\,\mathrm{Re}F_1F_2{}^*[(E_bE_l\\&-(p_b.p_l)]+2\,\mathrm{Re}F_1G_1{}^*(p_b.p_l)-\mathrm{Re}F_2G_2{}^*[2E_lE_\nu\\&-(p_l.p_\nu)]-2\,\mathrm{Re}F_1G_2{}^*[2E_lE_\nu-(p_l.p_\nu)\\&+(E_b-m_b)E_l]-2\,\mathrm{Re}F_1G_3{}^*(m_l{}^2/m_B)\\&\times[E_b-m_b+E_\nu]-2\,\mathrm{Re}F_2G_3{}^*(m_l{}^2/m_B)E_\nu\\&-\mathrm{Re}F_3G_3{}^*(m_l{}^2/m_B)(p_l.p_\nu)+\text{symmetric.}\end{aligned}\quad (12)$$

$$\begin{aligned}C_4=&2\,\mathrm{Im}F_1F_2{}^*(E_\nu-E_l)+2\,\mathrm{Im}F_1G_2{}^*m_b\\&+2\,\mathrm{Im}F_1G_2{}^*(E_b-m_b)-2\,\mathrm{Im}F_1F_3{}^*(m_l{}^2/m_B)\\&+\text{symmetric.}\end{aligned}\quad (13)$$

The "+symmetric" at the end of each expression means that one should add to the expression given the expression obtained from it by making the substitutions indicated in (8).

From four-momentum conservation we find

$$E_\nu=\frac{m_B(E_l{}^{\max}-E_l)}{m_B-E_l+|\mathbf{p}_l|\cos\theta_{l\nu}},\quad (14)$$

where

$$E_l{}^{\max}=\frac{m_B{}^2+m_l{}^2-m_b{}^2}{2m_B}.\quad (15)$$

We can thus regard the C_i as functions of E_l and $\cos\theta_{l\nu}$.

It should be noted that when $m_l=0$ the above expression for the decay rate agrees with a theorem given by Weinberg[8]: in the scalar terms $[C_1+C_e\mathbf{P}\cdot(\mathbf{p}_l\times\mathbf{p}_\nu)]$ the F-G interference terms are antisymmetric under interchange of l and ν while the remaining terms are symmetric. In the pseudoscalar terms $(C_2\,\mathbf{p}_l\cdot\mathbf{P}+C_3\mathbf{p}_\nu\cdot\mathbf{P})$ the situtation is reversed. Another theorem by Weinberg[8] states that the F_1G_1 term in C_1 will not contribute to the total decay rate even when $m_l\neq 0$.

Expression (9), in somewhat different form, has been obtained by Albright[4] for electron modes.

III. LEPTON SPECTRUM AND ASYMMETRY

The above expression for the complete distribution is not of much immediate use. One would like to obtain expressions for simpler observables by integrating over some of the variables. A difficulty arises, however, because of the unknown dependence of the form factors upon k^2. Fortunately, it is likely that this dependence is small. If a reasonably simple form is assumed for the strong current, J^μ, the k^2 dependence of the form factors will be due entirely to the renormalization effects of the strong interactions. Since the maximum value of k^2 is small compared to the masses of the possible

[8] S. Weinberg, Phys. Rev. **115**, 481 (1959).

TABLE I. Spectrum factor $a(x)=[1-4Rx+5\frac{1}{3}R^2x^2]|f_1|^2+\cdots$. The lepton energy spectrum is proportional to $\beta x^2(1-x)^2(1+\epsilon-2Rx)^{-3}a(x)$.

Form factors	x^{-1}	1	x	x^2	x^3
$\lvert f_1\rvert^2$	$\cdots$	1	$-4R$	$5\frac{1}{3}R^2$	$\cdots$
$\lvert g_1\rvert^2$	$-2\epsilon R^{-1}$	$3-2E$	$-8R$	$5\frac{1}{3}R^2$	$\cdots$
$\lvert f_2\rvert^2$	$2\frac{2}{3}\epsilon-3\frac{1}{3}\epsilon R+2\epsilon^2R^{-1}$	$2R^2-4\frac{2}{3}\epsilon+6\frac{2}{3}\epsilon R$	$-6\frac{2}{3}R^2+1\frac{1}{3}R^3-1\frac{1}{3}\epsilon R$	$6\frac{2}{3}R^2-2\frac{2}{3}R^3$	$-2\frac{2}{3}R^3$
$\lvert g_2\rvert^2$	$1\frac{1}{3}\epsilon-2\epsilon R-\epsilon^2R^{-1}$	$2R^2-\frac{1}{3}\epsilon-5\frac{2}{3}\epsilon R$	$-1\frac{1}{3}R^2-4R^3+\frac{2}{3}\epsilon R$	$1\frac{1}{3}R^2+2\frac{2}{3}R^3$	$-2\frac{2}{3}R^3$
$\mathrm{Re}f_1f_2^*$	$2\frac{1}{3}\epsilon-2\frac{1}{6}\epsilon R-2\frac{1}{2}\epsilon^2R^{-1}$	$2R^2+R^3-4\frac{1}{3}\epsilon+5\frac{2}{3}\epsilon R$	$-5\frac{1}{3}R^2-1\frac{1}{3}R^3-2\epsilon R$	$5\frac{1}{3}R^2-2\frac{2}{3}R^3$	$\cdots$
$\mathrm{Re}g_1g_2^*$	$-2\epsilon R^{-1}+5\epsilon$	$-4R+2R^2+9\epsilon$	$8R^2$	$\cdots$	$\cdots$
$\mathrm{Re}f_1g_1^*$	$-2\epsilon R^{-1}+2\frac{2}{3}\epsilon$	$-2R+1\frac{1}{3}\epsilon$	$4R+1\frac{1}{3}R^2$	$-5\frac{1}{3}R^2$	$\cdots$
$\mathrm{Re}f_2g_2^*$	$4\epsilon-5\frac{1}{3}\epsilon R-2\epsilon^2R^{-1}$	$4R^2-4\frac{2}{3}\epsilon R$	$-8R^2-2\frac{2}{3}R^3+4\epsilon R$	$10\frac{2}{3}R^3$	$\cdots$
$\mathrm{Re}f_1g_2^*$	$2\epsilon-1\frac{2}{3}\epsilon R-\epsilon^2R^{-1}$	$2R^2+R^2-1\frac{1}{3}\epsilon R$	$-4R^2-3\frac{1}{3}R^3+2\epsilon$	$5\frac{1}{3}R^3$	$\cdots$
$\mathrm{Re}g_1f_2^*$	$-4\epsilon R^{-1}+7\frac{1}{3}\epsilon$	$-4R+2R^2+2\frac{2}{3}\epsilon$	$8R-1\frac{1}{3}R^2$	$10\frac{2}{3}R^2$	$\cdots$
$\mathrm{Re}f_1f_3^*$	$2\epsilon R^{-1}-\epsilon$	$-7\epsilon+3\epsilon R$	$6\epsilon R$	$\cdots$	$\cdots$
$\mathrm{Re}g_1g_3^*$	$-\epsilon-\frac{1}{2}\epsilon R-\frac{1}{2}\epsilon^2R^{-1}$	$\epsilon+3\epsilon R$	$-2\epsilon R$	$\cdots$	$\cdots$
$\mathrm{Re}f_2f_3^*$	$\epsilon^2R^{-1}-1\frac{1}{3}\epsilon^2$	$\epsilon R-\frac{2}{3}\epsilon^2$	$-2\epsilon R-\frac{2}{3}\epsilon R^2$	$2\frac{2}{3}\epsilon R^2$	$\cdots$
$\mathrm{Re}g_2g_3^*$	$\epsilon^2R^{-1}-1\frac{1}{3}\epsilon^2$	$\epsilon R-\frac{2}{3}\epsilon^2$	$-2\epsilon R-\frac{2}{3}\epsilon R^2$	$2\frac{2}{3}\epsilon R^2$	$\cdots$
$\lvert f_3\rvert^2$	$-\epsilon^2R^{-1}$	$\epsilon-\epsilon R$	$-2\epsilon R$	$\cdots$	$\cdots$
$\lvert g_3\rvert^2$	$\frac{1}{6}\epsilon^2-\frac{1}{4}\epsilon^2R+\frac{1}{4}\epsilon^3R^{-1}$	$\frac{1}{4}\epsilon R^2+1/12\epsilon^2+\epsilon^2R$	$-\frac{2}{3}\epsilon R^2-\frac{1}{2}\epsilon R^3-\frac{1}{2}\epsilon^2R$	$\frac{2}{3}\epsilon R^2$	$\cdots$

intermediate states, the form factors should not vary by more than 5 or 10% over the range of k^2 for the decays considered in Table III. More precise estimates of this variation can be made by considering the range of variation of k^2 for each decay. To this accuracy, then, the form factors may be treated as constants and the integrations mentioned above may be performed.

Two observables which should be measurable are the lepton energy spectrum and the lepton asymmetry measured relative to the polarization of the initial hyperon (hyperons can be easily produced with large polarizations).[9] Integrating over the neutrino variables, and treating the form factors as constants, we obtain for the decay rate into $dE_ld(\cos\theta_l)$:

$$\omega(E_l,\cos\theta_l)dE_ld(\cos\theta_l)=\frac{2|G|^2}{(2\pi)^3}(m_BR)^4$$
$$\times\frac{\beta x^2(1-x)^2}{(1+\epsilon-2Rx)^3}[a(x)+b(x)\beta P\cos\theta_l]dE_ld(\cos\theta_l), \quad (16)$$

TABLE II. Asymmetry factor
$b(x)=[(-\frac{2}{3}R-\frac{1}{2}R^2+\frac{1}{2}\epsilon)+(\frac{2}{3}R+2\frac{2}{3}R^2)x+(-2\frac{2}{3}R^2)x^2]|f_1|^2+\cdots$.
The electron asymmetry is given by $\beta Pb(x)/a(x)$.

Form factors	1	x	x^2	x^3
$\lvert f_1\rvert^2$	$-\frac{2}{3}R-\frac{1}{2}R^2+\frac{1}{2}\epsilon$	$\frac{2}{3}R+2\frac{2}{3}R^2$	$-2\frac{2}{3}R^2$	$\cdots$
$\lvert g_1\rvert^2$	$-2+\frac{1}{3}R$	$4\frac{2}{3}R$	$\cdots$	$\cdots$
$\lvert f_2\rvert^2$	$-\frac{2}{3}R^2+2\epsilon-6\epsilon R$	$4R^2-4R^3+2\epsilon R$	$-5\frac{1}{3}R^2+8R^3$	$\cdots$
$\lvert g_2\rvert^2$	$-\frac{2}{3}R^2+1\frac{1}{3}\epsilon R$	$-1\frac{1}{3}R^2+1\frac{1}{3}R^3+\frac{2}{3}\epsilon R$	$2\frac{2}{3}R^3$	$\cdots$
$\mathrm{Re}f_1f_2^*$	$-1\frac{1}{3}R-\frac{2}{3}R^2+2\epsilon$	$-\frac{1}{3}R+6\frac{2}{3}R^2$	$-8R^2$	$\cdots$
$\mathrm{Re}g_1g_2^*$	$2\frac{2}{3}R-\frac{2}{3}R^2-2\epsilon$	$1\frac{1}{3}R-6\frac{2}{3}R^2$	$-2\frac{2}{3}R^2$	$\cdots$
$\mathrm{Re}f_1g_1^*$	$2+\frac{2}{3}R$	$-10\frac{2}{3}R$	$10\frac{2}{3}R^2$	$\cdots$
$\mathrm{Re}f_2g_2^*$	$-1\frac{1}{3}R^2-3\epsilon R$	$2\frac{2}{3}R^2-2\frac{2}{3}R^3-4\epsilon R$	$2\frac{2}{3}R^2$	$-5\frac{1}{3}R^3$
$\mathrm{Re}f_1g_2^*$	$-1\frac{1}{3}R-\frac{2}{3}R^2+\epsilon$	$1\frac{1}{3}R+5\frac{1}{3}R^2$	$-2\frac{2}{3}R^2$	$\cdots$
$\mathrm{Re}g_1f_2^*$	$2\frac{2}{3}R-\frac{2}{3}R^2+5\epsilon$	$-6\frac{2}{3}R+2\frac{2}{3}R^2$	$8R^2$	$\cdots$
$\mathrm{Re}f_1g_3^*$	$-\epsilon+R\epsilon$	$2\epsilon R$	$\cdots$	$\cdots$
$\mathrm{Re}g_1f_3^*$	$-\epsilon+R\epsilon$	$2\epsilon R$	$\cdots$	$\cdots$
$\mathrm{Re}f_2g_3^*$	$\frac{2}{3}\epsilon R+\frac{1}{2}\epsilon R^2$	$-\frac{2}{3}\epsilon R-2\frac{2}{3}\epsilon R^2$	$2\frac{2}{3}\epsilon R^2$	$\cdots$
$\mathrm{Re}g_2f_3^*$	$2\epsilon-\frac{1}{3}\epsilon R$	$-4\frac{2}{3}\epsilon R$	$\cdots$	$\cdots$
$\mathrm{Re}f_3g_3^*$	$\frac{1}{3}\epsilon R+\epsilon^2$	$-1\frac{1}{3}\epsilon R+\frac{2}{3}\epsilon R^2$	$1\frac{1}{3}R^2$	$\cdots$

[9] J. W. Cronin *et al.*, Bull. Am. Phys. Soc. **5**, 11 (1960); R. L. Cool *et al.*, Phys. Rev. **114**, 912 (1959).

where P is the polarization of B, θ_l is the angle between $\mathbf{P}$ and $\mathbf{p}_l$, and where β, ϵ, R, and x are dimensionless quantities defined by

$$\beta=|\mathbf{p}_l|/E_l,\quad \epsilon=(m_l/m_B)^2,$$
$$R=\frac{E_l^{\max}}{m_B}=\frac{m_B^2+m_l^2-m_b^2}{2m_B^2},\quad x=\frac{E_l}{E_l^{\max}}. \quad (17)$$

E_l varies between m_l and $E_l^{\max}$ so that $\epsilon^{\frac{1}{2}}/R\leq x\leq 1$. The functions $a(x)$ and $b(x)$ are quadratic forms in the 6 form factors, with coefficients that depend upon x. They are given in Tables I and II. To simplify the expressions, terms which do not introduce corrections of more than a few percent have been dropped. Notice that no assumptions have been made concerning the relative magnitudes of the form factors, i.e., no combination of form factors has been dropped because it has a small coefficient.

Values for R and ϵ for several decays are given in Table III. The decay rate (16) also holds for the β decay of the neutron. In this case R is quite small, and charge symmetry requires that f_3 and g_2 vanish, so that the expressions for $a(x)$ and $b(x)$ can be simplified

TABLE III. Values of R and ϵ for specific decays.

Decay	R	ϵ
$n\to p+e^-+\bar{\nu}$	0.0014	$0.3+10^{-6}$
$\Sigma^-\to\Lambda+\left\{\begin{matrix}\mu^-\\ e^-\end{matrix}\right\}+\bar{\nu}$	0.067 0.063	0.79×10^{-2} 0.2×10^{-2}
$\Lambda\to p+\left\{\begin{matrix}\mu^-\\ e^-\end{matrix}\right\}+\bar{\nu}$	0.150 0.146	0.90×10^{-2} 0.2×10^{-6}
$\Sigma^-\to n+\left\{\begin{matrix}\mu^-\\ e^-\end{matrix}\right\}+\bar{\nu}$	0.203 0.199	0.79×10^{-2} 0.2×10^{-6}
$\Xi\to\Lambda+\left\{\begin{matrix}\mu^-\\ e^-\end{matrix}\right\}+\bar{\nu}$	0.146 0.143	0.64×10^{-2} 0.2×10^{-6}
$\Xi\to\Sigma^0+\left\{\begin{matrix}\mu^-\\ e^-\end{matrix}\right\}+\bar{\nu}$	0.097 0.094	0.64×10^{-2} 0.2×10^{-6}

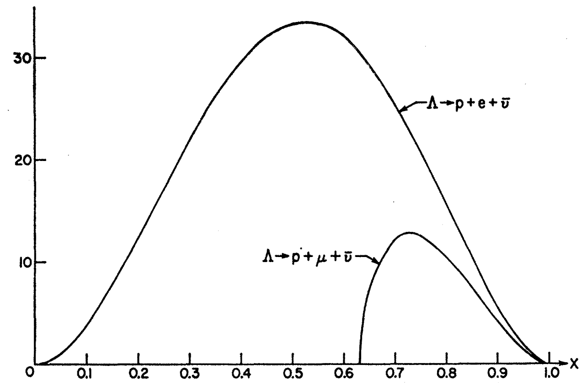

FIG. 1. Lepton energy spectra for the pure f_1 case, for the decay $\Lambda \rightarrow p+l+\bar{\nu}$. Ordinate scale is arbitrary.

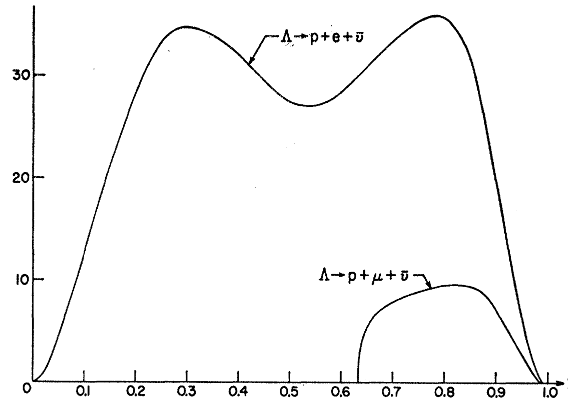

FIG. 2. Lepton energy spectra for the pure f_2 case, for the decay $\Lambda \rightarrow p+l+\bar{\nu}$. Ordinate scale is arbitrary.

considerably. This case has been treated by Bilenky *et al.*[10]

IV. DISCUSSION

This section contains some observations about what one might hope to learn by comparing experimental results with the above expressions. There is, of course, the possibility that no choice of form factors will fit the experiments. This would indicate that one of our assumptions is not valid; that either the interaction does not have the form $GJ^\mu(x)L_\mu(x)$ or that the form factors have a strong k^2 dependence. Assuming that this will not be the case, one would like to be able to use the experimental data to determine the form factors or at least to check conjectures as to which form factors are dominant.

Each combination of form factors predicts a definite value for the ratio of total decay rates,[11]

$$\frac{\omega_{B\rightarrow b+e+\bar{\nu}}}{\omega_{B\rightarrow b+\mu+\bar{\nu}}}.$$

For example, in the decays

$$\Lambda \rightarrow p+\left\{\begin{matrix} e \\ \mu \end{matrix}\right\}+\bar{\nu}$$

this ratio is about 6.5 for pure f_1 and about 10 for pure f_2. All terms containing as a factor either f_3 or g_3 are proportional to ϵ and are thus about 10^5 times as large for muon modes as for electron modes. Since $\Lambda \rightarrow p+e+\bar{\nu}$ has been seen, but not $\Lambda \rightarrow p+\mu+\bar{\nu}$, these terms should be completely negligible for the electron modes in Λ decay.

Each combination of form factors also corresponds to a definite shape for the lepton energy spectrum. Unfortunately, since $a(x)$ is always a fairly smooth function, the shape of the spectrum is dominated by the phase-space factor $\beta x^2(1-x)^2$ so that fairly accurate experiments will be needed to distinguish between different sets of form factors. Figures 1 and 2 show the spectra corresponding to pure f_1 and pure f_2. Finally we have the asymmetry, b/a, to compare with experimental results. Here the difference between different combinations of form factors can be more striking. For $\Lambda \rightarrow p+e+\bar{\nu}$ at $x=0.6$ (near the peak of the spectrum), b/a is about -0.06 for pure f_1, -0.76 for pure g_1 and -0.86 for pure g_2. If enough events near the ends of the spectrum can be observed, the energy dependence of b/a can also provide a means of distinguishing between combinations of form factors.

In conclusion, it appears unlikely that an unambiguous experimental determination of the 6 form factors will be made in the near future. Experimental studies of hyperon lepton decay should prove extremely valuable, however, as a check on theoretical predictions: those following from the general assumptions about the form of the interaction made above as well as the more specific predictions given by more detailed treatments of the baryon vertex.

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